

# Assignment 1

Make sure to write complete proofs. Try to avoid skipping steps. Write clear sentences.

1. A “multiplicative inverse” for a number  $x$  is a number  $y$  such that  $xy = 1$ .
  - i. Show that every non-zero rational number has a multiplicative inverse (that is also a rational number).
  - ii. If  $z$  is a complex number  $a + bi$ , then we define the *conjugate*  $\bar{z}$  as  $\bar{z} = a - bi$ . Show that the product of a complex number with its conjugate is a real number.
  - iii. Show that every non-zero complex number has a multiplicative inverse.

2. For this question assume the following for integeres, which is the analog of “every integer is odd or even” but using 3 instead of 2 as a factor. We have not proven this assumption, but you may, and will need to, use it. The assumption is: Every integer  $n$  can be written in exactly one of the following 3 ways:

**Type A** In the form  $3k$  where  $k$  is some integer.

**Type B** In the form  $3k + 1$  where  $k$  is some integer.

**Type C** In the form  $3k + 2$  where  $k$  is some integer.

Answer the following questions:

- i. Show that if  $n$  is of type C, then  $2n$  is of type B.

ii. Show that if  $m$  is of type A and  $n$  is any integer, then  $mn$  is also of type A.

iii. Show that if  $n$  is an integer, then  $n^2$  cannot be of type C.

iv. Show that for any integer  $n$ , the product  $(2n + 1)(n + 1)n$  is of type A.