Assignment 9

1. Consider the language ALL_{DFA} consisting of the string representations of all DFAs D such that $L(D) = \Sigma^*$ is the language of all strings in the alphabet (the alphabet defined by that D). Show that this is a decidable language.

2. Consider the language $L = \{\langle M, N\rangle \mid M, N \text{ are TMs and } L(M) \cup L(N) = \emptyset\}$.

   a. Show that L is not decidable.

   b. Show that L is co-Turing-recognizable.

   c. Show that L is not Turing-recognizable.

3. Consider the language $L = \{\langle M, w\rangle \mid M \text{ is TM and does not halt on } w\}$. Determine which of the following classifications is the correct one for L:

   - decidable
   - Turing-recognizable but not decidable
   - co-Turing-recognizable but not decidable
   - neither Turing-recognizable nor co-Turing-recognizable.

4. True or False (provide proof or counter-example as appropriate):

   a. If $A \subset B$ are two languages, then if A is decidable then B is also decidable.

   b. If $A \subset B$ are two languages, then if B is decidable then A is also decidable.

5. Suppose A is a decidable language, and consider the language $L_A = \{\langle M, w\rangle \mid M \text{ is a TM and it acc}$ For each of the following determine if it is true or false. Provide proof or counter-example as appropriate.

   a. For every decidable language A, we have L_A is Turing-recognizable.

   b. For every decidable language A, we have L_A is decidable.

   c. There is a decidable language A for which L_A is decidable.