John Scebold
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MiniMax vs Alpha Beta Pruning

Example 1. Both algorithms gave the same output position and estimate (-11) searching to a depth of 2, but AB pruning did it in just 90 evaluations, vs 137 by MiniMax.

MiniMaxGame.move(p3, 2)
Input position: WBxBWxWWWxWBxxxBxBBBx,
Output position: WBWBWxWxWxWBxxxBxBBxx,
Positions evaluated by static: 137, MINIMAX estimate: -11

ABGame.move(p3, 2)
Input position: WBxBWxWWWxWBxxxBxBBBx,
Output position: WBWBWxWxWxWBxxxBxBBxx,
Positions evaluated by static: 90, AlphaBeta estimate: -11

Example 2. Again, both algorithms gave the same output position and estimate (10000) searching to a depth of 3, but AB pruning did it in just 893 evaluations, vs 6649 by MiniMax.

MiniMaxGame.move(p2, 3)
Input position: WxxBWxWWWxxBxWxxxBxWx,
Output position: WxWxWxWxWxxBxWxxxBxWx,
Positions evaluated by static: 6649, MINIMAX estimate: 10000

ABGame.move(p2, 3)
Input position: WxxBWxWWWxxBxWxxxBxWx,
Output position: WxWxWxWxWxxBxWxxxBxWx,
Positions evaluated by static: 893, AlphaBeta estimate: 10000

Improved vs Standard Evaluation

Example 1. The standard evaluation chose to place W to close a mill and removed a B piece. The improved evaluation chose to place W to block Black from closing a mill in the next move. Both searched to a depth of 3.

MiniMaxOpening.move(p3, 3)
Input position: WBxBWxWWWxWBxxxBxBBBx,
Output position: WxWBWxWWWxWBxxxBxBBBx,
Positions evaluated by static: 2503, MINIMAX estimate: 1

MiniMaxOpeningImproved.move(p3, 3)
Input position: WBxBWxWWWxWBxxxBxBBBx,
Output position: WBxBWxWWWxWBxxxBxBBBW,
Positions evaluated by static: 2503, MINIMAX estimate: 2

Example 2.

MiniMaxGame.move(p6, 3)
Input position: xxxBWxWWWxBBxxxxWBxWx,
Output position: xxWBxxWWWxBBxxxxWBxWx,
Positions evaluated by static: 535, MINIMAX estimate: 2964

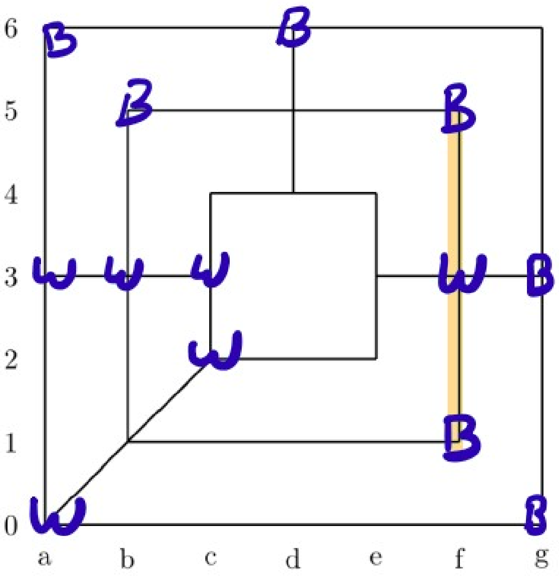
MiniMaxGameImproved.move(p6, 3)
Input position: xxxBWxWWWxBBxxxxWBxWx,
Output position: xxxBWxWWxxBBWxxxWBxWx,
Positions evaluated by static: 535, MINIMAX estimate: 3262.0

Discussion of Improved Evaluation Function

My opening and mid/end game functions take into account the advantage of certain board configurations. They quantify these and add points to what the standard evaluation functions calculate. So my total improved functions are sums of the original plus my own.

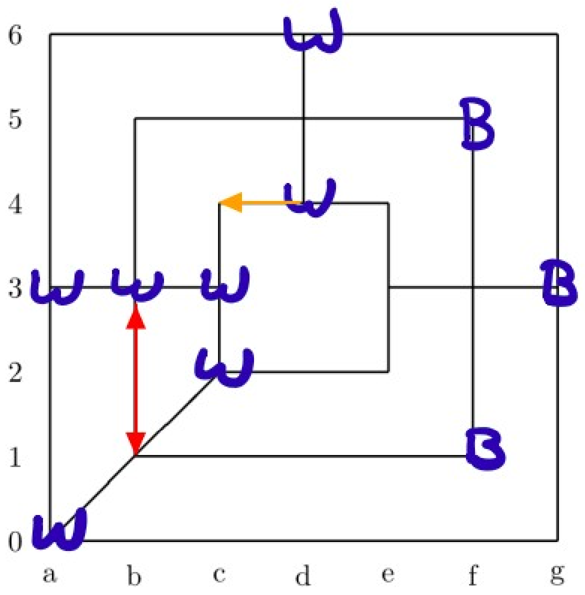
For the game opening phase if there is a mill that Black has two pieces on, but White blocking the third location, White gets 100 points. So even if Black has more pieces on the board at that time, White will see it as an advantageous position. White gets 100 points per mill it blocks Black from closing. See Fig 1. Black has 7 pieces while White has 6. This would give a score of -1 for White, but now it gives 99 telling White this is advantageous.

Fig 1



For the mid end game I also take into account if White blocks a Black mill from closing. In addition I take into account if White can close a mill in a single move. If so I give 200 points per mill that could closed in the next move. See the orange arrow in fig2. If White has to break a mill to close another mill in the next turn, it means he can move the piece right back later to close the old mill again, and thus close a mill every turn; provided Black doesn't pick off a piece. For that kind of setup I give 300 points. See the red arrow in fig2. So the configuration in fig2 would give 500 points on top of whatever the standard evaluation scores.

Fig 2



Finally I ran an experiment.
White and Black are both computer players. White gets to use the improved estimation functions and get 0.1 seconds per move. Meanwhile Black uses the old estimation functions, but gets 1.0 second per move. Black can consistently search deeper, but White still wins by virtue of better estimations.

Next I let Black also use the improved estimation and retain its 10x advantage in allotted time. This time Black won. This shows that the improved estimation function is truly an improvement.

Output of the White victory follows. White won after 45 moves in 38.43 seconds.

```
Opening phase xxxxxxxxxxxxxxxxxxxxx
white move Wxxxxxxxxxxxxxxxxxxxx 1 5
black move WBxxxxxxxxxxxxxxxxxxx 0 7
white move WBxxxxxWxxxxxxxxxxxxx 2 5
black move WBxxxxxWxxxBxxxxxxxxx 0 7
white move WBxxxxxWxxxBxxxxxxxxW 1 4
black move WBxxxxxWxBxBxxxxxxxxW 0 7
white move WBxxxxxWxBWBxxxxxxxxW 4 5
black move WBxxxxxWxBWBxxBxxxxxW -1 6
white move WBxxxWxWxBWBxxBxxxxxW 5 5
black move WBxxxWxWxBWBBxBxxxxxW 0 5
white move WBxxxWxWxBWBBWBxxxxxW 4 4
black move WBxxBWxWxBWBBWBxxxxxW -1 4
white move WBxxBWxWWBWBBWBxxxxxW 6 3
black move WBxxBWBWWBWBBWBxxxxxW 0 3
white move WBWxBWBWWBWBBWBxxxxxW 5 2
black move WBWxBWBWWBWBBWBBxxxxW -1 2
white move WBWWBWBWWBWBBWBBxxxxW 6 1
black move WBWWBWBWWBWBBWBBBxxxW 0 1
Game phase
white move WBWWBWBWWBxBBWBBBWxxW 1 2
black move WBWWBWBWWxBBBWBBBWxxW -2 4
white move WBWWBxBWWWBBBWBBBWxxW 2 2
black move WBWWBxBWWWBBBWBBxWxBW -3 3
white move WBWWBxBWWWBBBWBBWxxBW 0 2
black move WBWWBxBWWWxBBWBBWBxBW -3 3
white move WBWxBxBWWWWBBWBBWBxBW 201.0 2
black move WBWxxBBWWWWBBWBBWBxBW -12 3
white move WBWxWBBWxWWBBWBBWBxxW 1202.0 1
black move WBWxWBBWBWWBxWBBWBxxW -1012 3
white move WBWxWBBWBWWBxWBxWBxWx 2299.0 1
black move WBWxWBBWBWWxxWBxWBxWB -2013 3
white move WxWxWBBWBWWxxWBWxBxWB 3298.0 1
black move WxWxWBBWBWWxxWBWBxxWB -3007 3
white move WxxWWBBWBWWxxWBWBxxWB 3998.0 2
black move WxxWWBBWBWWBxWBWBxxWx -3029 2
white move WxWxWBBWBWWBxWBWxxxWx 4298.0 1
black move WBWxWBBWBWWxxWBWxxxWx -4014 3
white move WxWxWBBWBWWxxWBxWxxWx 5300.0 1
black move WxWxWBxWBWWxxWBxWxBWx -5015 3
white move WxWxWBxWxWWxxWBWxxBWx 6274.0 2
black move WxWxWBxWxWWxxWBWBxxWx -6012 2
white move xxWxWBWWxWWxxWBWBxxWx 10300.0 3
black move BxWxWxWWxWWxxWBWBxxWx -6014 2
white move BxWxWxWxWWWxxWBWBxxWx 10500.0 3
black move BxWxWxWBWWWxxWxWBxxWx -6015 2
white move BxWxWxWxWWWxWxxWBxxWx 10900.0 3
Game over, white victory BxWxWxWxWWWxWxxWBxxWx
45 38.43748927116394
```