Assignment 1

Make sure to write complete proofs. Try to avoid skipping steps. Write clear sentences.

- 1. A "multiplicative inverse" for a number x is a number y such that xy = 1.
- i. Show that every non-zero rational number has a multiplicative inverse (that is also a rational number).

ii. If z is a complex number a+bi, then we define the *conjugate* \bar{z} as $\bar{z}=a-bi$. Show that the product of a complex number with its conjugate is a real number.

iii. Show that every non-zero complex number has a multiplicative inverse.

2. For this question assume the following for integers, which is the analog of "every integer is odd or even" but using 3 instead of 2 as a factor. We have not proven this assumption, but you may, and will need to, use it. The assumption is: Every integer n can be written in exactly one of the following 3 ways:

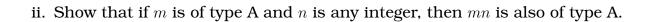
Type A In the form 3k where k is some integer.

Type B In the form 3k + 1 where k is some integer.

Type C In the form 3k + 2 where k is some integer.

Answer the following questions:

i. Show that if n is of type C, then 2n is of type B.



iii. Show that if n is an integer, then n^2 cannot be of type C.

iv. Show that for any integer n, the product (2n+1)(n+1)n is of type A.