Introduction to Number Systems

Reading

Sections 1.1, 1.2

Practice Problems

1.1 5, 8, 9, 10, 11
1.2 1, 2, 3, 9

Notes

First a short survey of number systems.

- We all know the *natural numbers*: 1, 2, 3, 4, ... Start from 1, and add it into itself over and over. These will be the main focus of this course.
- By including a zero and additive inverses, we get *negative integers*: -1, -2, 0
- By taking quotients of those we form *rational numbers*: $2/3$, $-42/11$
- In Calculus we learn about the *real numbers*: e, π, $\sqrt{2}$
- One of the fundamental results is that there are many real numbers that are not rational (*irrational numbers*). In fact almost every real number is not rational.
- *Complex numbers* have the form: $a + bi$ where $i = \sqrt{-1}$ represents a "square root of -1". They are an extension of the reals.
  - We can define addition and multiplication of complex numbers by extending the properties for reals.
  - There is a zero, $0 + 0i$.
  - Cool fact: Every polynomial equation has solutions in the complex numbers. For example $x^2 + 1 = 0$ has solutions i and $-i$.
- There is a special subset of the complex numbers, called *Gaussian integers*. These consist of all complex numbers $a + bi$ where a and b are both integers.
- The standard number systems:

  $\mathbb{N}$ The natural numbers: $\mathbb{N} = \{1, 2, 3, \ldots\}$
  $\mathbb{Z}$ The integers: $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, 3, \ldots\}$
  $\mathbb{Q}$ The rational numbers: $\mathbb{Q} = \left\{\frac{a}{b} \mid a, b \in \mathbb{Z}, b \neq 0\right\}$
  $\mathbb{R}$ The real numbers. Precisely defining them is more difficult.
  $\mathbb{C}$ The complex numbers: $\mathbb{C} = \{a + bi \mid a, b \in \mathbb{R}\}$