

## Assignment 4

Make sure to write complete proofs. Try to avoid skipping steps. Write clear sentences.

1. True or False: If  $p, q$  are prime, then  $\gcd(p, q)$  is equal to either  $p$  or  $q$ .
2. Show that if  $p > 3$  is a prime number then the remainder of dividing  $p$  by 6 is either 1 or 5. Provide examples to show that both remainders are indeed possible.
3. Suppose that  $M$  is a number that is relatively prime to 6 and is also a perfect square (i.e. it is  $M = n^2$  of some integer  $n$ ). Show that  $M$  divided by 6 leaves remainder 1.
4. Suppose that  $M$  is a perfect square. Show that its remainder when divided by 4 must be 0 or 1.
5. Use the previous problem to show that the number 1403 cannot possibly be the sum of two perfect squares.