Assignment 10

In the following questions answering a "True or False" always requires providing a proof or a counter-example, as appropriate.

1. True or False: If $A \subset_m B$ and B is co-Turing-recognizable, then A is also co-Turing-recognizable.
2. True or False: If $A \subset_m B$ and A is decidable, then B is also decidable.
3. True or False: If $A \subset_m B$ and A is Turing-recognizable, then B is also Turing-recognizable.
4. True or False: If $A \subset_m B$, for some languages A, B, then we must also have $B \subset_m A$.
5. Consider the language J consisting of all strings $0w$ where $w \in A_{\mathrm{TM}}$ and $1w$ where $w \in \overline{A_{\mathrm{TM}}}$.

   a. Show that $A_{\mathrm{TM}} \leq_m J$ and $\overline{A_{\mathrm{TM}}} \leq_m J$.
   b. Use this information to show that J is neither Turing-recognizable nor co-Turing-recognizable.

6. In this problem we address a number of questions related to having a language be mapping-reducible to its complement.

   a. Show that $A_{\mathrm{DFA}} \leq_m \overline{A_{\mathrm{DFA}}}$.
   b. Show that if $B \leq_m \bar{B}$, then B is Turing-recognizable if and only if it is co-Turing-recognizable.
   c. Show that if $B \leq_m \bar{B}$ and B is undecidable, then B is neither Turing-recognizable nor co-Turing-recognizable.
   d. Give an example of an undecidable language B such that $B \leq_m \bar{B}$.