Consequences of the Fundamental Theorem of Arithmetic

Reading

- Section 6.2

Practice Problems

6.2 1, 2, 4, 6, 11
Challenge (Optional) 8, 16, 22, 32-35

Notes

The fundamental theorem of arithmetic has two important consequences. One is regarding the divisors of a number, the other is regarding gcd and lcm.

> If $n = p_1^{a_1} \cdots p_k^{a_k}$ is the prime factorization of the number n, then any positive divisor d of n has the form:
>
> $$d = p_1^{b_1} \cdots p_k^{b_k}$$
>
> where each $b_i \leq a_i$.
>
> In particular, there are exactly $(a_1 + 1)(a_2 + 1) \cdots (a_k + 1)$ different positive divisors of n.

> Suppose $n = p_1^{a_1} \cdots p_k^{a_k}$ and $m = p_1^{b_1} \cdots p_k^{b_k}$ are prime number factorizations of n and m, where we have allowed some of the exponents to equal 0 to ensure we have the same list of prime numbers. Then we have:
>
> $$\gcd(n, m) = p_1^{\min(a_1,b_1)} p_2^{\min(a_2,b_2)} \cdots p_k^{\min(a_k,b_k)}$$
>
> $$lcm(n, m) = p_1^{\max(a_1,b_1)} p_2^{\max(a_2,b_2)} \cdots p_k^{\max(a_k,b_k)}$$

A direct consequence of this is the formula $nm = \gcd(n, m) lcm(n, m)$ that we saw earlier.