# Assignment 9

1. Consider the language $\text{ALL}_{\text{DFA}}$ consisting of the string representations of all DFAs $D$ such that $L(D) = \Sigma^*$ is the language of all strings in the alphabet (the alphabet defined by that $D$). Show that this is a decidable language.

2. Consider the language $L = \{\langle M, N\rangle \mid M, N \text{ are TMs and } L(M) \cup L(N) = \emptyset\}$.

   a. Show that $L$ is not decidable.

   b. Show that $L$ is co-Turing-recognizable.

   c. Show that $L$ is not Turing-recognizable.

3. Consider the language $L = \{\langle M, w\rangle \mid M \text{ is TM and does not halt on } w\}$. Determine which of the following classifications is the correct one for $L$:

   - decidable
   - Turing-recognizable but not decidable
   - co-Turing-recognizable but not decidable
   - neither Turing-recognizable nor co-Turing-recognizable.

4. True or False (provide proof or counter-example as appropriate):

   a. If $A \subset B$ are two languages, then if $A$ is decidable then $B$ is also decidable.

   b. If $A \subset B$ are two languages, then if $B$ is decidable then $A$ is also decidable.

5. Suppose $A$ is a decidable language, and consider the language $L_A = \{\langle M, w\rangle \mid M \text{ is a TM and it accepts } w \text{ and what is left on the tape at that point is an elemen}$ For each of the following determine if it is true or false. Provide proof or counter-example as appropriate.

   a. For every decidable language $A$, we have $L_A$ is Turing-recognizable.

   b. For every decidable language $A$, we have $L_A$ is decidable.

   c. There is a decidable language $A$ for which $L_A$ is decidable.