Chapter 2 Exercises: Combinational Logic and Boolean Algebra

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2.2 Write a Boolean equation in sum-of-products canonical form for each of the truth tables in Figure 2.81.

To determine this, look where Y = 1. Then just put complemented or not A, B, C, or D AND'ED together (Ex: If B = 0, complement it.). That is, turn the inputs to 1. And OR the results.

(a) $\overline{A}B = 1$, $A\overline{B} = 1$, $AB = 1$. Therefore
$Y = \overline{A}B + A\overline{B} + AB$

(b) $Y = \overline{A}\overline{B}C + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}\overline{C} + AB\overline{C}$

(c) $Y = \overline{A}\overline{B}C + AB\overline{C} + ABC$

(d) $Y = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}C\overline{D} + \overline{A}\overline{B}CD + \overline{A}BC\overline{D} + \overline{A}BCD + A\overline{B}\overline{C}\overline{D} + A\overline{B}C\overline{D}$

(e) $Y = \overline{A}\overline{B}CD + \overline{A}BC\overline{D} + \overline{A}BCD + A\overline{B}\overline{C}\overline{D} + A\overline{B}\overline{C}D + A\overline{B}C\overline{D} + A\overline{B}CD$

2.4 Write a Boolean equation in product-of-sums canonical form for the truth tables in 2.81.

To determine this, look where Y = 0. Then just put complemented or not A, B, C, or D OR'ED together (Ex: If B = 0, do not complement it.). That is, turn the inputs to 0. And AND the results.

(a) $Y = (A + B)$

(b) $Y = (A + B + C)(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + \overline{C})$

(c) $Y = (A + B + C)(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + B + C)(\overline{A} + B + \overline{C})$

(d) $Y = (A + B + C + \overline{D})(A + \overline{B} + C + D)(A + \overline{B} + C + \overline{D})(\overline{A} + B + C + \overline{D})(\overline{A}+B+\overline{C}+\overline{D})(\overline{A}+\overline{B}+C+D)(\overline{A}+\overline{B}+C+\overline{D})(\overline{A}+\overline{B}+\overline{C}+D)(\overline{A}+\overline{B}+\overline{C}+\overline{D})$

(e) $Y = (A + B + C + D)(A + B + C + \overline{D})(A + B + \overline{C} + D)(A + \overline{B} + C + D)(A+\overline{B}+C+\overline{D})(\overline{A}+\overline{B}+C+D)(\overline{A}+\overline{B}+C+\overline{D})(\overline{A}+\overline{B}+\overline{C}+D)(\overline{A}+\overline{B}+\overline{C}+\overline{D})$

2.6 Minimize each of the Boolean equations from Exercise 2.2.
(a) $Y = \overline{A}B + A\overline{B} + AB$
$= \overline{A}B + A(\overline{B} + B)$ T5'
$= \overline{A}B + A$

Finally, you can remove the $\overline{A}$. Due to T1 or T2. See the examples below for justification.
$= A + B$
Examples on why $\overline{A}$ can be eliminated.
A = 1, B = 1. $0 \, and \, 1 \, or \, 1 = 1 \Leftrightarrow 1 \, or \, 1 = 1$
A = 0, B = 1. $1 \, and \, 1 \, or \, 0 = 1 \Leftrightarrow 1 \, or \, 0 = 1$
A = 1, B = 0. $0 \, and \, 0 \, or \, 1 = 1 \Leftrightarrow 0 \, or \, 1 = 1$
A = 0, B = 0. $1 \, and \, 0 \, or \, 0 = 0 \Leftrightarrow 0 \, or \, 0 = 0$
See how they're the same? Since $\overline{A}$ and B are AND'ed together, $\overline{A}$ doesn't matter since it has A on the other side of an OR.

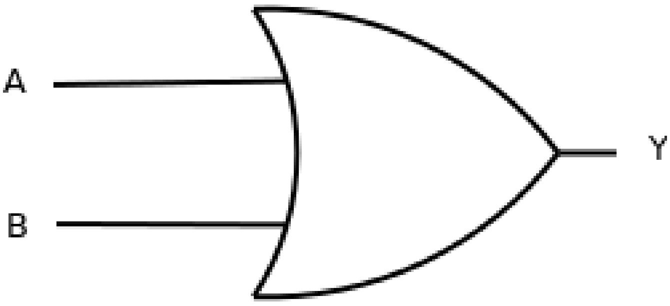
(b) $Y = \overline{AB}C + \overline{A}B\overline{C} + \overline{A}BC + A\overline{BC} + AB\overline{C}$
$= \overline{A}(\overline{B}C + B\overline{C} + BC) + \overline{C}(A\overline{B} + AB)$ T8
$= \overline{A}(C(\overline{B} + B) + B\overline{C}) + A\overline{C}$ T5, T3
$= \overline{A}(B\overline{C} + C) + A\overline{C}$
$= \overline{A}(B + C) + A\overline{C}$
$= \overline{A}B + \overline{A}C + A\overline{C}$

(c) $Y = \overline{AB}C + AB\overline{C} + ABC$
$= \overline{AB}C + A(B(\overline{C} + C))$
$= \overline{AB}C + AB$ T5, T3

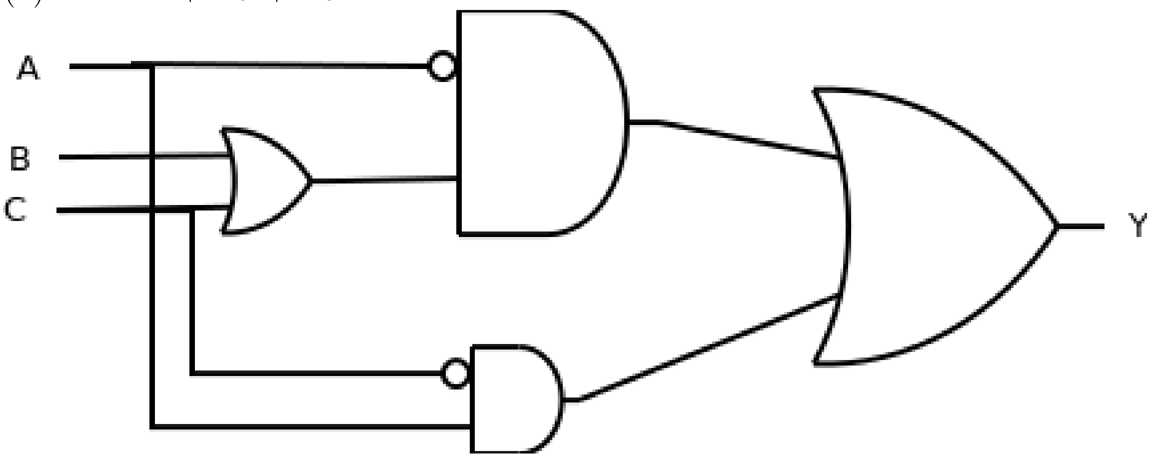
(d) $Y = \overline{ABCD} + \overline{AB}C\overline{D} + \overline{AB}CD + \overline{A}BC\overline{D} + \overline{A}BCD + A\overline{BCD} + A\overline{B}C\overline{D}$
$= \overline{AB}(\overline{CD} + C\overline{D} + CD) + \overline{A}BC(\overline{D} + D) + A\overline{BD}(\overline{C} + C)$
$= \overline{AB}(C + \overline{D}) + \overline{A}BC + A\overline{BD}$
$= \overline{AB}C + \overline{ABD} + \overline{A}BC + A\overline{BD}$
$= \overline{BD}(\overline{A} + A) + \overline{A}C(\overline{B} + B)$
$= \overline{A}C + \overline{BD}$

(e) $Y = \overline{AB}CD + \overline{A}BC\overline{D} + \overline{A}BCD + A\overline{BCD} + A\overline{BC}D + A\overline{B}C\overline{D} + A\overline{B}CD$
$= \overline{AB}CD + \overline{A}BC(\overline{D} + D) + A\overline{BC}(\overline{D} + D) + AC\overline{B}(\overline{D} + D)$
$= \overline{AB}CD + \overline{A}BC + A\overline{BC} + AC\overline{B}$
$= \overline{AB}CD + \overline{A}BC + A\overline{B}(\overline{C} + C)$
$= \overline{AB}CD + \overline{A}BC + A\overline{B}$
$= \overline{A}C(\overline{B}D + B) + A\overline{B}$
$= \overline{A}C(D + B) + A\overline{B}$
$= \overline{A}CD + \overline{A}BC + A\overline{B}$

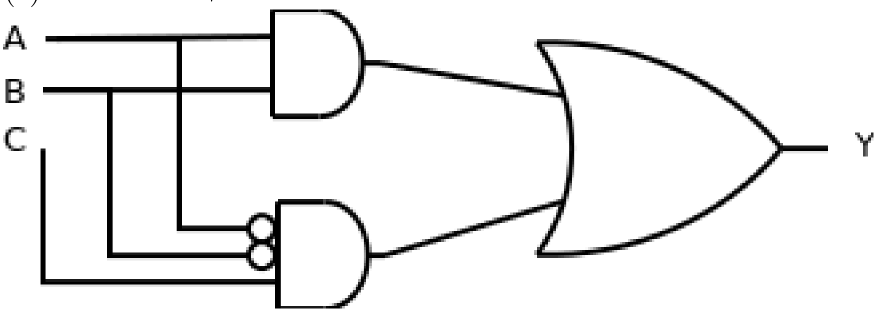
2.8 Sketch a reasonably simple combinational circuit implementing each of the functions from Exercise 2.6.
(a) $Y = A + B$



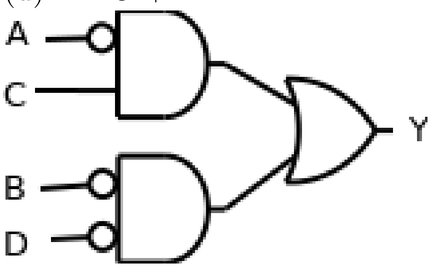
(b) $Y = \overline{A}B + \overline{A}C + A\overline{C}$



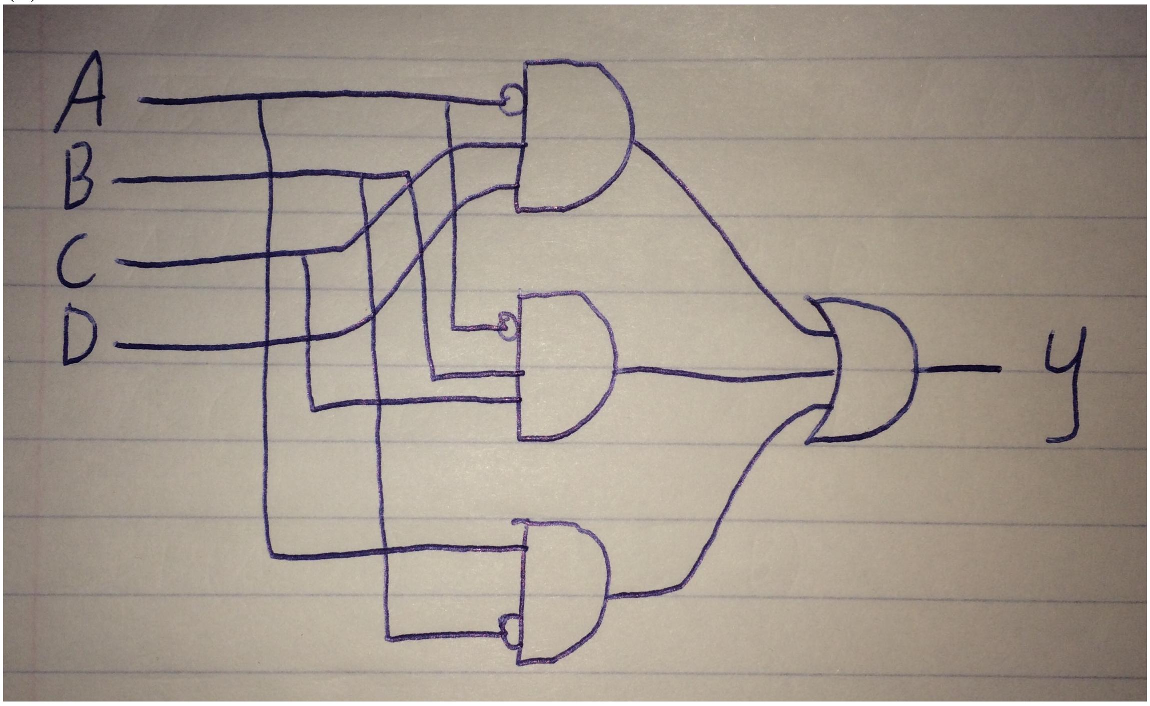
(c) $Y = \overline{AB}C + AB$



(d) $= \overline{A}C + \overline{BD}$



(e) $= \overline{A}CD + \overline{A}BC + A\overline{B}$



2.14 Simplify the following Boolean equations using Boolean theorems. Check for correctness using a truth table or K-map.

(a) $Y = \overline{A}BC + \overline{A}B\overline{C}$
$= \overline{A}B(C + \overline{C})$
$= \overline{A}B$

A	B	C	$\overline{A}BC + \overline{A}B\overline{C}$	$\overline{A}B$
1	1	1	0	0
1	1	0	0	0
1	0	1	0	0
1	0	0	0	0
0	1	1	1	1
0	1	0	1	1
0	0	1	0	0
0	0	0	0	0

(b) $Y = \overline{ABC} + A\overline{B}$
$= \overline{B}(\overline{AC} + A)$
$= \overline{B}(A + \overline{C})$

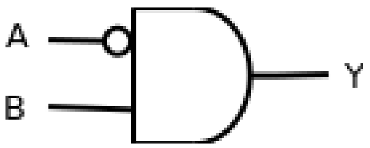
$= A\overline{B} + \overline{BC}$

A	B	C	$\overline{ABC} + A\overline{B}$	$A\overline{B} + \overline{BC}$
1	1	1	0	0
1	1	0	0	0
1	0	1	1	1
1	0	0	1	1
0	1	1	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	1

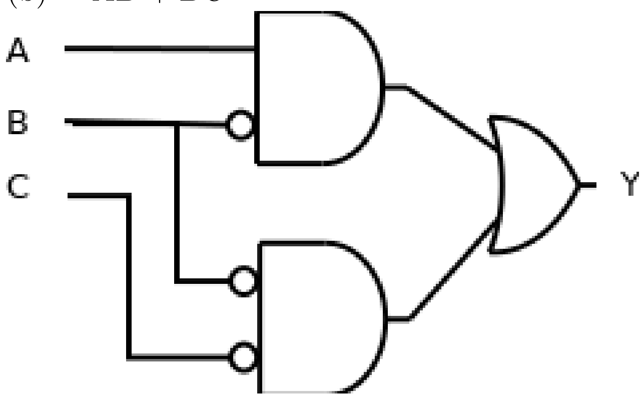
(c) $Y = ABC\overline{D} + A\overline{BCD} + (\overline{A + B + C + D})$
$= ABC\overline{D} + A\overline{BCD} + \overline{ABCD}$
$= \overline{D}(ABC + A\overline{BC} + \overline{ABC})$
$= \overline{D}(ABC + \overline{BC}(A + \overline{A}))$
$= \overline{D}(ABC + \overline{BC})$
$= ABC\overline{D} + \overline{BCD}$

A	B	C	D	$ABC\overline{D} + A\overline{BCD} + (\overline{A + B + C + D})$	$ABC\overline{D} + \overline{BCD}$
1	1	1	1	0	0
1	1	1	0	1	1
1	1	0	1	0	0
1	1	0	0	0	0
1	0	1	1	0	0
1	0	1	0	0	0
1	0	0	1	0	0
1	0	0	0	1	1
0	1	1	1	0	0
0	1	1	0	0	0
0	1	0	1	0	0
0	1	0	0	0	0
0	0	1	1	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	1

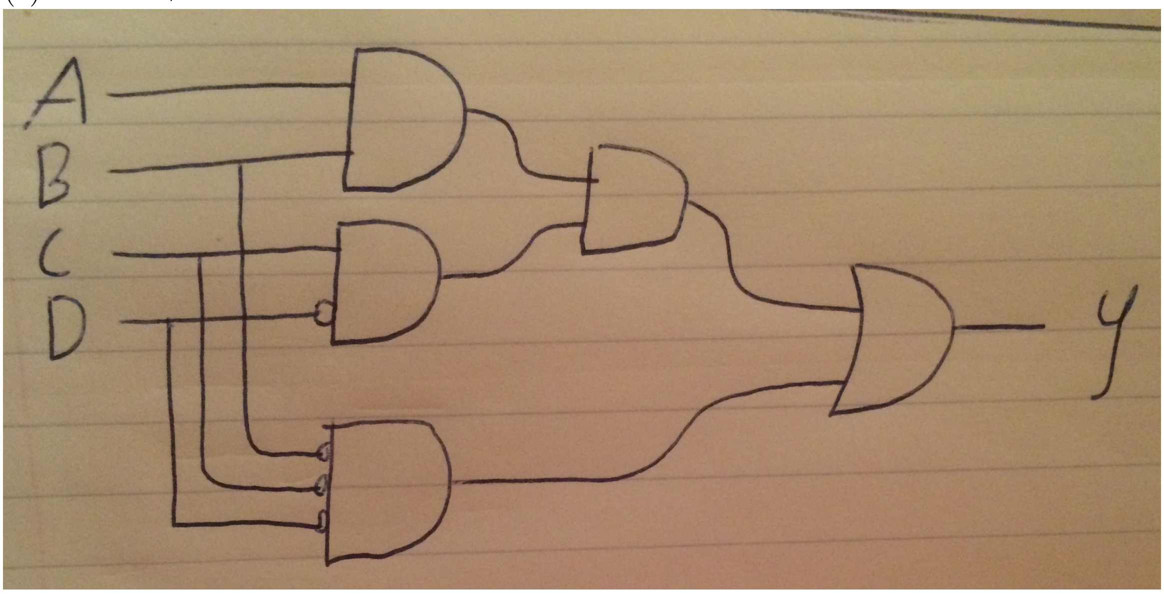
2.16 Sketch a reasonably simple combinational circuit implementing each of the functions from Exercise 2.14.
(a) $= \overline{A}B$



(b) $= A\overline{B} + \overline{BC}$



(c) $ABC\overline{D} + \overline{BCD}$



2.22 Prove that the following theorems are true using perfect induction. You need not prove their duals.

For these, build a truth table. Show every option. And so long as the theorem matches with what it claims, it is proved.

(a) The idempotency theorem (T3), $B \bullet B = B$
See how 'B' matches '$B \bullet B$'?

B	$B \bullet B$
0	0
1	1

(b) The distributivity theorem (T8), $(B \bullet C) + (B \bullet D) = B \bullet (C + D)$

B	C	D	$(B \bullet C) + (B \bullet D)$	$B \bullet (C + D)$
1	1	1	1	1
1	1	0	1	1
1	0	1	1	1
1	0	0	0	0
0	1	1	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	0	0

(c) The combining theorem (T10), $(B \bullet C) + (B \bullet \overline{C}) = B$

B	C	$(B \bullet C) + (B \bullet \overline{C})$	B
1	1	1	1
1	0	1	1
0	1	0	0
0	0	0	0