

Hamiltonian Circuits, Complete Graphs

Read the book chapters first, then make sure you can answer the questions in the notes. Following that, work on some skills-check problems and exercises. Then take the online quizzes.

Reading 2.1

Skills Check 1, 2, 5, 6, 7, 9, 13

Exercises 1, 2, 3, 4, 7, 8, 14, 21, 26, 27, 28

Quiz Take the quiz¹

Notes

2.1

- What circuit do we call a *Hamiltonian Circuit*?
- How do Hamiltonian Circuits differ from Euler Circuits?
- Do all graphs have Hamiltonian Circuits?
- Draw examples of graphs that have a HC but not a EC, as well as graphs that have a EC but not a HC.
- How many edges can we have in a Hamiltonian Circuit, in relation to the number of vertices in the graph?
- Examine the examples in Figure 2.2. How do we know for sure that these examples cannot have a Hamiltonian Circuit?
- The graph on Figure 2.2b shows an example of a graph with a “valid-2-coloring”: Each vertex is colored by one of two colors, in such a way that for every edge the two vertices corresponding to it have different colors.
 - Show how the graph on figure 2.1a can also be valid-2-colored.
 - When we can valid-2-color a graph, what condition would preclude such a graph from having a Hamiltonian Circuit?
 - Can you provide examples of graphs that cannot possibly be valid-2-colored?
 - Can you provide examples of graphs that cannot possibly be valid-3-colored?
- What graphs do we call *complete graphs*?
- Do complete graphs always have Hamiltonian Circuits?
- Do complete graphs always have Euler Circuits?
- If we add weights to each edge, what would be a *minimum-cost Hamiltonian Circuit*, and what would be a straightforward algorithm for finding such a circuit?
- What does the *fundamental principle of counting* say? What are some examples where we can apply it?
- In a complete graph on 5 vertices, how many different paths would the “method of trees” end up with?

¹<https://moodle.hanover.edu/mod/quiz/view.php?id=4666>

- When applying the method of trees to find the minimum-cost Hamiltonian Circuit, we get multiple paths in the tree with the same cost. Why is that?