1 Task one

Slot machine 3×3 is given, with following reel strips:

1. A B C B D E A A B D E C B D A B D E
2. E D E B C A B E D A B C D E A A B B B D D D C E D A D
3. B B D E A E D A C B E A B D E A C D

On every spin of the slot machine, a random position for each reel is chosen and a reel stops in this position. Player bets 1 credit before every spin. One central horizontal payline is active. Player wins if the following combinations appear on the payline: AAA$\rightarrow$ 100, BBB$\rightarrow$ 80, CCC$\rightarrow$ 60, DDD$\rightarrow$ 40, EEE$\rightarrow$ 20, ABC$\rightarrow$ 10, EDC$\rightarrow$ 5.
Calculate theoretical payback of this slot machine.

Solution. Let $p_i(K)$ be probability of getting K on reel i. Then:

$$
\begin{array}{lllll}
p_1(A) = {}^4/_{18}, & p_1(B) = {}^5/_{18}, & p_1(C) = {}^2/_{18}, & p_1(D) = {}^4/_{18}, & p_1(E) = {}^3/_{18}, \\
p_2(A) = {}^5/_{27}, & p_2(B) = {}^6/_{27}, & p_2(C) = {}^3/_{27}, & p_2(D) = {}^8/_{27}, & p_2(E) = {}^5/_{27}, \\
p_3(A) = {}^2/_{9}, & p_3(B) = {}^2/_{9}, & p_3(C) = {}^1/_{9}, & p_3(D) = {}^2/_{9}, & p_3(E) = {}^2/_{9}.
\end{array}
$$

Results on each reels are independent, so

$$p(XYZ) = p_1(X) \times p_2(Y) \times p_3(Z), \text{ where } X, Y, Z \in \{A, B, C, D, E\}.$$

The expected payback of this slot machine (X) can be calculated as follows:

$$
\begin{aligned}
\mathbf{E}[X] = \quad & 100 \times p(AAA) + 80 \times p(BBB) + 60 \times p(CCC) + 40 \times p(DDD) \\
+ \quad & 20 \times p(EEE) + 10 \times p(ABC) + 5 \times p(EDC) - 1 \\
= \quad & 100 \times p_1(A) \times p_2(A) \times p_3(A) + 80 \times p_1(B) \times p_2(B) \times p_3(B) \\
+ \quad & 60 \times p_1(C) \times p_2(C) \times p_3(C) + 40 \times p_1(D) \times p_2(D) \times p_3(D) \\
+ \quad & 20 \times p_1(E) \times p_2(E) \times p_3(E) + 10 \times p_1(A) \times p_2(B) \times p_3(C) \\
+ \quad & 5 \times p_1(E) \times p_2(D) \times p_3(C) - 1 \\
= \quad & \Big(100 \times 4 \times 5 \times 2 + 80 \times 5 \times 6 \times 2 + 60 \times 2 \times 3 \times 1 + 40 \times 4 \times 8 \times 2 \\
+ \quad & 20 \times 3 \times 5 \times 2 + 10 \times 4 \times 6 \times 1 + 5 \times 3 \times 8 \times 1\Big) \times \Big({}^1/_{18} \times {}^1/_{27} \times {}^1/_{9}\Big) - 1 \\
= \quad & \Big(4000 + 4800 + 360 + 2560 + 600 + 240 + 120\Big) \times \Big(18 \times 27 \times 9\Big)^{-1} - 1 \\
= \quad & 12680/4374 - 1 \approx 1.8989
\end{aligned}
$$