Graphs, Euler Circuits

Read the book chapters first, then make sure you can answer the questions in the notes. Following that, work on some skills-check problems and exercises. Then take the online quizzes.

Reading 1.1-1.3
Skills Check 1-4, 7-10, 12-14, 18, 19, 21, 23, 25, 27
Exercises 2, 7, 11, 13, 14, 18, 19, 23, 30, 37, 39, 40, 43, 46, 59

Notes

1.1

- What is a *graph*? What are the vertices and edges in a graph?
- Give examples of graphs with 4 vertices and 4 edges.
- Provide at least 3 different examples of real-world scenarios from your experience that you might model using a graph.
- What is a *path* in a graph? What is a *circuit*?
- Could we have two distinct edges that both connect the same two vertices?
- Could we have a vertex that is not connected to any other vertex?
- Could we have an edge that does not connect two vertices?
- What circuits do we call *Euler circuits*?
- Why are we interested in Euler circuits?
- What did you find most interesting about Leonhard Euler?

1.2

- What are the two key questions we would want to ask about Euler circuits? How do they differ?
- What is the *valence* of a vertex?
- Can two vertices have the same valence?
- Can two vertices have different valences?
- Can a vertex have a valence of 0? What would that mean for it?
- Observe that every edge in a graph contributes 1 point each to two vertices' valences. With that in mind, what can we say about the sum of all the valences?
- Why can't we have a graph that has exactly one odd valence?
- When do we say that a graph is *connected*?
- If all the valences in a graph are positive, why does that not mean that the graph is connected? Provide an example.
- What does the *Euler Circuit Theorem* say?
- Demonstrate the ECT in some examples.

- Intuitively, why does it make sense that a graph that is not connected cannot have an EC?
- Intuitively, why does it make sense that a graph that has an odd valence cannot have an EC?
- What is the key guiding principle to keep in mind when trying to find an Euler Circuit in a systematic way?

1.3

- Describe the *Chinese Postman Problem*. What are some real-world examples where it might come up?
- What does *eulerizing a graph* mean?
- Can we add any kinds of edges when we are trying to "eulerize" a graph? Explain.
- In *rectangular networks* what systematic method can we follow to eulerize them?