

## Assignment 10

In the following questions answering a “True or False” always requires providing a proof or a counter-example, as appropriate.

1. True or False: If  $A \leq_m B$  and  $B$  is co-Turing-recognizable, then  $A$  is also co-Turing-recognizable.
2. True or False: If  $A \leq_m B$  and  $A$  is decidable, then  $B$  is also decidable.
3. True or False: If  $A \leq_m B$  and  $A$  is Turing-recognizable, then  $B$  is also Turing-recognizable.
4. True or False: If  $A \leq_m B$ , for some languages  $A, B$ , then we must also have  $B \leq_m A$ .
5. Consider the language  $J$  consisting of all strings  $0w$  where  $w \in A_{\text{TM}}$  and  $1w$  where  $w \in \overline{A_{\text{TM}}}$ .
  - a. Show that  $A_{\text{TM}} \leq_m J$  and  $\overline{A_{\text{TM}}} \leq_m J$ .
  - b. Use this information to show that  $J$  is neither Turing-recognizable nor co-Turing-recognizable.
6. In this problem we address a number of questions related to having a language be mapping-reducible to its complement.
  - a. Show that  $A_{\text{DFA}} \leq_m \overline{A_{\text{DFA}}}$ .
  - b. Show that if  $B \leq_m \bar{B}$ , then  $B$  is Turing-recognizable if and only if it is co-Turing-recognizable.
  - c. Show that if  $B \leq_m \bar{B}$  and  $B$  is undecidable, then  $B$  is neither Turing-recognizable nor co-Turing-recognizable.
  - d. Give an example of an undecidable language  $B$  such that  $B \leq_m \bar{B}$ .