

The Principle of Mathematical Induction

We discuss here the Principle of Mathematical Induction, usually simply called “Induction”, which is a fundamental technique for proving statements about all natural numbers.

Reading

Section 2.1

Practice Problems

2.1 1, 2, 3, 4, 6, 7, 11, 21, 23

Challenge 2.1 (optional) 12, 13, 14, 22

Notes

- The principle of induction says the following: In order to prove a statement $P(n)$ for all natural numbers n , it is enough to prove the following two statements:

base case $P(1)$ is true

inductive step If $P(k)$ was true for some particular k , then $P(k + 1)$ would also have to be true.

- This is helpful because we no longer have to prove things for all natural numbers directly. We just need one special case, 1, and then under the assumption that $P(k)$ is true we have to be able to “move to the next step”. Being able to assume $P(k)$ to be true while trying to prove $P(k + 1)$ is very powerful.
- The motivation behind this principle, is that since we can reach every natural number by starting from 1 and just adding 1 over and over again, the two statements above would allow us to reach any $P(n)$ we like (though it may take a while to write it out if we have to say make it to $P(100000)$).
- Induction effectively allows us to say: “ $P(1)$ is true, so $P(2)$ is true, so $P(3)$ is true, so ... so $P(n)$ is true”
- The book has a number of nice examples, you should read them. We will do a slightly different example here. We will use induction to prove the statement $P(n)$ for all natural numbers, where $P(n)$ = “ n is either even or odd”. So we will effectively be proving that every natural number is either even or odd.
 - Base case: $P(1)$ is true, since 1 is in fact an odd number, as it is $2 \times 0 + 1$.
 - Inductive case: Suppose $P(k)$ is true, so k is either even or odd.
 - * We need to show that $P(k + 1)$ is also true.

- * In other words we need to show that $k + 1$ is either even or odd.
 - * We take cases on k . If k is even, then $k = 2i$, and $k + 1 = 2i + 1$ is odd.
 - * If k is odd, then $k = 2i + 1$ and $k + 1 = 2i + 1 + 1 = 2i + 2 = 2(i + 1)$ is even.
 - * So in either case, $k + 1$ is either odd or even. This proves the inductive step.
- The principle of induction now tells us that the statement must be true for all natural numbers.
- As practice: Prove that every natural number n is greater than or equal to 1.
 - Food for thought: Why can't we use induction to prove things for all real numbers? Why isn't the same principle true for all real numbers?