

Consequences of the Fundamental Theorem of Arithmetic

Reading

- Section 6.2

Practice Problems

6.2 1, 2, 4, 6, 11

Challenge (Optional) 8, 16, 22, 32-35

Notes

The fundamental theorem of arithmetic has two important consequences. One is regarding the divisors of a number, the other is regarding gcd and lcm.

If $n = p_1^{a_1} \cdots p_k^{a_k}$ is the prime factorization of the number n , then any positive divisor d of n has the form:

$$d = p_1^{b_1} \cdots p_k^{b_k}$$

where each $b_i \leq a_i$.

In particular, there are exactly $(a_1 + 1)(a_2 + 1) \cdots (a_k + 1)$ different positive divisors of n .

Suppose $n = p_1^{a_1} \cdots p_k^{a_k}$ and $m = p_1^{b_1} \cdots p_k^{b_k}$ are prime number factorizations of n and m , where we have allowed some of the exponents to equal 0 to ensure we have the same list of prime numbers. Then we have:

$$\gcd(n, m) = p_1^{\min(a_1, b_1)} p_2^{\min(a_2, b_2)} \cdots p_k^{\min(a_k, b_k)}$$

$$\text{lcm}(n, m) = p_1^{\max(a_1, b_1)} p_2^{\max(a_2, b_2)} \cdots p_k^{\max(a_k, b_k)}$$

A direct consequence of this is the formula $nm = \gcd(n, m)\text{lcm}(n, m)$ that we saw earlier.