

Midterm 2 Study Guide

Material covered

Chapters 7 through 9, and 10.1.

- Definitions to know:
 - What it means for two numbers to be congruent modulo n . This has 3-4 variations.
 - Congruence classes.
 - How we define addition and multiplication for congruence classes.
 - The notions of ring, integral domain, field.
 - “reduced residues”.
 - Euler’s ϕ function.
 - Order of an element in \mathbb{Z}_n
- You should know all theorem and lemma statements. Especially:
 - Two numbers are congruent modulo n if and only if they have the same remainder when divided by n .
 - Addition and multiplication are well defined for congruence classes.
 - Chinese Remainder Theorem (versions 7.4.1 and 7.4.2).
 - \mathbb{Z}_n forms a ring.
 - Equivalent conditions for congruence (8.2.4).
 - Which elements mod n have multiplicative inverses.
 - The only solutions to $x^2 = 1 \pmod{n}$ are ± 1 .
 - Wilson’s theorem.
 - Formulas for computing $\phi(n)$.
 - Fermat’s Little Theorem.
 - Euler’s Theorem.
 - Encryption/Decryption via exponentiation (9.4.1).
 - Public Key Cryptography and RSA.
 - Order of elements modulo p divides $p - 1$.
 - Order of a power of an element divides order of the element.
- Theorems you should know how to prove:
 - Why addition and multiplication are well-defined operations for congruence classes.
 - Why congruence is an equivalence relation.
 - A finite integral domain is necessarily a field.
 - \mathbb{Z}_n has zero-divisors if and only if n is not prime if and only if \mathbb{Z}_n is not a field.
 - Every invertible element is cancellable. (Also show by example the converse is not in general true)
 - Fermat’s Little Theorem.
 - Order of a power of an element divides order of the element.

Practice Problems

- Know very well all the turned-in assignments (6-8)
- Know how to do the non-optional practice problems
- Be ready for true/false questions
- Know how to compute multiplicative inverses using the Euclidean algorithm for the gcd.
- Know how to compute the common solution to a Chinese Remainder Theorem situation.
- Know how to encrypt/decrypt via addition/multiplication modulo 26.
- Know how to perform fast exponentiation.