22/5/25

- Conditional Probability of A

(probability of A as B already occured)

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

already occured

Examples

1.6
1.7
1.8
1.9

$$P(B|A) = \frac{P(A\cap B)}{P(A)}$$

$$\boxed{P(A\cap B) = \overbrace{P(A|B)\cdot P(B)}^{\text{multiplication block}} = P(B|A)\cdot P(A)} \Rightarrow \text{Simple multiplication law}$$

$P(A|B)$ → Conditional Probability

$P(B)$ → Unconditional Probability

Multiplication Law:

Assuming that all of the conditioning events have positive probability, we have

$$P\left(\bigcap_{i=1}^{n} A_i\right) = P(A_1\cap A_2\cap A_3\cap \dots \cap A_n)$$

$$= P(A_1)\,P(A_2|A_1)\,P(A_3|A_1\cap A_2)\,P(A_4|A_1\cap A_2\cap A_3)\dots P\left(A_n\middle|\bigcap_{i=1}^{n-1} A_i\right)$$

Problem #2: A family has 2 children. What is the conditional probability that both of them are boys, given that at least one of them is a boy.

Uncondition

condition

Solution: $S = \{(b,b), (b,g), (g,b), (g,g)\}$

$A = \{(b,b)\}$

$B = \{(b,b), (b,g), (g,b)\} \Rightarrow$ condition

$A \cap B = \{(b,b)\}$

$n(S) = 4$

$n(A) = 1$

$n(B) = 3$

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{1}{3} \left[\frac{\frac{n(A\cap B)}{n(S)}}{\frac{n(B)}{n(S)}} \right]$$

Problem #3: The probability that a married man watches a certain TV show is 0·4 and that of his wife is 0·5. The probability that a man watches the show given that ~~*~~ his wife does is 0·7. Now, find the following probabilities:

(a) The probability that the married couple watches the TV show.

(b) The probability that his wife watches the show given that her husband does.

(c) the probability that at least one of them watches the show.

$\underbrace{\text{at least}}_{\text{union}}$

soln: H: | $P(H) = 0.4$ | H = event husband watches the show
W: | $P(W) = 0.5$ | W = event wife watches the show.
$P(H|W) = 0.7$

(a) $P(W \cap H) = P(H \cap W) = P(W) \cdot P(H|W) = 0.5 \times 0.7$
$= 0.35$ (Ans)

(b) $P(W|H) = \dfrac{P(W \cap H)}{P(H)} = \dfrac{0.35}{0.4} = 0.875$

(c) $P(H \cup W) = P(H) + P(W) - P(H \cap W)$
$= 0.4 + 0.5 - 0.35$
$= 0.55$ (Ans)

Problem #4: A box contains 7 red balls and 3 black balls. The balls are drawn from the box one after another. Find the probability that the first two balls are red and the third one is black if

(a) The balls are replaced before the next draw. [Place the picked ball into the box again] [Balls constant]

(b) The balls are not replaced. [Balls not constant]

$R_1 R_2 B_3$ } sequence

Solution: (a) $P(R_1 \cap R_2 \cap B_3)$

$n(R) = 7$

$n(B) = 3$

10

$= P(R_1) \cdot P(R_2) \cdot P(B_3)$

$= \left(\frac{7}{10}\right)\left(\frac{7}{10}\right)\left(\frac{3}{10}\right) = \frac{147}{1000}$

(b) $P(R_1 \cap R_2 \cap B_3) = P(R_1) \cdot P(R_2 \mid R_1) \cdot P(\cancel{R}\, B_3 \mid R_1 \cap R_2)$

$= \left(\frac{7}{10}\right)\left(\frac{\cancel{6}^{3}}{\cancel{9}_{3}}\right)\left(\frac{3}{8}\right)$

$= \frac{126}{720}$

$= \frac{7}{40}$

* *

Problem#5: A coin is tossed until a head appeared or it has been ~~toos~~ tossed three times. Given that the head does not appear on the first toss, what is the probability that the coin is tossed 3 times?

~~H, HT,~~

$S = \{H, TH, TTH, TTT\}$

breaks homogenety here e.g. 1, 2, 3, 3 So we need to find individual events. Here, Total number of points does not remain same.

If unbaised or fair coin,

$P(H) = P(T) = 1/2$

$P(TH) = 1/4$

$P(TTH) = 1/8$

$P(TTT) = 1/8$

	HH	HT	~~H~~TA	TT
H	~~HHH~~	HHT	HTH	HTT
T	~~THH~~	THT	~~THT~~ TTH	TTT

A : The event that the coin is tossed three times

B : No head ~~appe~~ appeard on the first toss.

$A = \{TTH, TTT\}$

$B = \{TH, TTH, TTT\}$

$(A \cap B) = \{TTH, TTT\}$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$\neq \left[\frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} \right]$ → cannot use it as samples is not 8 all the time as "head appeared"

$= \frac{1/4}{1/2} = \frac{1}{4} \times 2 = 1/2$] Not correct

$= \frac{2/4}{3/4} = 2/3$

$P(B) = \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = 1/2$

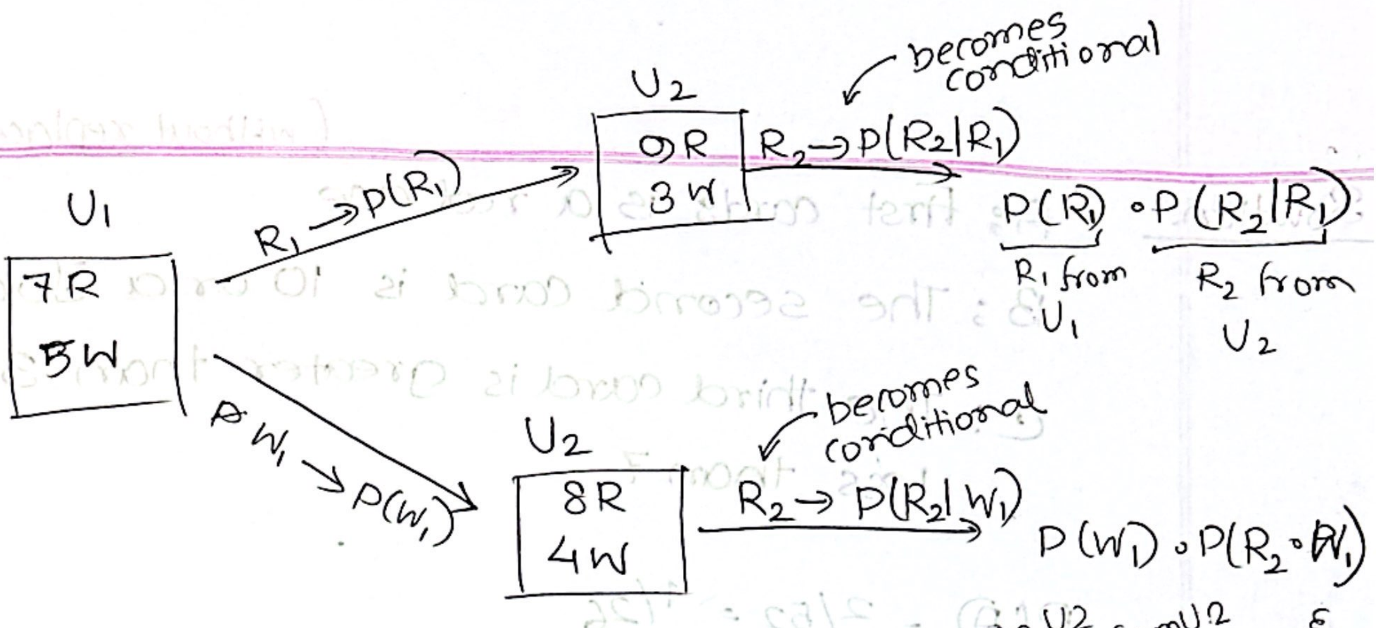
$P(A \cap B) = \frac{1}{8} + \frac{1}{8} = 1/4$

$P(A|B) = \frac{1/4}{1/2} = \frac{1}{4} \times 2 = 1/2$

Need to do individual events to calculate.

one's appearance won't hamper another's occurence

Problem #6: Suppose we have 2 Urns, U_1 and U_3. The first urn contains 5 white balls and 7 red balls, while the second urn contains 3 white balls and 8 red balls. One ball is transferred from the first urn to the second urn unseen and then a ball is drawn from the second urn. What is the probability that the picked ball is red?



placed red in U2
picked red from U2
or
placed white in U_2
picked white from U_2

$$P(\text{Red Ball from Urn 2}) = P(R_1 \cap R_2) + W(W_1 \cap R_2)$$

$$= P(R_1) \cdot P(R_2|R_1) + P(W_1) \cdot P(R_2|W_1)$$

$$= \left(\frac{7}{12}\right) \times \left(\frac{9}{12}\right) + \left(\frac{5}{12}\right) \times \left(\frac{8}{12}\right)$$

$$= \frac{103}{144}$$

Problem #7: Three cards are drawn in succession without replacement from an ordinary deck of playing cards. Find the probability that the first card is a red ace, the second card is a 10 or a Jack and the third card is greater than 3 but less than 7.

(without replacement)

Solution: A; First cards is a red ace

B; The second card is 10 or a Jack

C: The third card is greater than 3 but less than 7.

$$P(A) = 2/52 = 1/26$$

$$P(B) = 8/51$$

$$P(C) = 12/50 = 6/25$$

nested conditional loop

$$P(A \cap B \cap C) = P(A) \cdot P(B|A) \cdot P(C \,|\, \cancel{B \cap A}\, (A \cap B))$$

$$= \left(\frac{1}{26}\right) \cdot \left(\frac{8}{51}\right) \cdot \left(\frac{6}{25}\right)$$