

Chapter 2 Exercises: Combinational Logic and Boolean Algebra

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2.2 Write a Boolean equation in sum-of-products canonical form for each of the truth tables in Figure 2.81.

To determine this, look where $Y = 1$. Then just put complemented or not A, B, C, or D AND'ED together (Ex: If $B = 0$, complement it.). That is, turn the inputs to 1. And OR the results.

(a) $\overline{A}B = 1$, $A\overline{B} = 1$, $AB = 1$. Therefore
 $Y = \overline{A}B + A\overline{B} + AB$

(b) $Y = \overline{A}\overline{B}C + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}\overline{C} + ABC$

(c) $Y = \overline{A}\overline{B}C + A\overline{B}\overline{C} + ABC$

(d) $Y = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}C\overline{D} + \overline{A}BCD + \overline{A}B\overline{C}\overline{D} + \overline{A}BCD + A\overline{B}\overline{C}\overline{D} + A\overline{B}C\overline{D}$

(e) $Y = \overline{A}\overline{B}CD + \overline{A}B\overline{C}\overline{D} + \overline{A}BCD + A\overline{B}\overline{C}\overline{D} + A\overline{B}C\overline{D} + A\overline{B}\overline{C}\overline{D} + A\overline{B}CD$

2.4 Write a Boolean equation in product-of-sums canonical form for the truth tables in 2.81.

To determine this, look where $Y = 0$. Then just put complemented or not A, B, C, or D OR'ED together (Ex: If $B = 0$, do not complement it.). That is, turn the inputs to 0. And AND the results.

(a) $Y = (A + B)$

(b) $Y = (A + B + C)(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + \overline{C})$

(c) $Y = (A + B + C)(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + B + C)(\overline{A} + B + \overline{C})$

(d) $Y = (A + B + C + \overline{D})(A + \overline{B} + C + D)(A + \overline{B} + C + \overline{D})(\overline{A} + B + C + \overline{D})(\overline{A} + B + \overline{C} + \overline{D})(\overline{A} + \overline{B} + C + D)(\overline{A} + \overline{B} + C + \overline{D})(\overline{A} + \overline{B} + \overline{C} + D)(\overline{A} + \overline{B} + \overline{C} + \overline{D})$

(e) $Y = (A + B + C + D)(A + B + C + \overline{D})(A + B + \overline{C} + D)(A + \overline{B} + C + D)(A + \overline{B} + C + \overline{D})(\overline{A} + B + C + D)(\overline{A} + B + C + \overline{D})(\overline{A} + \overline{B} + C + D)(\overline{A} + \overline{B} + C + \overline{D})(\overline{A} + \overline{B} + \overline{C} + D)(\overline{A} + \overline{B} + \overline{C} + \overline{D})$

2.6 Minimize each of the Boolean equations from Exercise 2.2.

(a) $Y = \overline{A}B + A\overline{B} + AB$
 $= \overline{A}B + A(\overline{B} + B)$ T5'
 $= \overline{A}B + A$

Finally, you can remove the \overline{A} . Due to T1 or T2. See the examples below for justification.

$$= A + B$$

Examples on why \overline{A} can be eliminated.

$$A = 1, B = 1. \text{ 0 and 1 or 1 = 1 } \Leftrightarrow \text{1 or 1 = 1}$$

$$A = 0, B = 1. \text{ 1 and 1 or 0 = 1 } \Leftrightarrow \text{1 or 0 = 1}$$

$$A = 1, B = 0. \text{ 0 and 0 or 1 = 1 } \Leftrightarrow \text{0 or 1 = 1}$$

$$A = 0, B = 0. \text{ 1 and 0 or 0 = 0 } \Leftrightarrow \text{0 or 0 = 0}$$

See how they're the same? Since \overline{A} and B are AND'ed together, \overline{A} doesn't matter since it has A on the other side of an OR.

$$\begin{aligned} \text{(b) } Y &= \overline{A}\overline{B}C + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}\overline{C} + AB\overline{C} \\ &= \overline{A}(\overline{B}C + B\overline{C} + BC) + \overline{C}(A\overline{B} + AB) \text{ T8} \\ &= \overline{A}(C(\overline{B} + B) + B\overline{C}) + A\overline{C} \text{ T5, T3} \\ &= \overline{A}(B\overline{C} + C) + A\overline{C} \\ &= \overline{A}(B + C) + A\overline{C} \\ &= \overline{A}B + \overline{A}C + A\overline{C} \end{aligned}$$

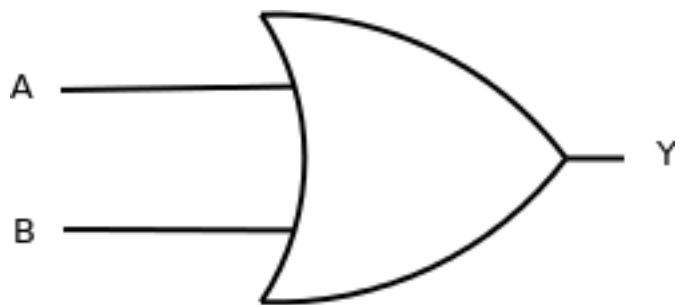
$$\begin{aligned} \text{(c) } Y &= \overline{A}\overline{B}C + AB\overline{C} + ABC \\ &= \overline{A}\overline{B}C + A(B(\overline{C} + C)) \\ &= \overline{A}\overline{B}C + AB \text{ T5, T3} \end{aligned}$$

$$\begin{aligned} \text{(d) } Y &= \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}C\overline{D} + \overline{A}B\overline{C}\overline{D} + \overline{A}BC\overline{D} + \overline{A}B\overline{C}D + \overline{A}BCD + A\overline{B}\overline{C}\overline{D} + A\overline{B}C\overline{D} \\ &= \overline{A}\overline{B}(\overline{C}\overline{D} + C\overline{D} + CD) + \overline{A}BC(\overline{D} + D) + A\overline{B}D(\overline{C} + C) \\ &= \overline{A}\overline{B}(C + \overline{D}) + \overline{A}BC + A\overline{B}D \\ &= \overline{A}\overline{B}C + \overline{A}B\overline{D} + \overline{A}BC + A\overline{B}D \\ &= \overline{B}\overline{D}(\overline{A} + A) + \overline{A}C(\overline{B} + B) \\ &= \overline{A}C + \overline{B}\overline{D} \end{aligned}$$

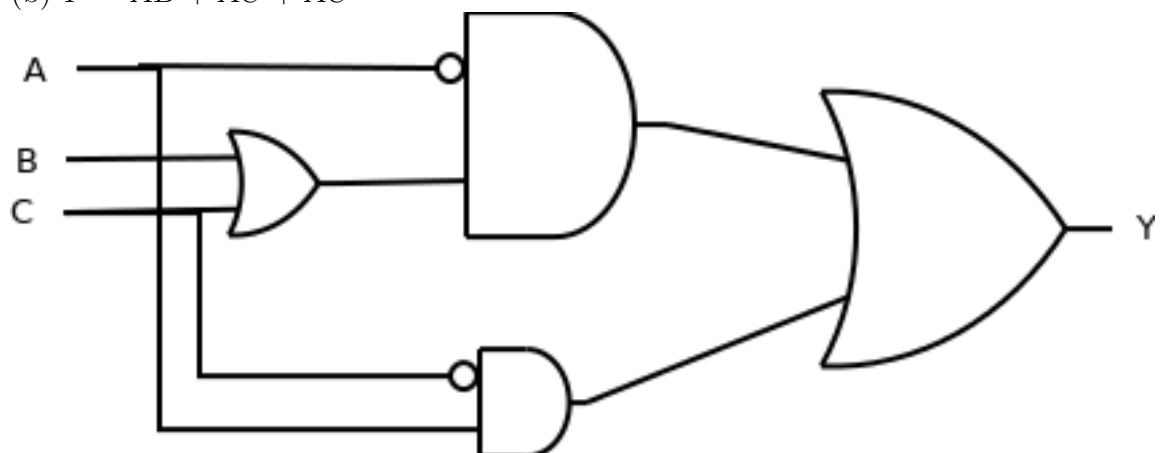
$$\begin{aligned} \text{(e) } Y &= \overline{A}\overline{B}C\overline{D} + \overline{A}B\overline{C}\overline{D} + \overline{A}BCD + A\overline{B}\overline{C}\overline{D} + A\overline{B}C\overline{D} + A\overline{B}C\overline{D} + A\overline{B}CD \\ &= \end{aligned}$$

2.8 Sketch a reasonably simple combinational circuit implementing each of the functions from Exercise 2.6.

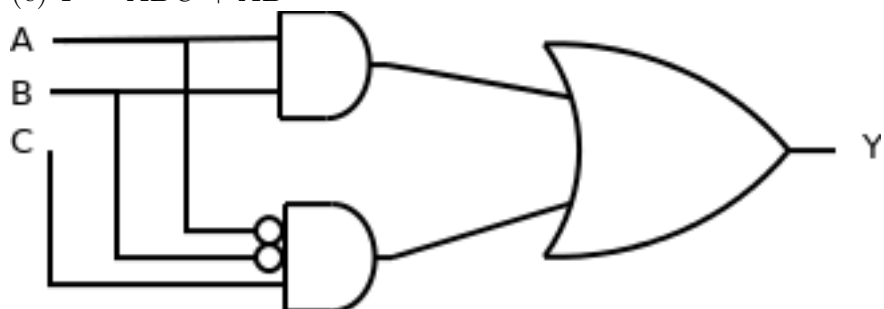
$$\text{(a) } Y = A + B$$



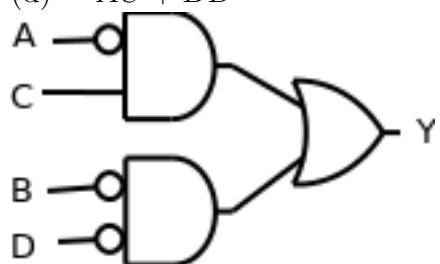
(b) $Y = \overline{A}B + \overline{A}C + A\overline{C}$



(c) $Y = \overline{A}\overline{B}C + AB$



(d) $Y = \overline{A}C + \overline{B}D$



(e)

2.14 Simplify the following Boolean equations using Boolean theorems. Check for correctness using a truth table or K-map.

$$\begin{aligned}\text{(a)} \quad Y &= \overline{A}BC + \overline{A}B\overline{C} \\ &= \overline{A}B(C + \overline{C}) \\ &= \overline{A}B\end{aligned}$$

A	B	C	$\overline{A}BC + \overline{A}B\overline{C}$	$\overline{A}B$
1	1	1	0	0
1	1	0	0	0
1	0	1	0	0
1	0	0	0	0
0	1	1	1	1
0	1	0	1	1
0	0	1	0	0
0	0	0	0	0

$$\begin{aligned}\text{(b)} \quad Y &= \overline{A}BC + A\overline{B} \\ &= \overline{B}(\overline{A}C + A) \\ &= \overline{B}(A + \overline{C}) \\ &= A\overline{B} + \overline{B}\overline{C}\end{aligned}$$

A	B	C	$\overline{A}BC + A\overline{B}$	$A\overline{B} + \overline{B}\overline{C}$
1	1	1	0	0
1	1	0	0	0
1	0	1	1	1
1	0	0	1	1
0	1	1	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	1

$$\begin{aligned}\text{(c)} \quad Y &= ABC\overline{D} + \overline{A}BC\overline{D} + \overline{(A + B + C + D)} \\ &= ABC\overline{D} + \overline{A}BC\overline{D} + \overline{A}B\overline{C}\overline{D} \\ &= \overline{D}(ABC + \overline{A}BC + \overline{A}B\overline{C}) \\ &= \overline{D}(ABC + \overline{B}C(A + \overline{A})) \\ &= \overline{D}(ABC + \overline{B}C) \\ &= ABC\overline{D} + \overline{B}C\overline{D}\end{aligned}$$

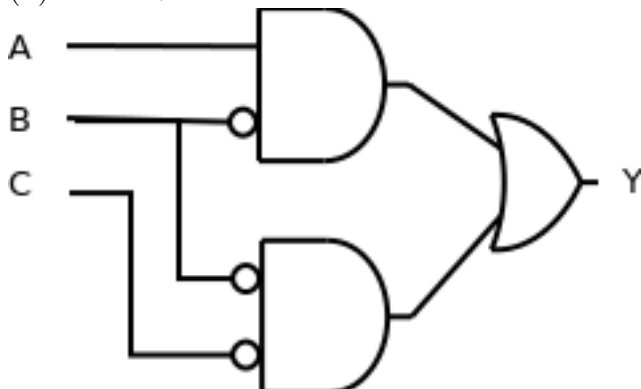
A	B	C	D	$ABCD + \overline{ABCD} + (\overline{A+B+C+D})$	$ABCD + \overline{BCD}$
1	1	1	1	0	0
1	1	1	0	1	1
1	1	0	1	0	0
1	1	0	0	0	0
1	0	1	1	0	0
1	0	1	0	0	0
1	0	0	1	0	0
1	0	0	0	1	1
0	1	1	1	0	0
0	1	1	0	0	0
0	1	0	1	0	0
0	1	0	0	0	0
0	0	1	1	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	1

2.16 Sketch a reasonably simple combinational circuit implementing each of the functions from Exercise 2.14.

(a) $= \overline{A}B$



(b) $= A\overline{B} + \overline{B}C$



(c) $ABCD + \overline{BCD}$

TODO! Add the picture

2.22 Prove that the following theorems are true using perfect induction. You need not prove their duals.

For these, build a truth table. Show every option. And so long as the theorem matches with what it claims, it is proved.

(a) The idempotency theorem (T3), $B \bullet B = B$
See how 'B' matches ' $B \bullet B$ '?

B	$B \bullet B$
0	0
1	1

(b) The distributivity theorem (T8), $(B \bullet C) + (B \bullet D) = B \bullet (C + D)$

B	C	D	$(B \bullet C) + (B \bullet D)$	$B \bullet (C + D)$
1	1	1	1	1
1	1	0	1	1
1	0	1	1	1
1	0	0	0	0
0	1	1	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	0	0

(c) The combining theorem (T10), $(B \bullet C) + (B \bullet \overline{C}) = B$

B	C	$(B \bullet C) + (B \bullet \overline{C})$	B
1	1	1	1
1	0	1	1
0	1	0	0
0	0	0	0