

## Assignment 8

True/False questions, not to turn in:

- $\phi(n)$  is always strictly less than  $n$ , when  $n \geq 2$ .
- If  $a, b$  are reduced residues modulo  $n$ , then so is  $a + b$ .
- If  $a, b$  are reduced residues modulo  $n$ , then so is  $ab$ .
- If  $a$  is a reduced residue modulo  $n$ , and  $m|n$ , then  $a$  is also a reduced residue modulo  $m$ .
- The multiplicative inverse of an element  $a$  modulo a prime  $p$  is a power of the element.
- $\phi(n)$  is always an even number.
- $\phi(n)$  is always divisible by 4.
- There are  $n > 2$  and  $a$  such that the multiplicative inverse of  $a$  modulo  $n$  is equal to  $-a (= n - a)$ .

Questions to turn in:

1. Using the formulas we have learned for  $\phi$ , compute  $\phi(215)$ . Also determine at least 5 elements that are reduced residues modulo 215.
2. Using Euler's theorem followed by fast exponentiation, compute  $11^{214563} \bmod 215$ . Only use your calculator for computing the product of two numbers and for reducing numbers modulo another (i.e. don't just give something like  $11^{313}$  to the calculator — exponent made up for illustrative purposes).
3. Use some of the work from problem 2 to find the multiplicative inverse of 11 modulo 215.
4. Find all reduced residues in  $\mathbb{Z}_{16}$ , and for each residue find the first power that equals 1. Determine if there is a reduced residue, such that looking at its powers produces all the other reduced residues.
5. Do the same for  $\mathbb{Z}_{18}$ .