Chapter 1 Exercises: Number Systems

Kyle Swanson

February 5, 2015

1.8 What is the largest 32-bit unsigned number?
$2^{32} - 1 = 4,294,967,295$

1.10 What is the largest 32-bit binary number that can be represented with
(a) unsigned numbers?
$2^{32} - 1 = 4,294,967,295$
(b) two's complement numbers?
$2^{31} - 1 = 2,147,483,647$
(c) sign/magnitude numbers?
$2^{31} - 1 = 2,147,483,647$

1.12 What is the smallest (most negative) 32-bit binary number that can be represented with
(a) unsigned numbers?
If you can't have a sign, either it's impossible to have a negative value, or you assume that all values are negative. A negative-int data type I suppose.
$-2^{32} - 1 = -4,294,967,295$
(b) two's complement numbers?
$-2^{31} = -2,147,483,648$
(c) sign/magnitude numbers?
$-2^{31} - 1 = -2,147,483,647$

1.14 Convert the following unsigned binary numbers to decimal.
(a) 1110_2
$2^3 + 2^2 + 2^1 + 0 = 14$
(b) 100100_2
$2^5 + 2^2 = 36$
(c) 11010111_2
$2^7 + 2^6 + 2^4 + 2^2 + 2^1 + 20 = 215$
(d) 011101010100100_2
$2^{13} + 2^{12} + 2^{11} + 2^9 + 2^7 + 2^5 + 2^2 = 15,012$

1.16 Repeat 1.14, but convert to hexadecimal.
Since they are split into 4 bit sections, just match each section with it's hex digit.
(a) 1110_2
$0xE$
(b) $10\,0100_2$
$0x24$
(c) $1101\,0111_2$
$0xD7$

(d) $011\ 1010\ 1010\ 0100_2$
$0x3AA4$

1.18 Convert the following hexadecimal numbers to decimal.
(a) $0x4E$
$4 * 16^1 + 14 * 16^0 = 78$
(b) $0x7C$
$7 * 16^1 + 12 * 160 = 124$
(c) $0xED3A$
$14 * 16^3 + 13 * 16^2 + 3 * 16^1 + 10 * 16^0 = 60,730$
(d) $0x403FB001$
$4 * 16^7 + 3 * 16^5 + 15 * 16^4 + 11 * 16^3 + 1 * 16^0 = 1,077,915,649$

1.20 Repeat 1.18, but convert to unsigned binary.
Simply take each digit, and match it to it's binary equivalent. Then push the resulting sections together.
(a) $0x4E$
0100 1110
(b) $0x7C$
0111 1100
(c) $0xED3A$
1110 1101 0011 1010
(d) $0x403FB001$
0100 0000 0011 1111 1011 0000 0000 0001

1.22 Convert the following two's complement binary numbers to decimal.
(a) 1110_2
The left most bit is 1, flip the bits, $1110 = 0001$ then add 1. $0001 + 0001 = 0010 = 2$
Since the left most bit was 1, the result is negative. -2
(b) 100011_2
$011100 + 000001 = 011101$
$2^4 + 2^3 + 2^2 + 2^0 = -29$
(c) 01001110_2
The left most bit is 0, so this is a positive number. Continue as normal.
$2^6 + 2^3 + 2^2 + 2^1 = 78$
(d) 10110101_2
$01001010 + 00000001 = 01001011$
$2^6 + 2^3 + 2^1 + 2^0 = -75$

1.26 Convert the following decimal numbers to unsigned binary numbers.

(a) 14
$14/2 = 7r0$
$7/2 = 3r1$
$3/2 = 1r1$
$1/2 = 0r1$
$14 = 1110$

(b) 52
$52/2 = 26r0$
$26/2 = 13r0$
$13/2 = 6r1$
$6/2 = 3r0$
$3/2 = 1r1$
$1/2 = 0r1$
$52 = 110100$

(c) 339
$339/2 = 169r1$
$169/2 = 84r1$
$84/2 = 42r0$
$42/2 = 21r0$
$21/2 = 10r1$
$10/2 = 5r0$
$5/2 = 2r1$
$2/2 = 1r0$
$1/2 = 0r1$
$339 = 101010011$

(d) 711
$711/2 = 355r1$
$355/2 = 177r1$
$177/2 = 88r1$
$88/2 = 44r0$
$44/2 = 22r0$
$22/2 = 11r0$
$11/2 = 5r1$
$5/2 = 2r1$
$2/2 = 1r0$
$1/2 = 0r1$
$711 = 1011000111$

1.28 Repeat Exercise 1.26, but convert to hexadecimal.
Since it's converted to binary already, you can start with that, then go to hex. (a) 14
$14 = 1110 = 0xE$

(b) 52
$52 = 0011\ 0100 = 0x34$

(c) 339
$339 = 0001\ 0101\ 0011 = 0x153$

(d) 711

1.30 Convert the following decimal numbers to 8-bit two's complement numbers or indicate that the decimal number would overflow the range.

1.34 Convert the following 4-bit two's complement numbers to 8-bit two's complement numbers.

1.36 Repeat Exercise 1.34 if the numbers are unsigned rather than two's complement.