1 Task three

Random number generator generates a sequence of random numbers between 0 and 100.

- Player bets 10 credits before every new number is generated.
- Player wins 20 credits if a random number is in a range [25, 40].
- If a random number is in a range [45, 60] the price for the next 10 rounds is only 5 credits.

Calculate theoretical payback of this game.

Solution. Each number is generated independently, so probability of winning $p(W)$ is the same in each round, so is probability of getting a discount $p(D)$, both equal to ${}^{16}/_{101}$.
Expected cost C_i of round i-indexed, depends on whether an outcome of at least one of previous 10 rounds (if $i \leq 10$, then outcome of at least one of previous $i-1$ rounds) was in range [45, 60] - if yes, than it's 5 credits, if no - 10 credits. Let D' be complement of D. Then C_i can be represented as follows:

$$C_i = \begin{cases} 10 \times p(D')^{i-1} + 5 \times \left(1 - p(D')^{i-1}\right) & \text{if } i \leq 10 \\ 10 \times p(D')^{10} + 5 \times \left(1 - p(D')^{10}\right) & \text{if } i > 10 \end{cases}$$

So, expected payback of this game can be represented by an equation:

$$E[P_n] = n \times 20 \times p(W) - \sum_{i=1}^{n} C_i, \text{ where } n \text{ - number of rounds}$$

With infinite number of rounds, expected payback per round can be calculated as follows:

$$E[P] = \lim_{n\to\infty} \left(\frac{n \times 20 \times p(W) - \sum_{i=1}^{n} C_i}{n} \right) = \lim_{n\to\infty} \left(20p(W) - \left(\frac{1}{n} \sum_{i=1}^{10} C_i + \frac{(n-10)}{n} C_{11} \right) \right)$$

$$= \lim_{n\to\infty} 20p(W) - \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{10} C_i - \lim_{n\to\infty} \frac{(n-10)}{n} C_{11}$$

$$= 20p(W) - C_{11} = \frac{320}{101} - \left(10 \times \left({}^{85}/_{101}\right)^{10} + 5 \times \left(1 - \left({}^{85}/_{101}\right)^{10}\right) \right) \approx -2.7228$$