Assignment 5

Make sure to write complete proofs. Try to avoid skipping steps. Write clear sentences.

1. True or False: If the remainder of doing Euclidean Division of a by c is r, i.e. if $a = qc + r$ with certain conditions on r, and the remainder of doing Euclidean Division of b by c is s, then the remainder of doing Euclidean Division of $a + b$ by c is $r + s$. Prove or provide counter-example.
2. True or False: If $\gcd(a, b) = 1$ and $\gcd(a, c) = 1$, then $\gcd(a, bc) = 1$. Prove or provide counter-example.
3. Using the Euclidean algorithm, find concrete numbers x and y such that $245x + 356y = 1$.
4. True or False: For given a, b, if we can solve $ax + by = c$, then we can also solve $ax + by = d$ for any $d \geq c$. Prove or provide counter-example.
5. True or False: The equation $2x + by = c$ has a solution for all c if and only if b is an odd number. Prove or provide counter-example.