exercises

1.4 An analog voltage is in the range of 0-5V. If it can be measured with an accuracy of ± 50mV, at most, how many bits of information does it convey?

Find the number of discrete states:

The voltage can be measured to nearest 100mV

$$N = 5\cancel{V} / 0.1\cancel{V}$$

$$N = 50$$

Note: See how the units of Volts cancels out? The number of discrete states is unitless.

This may be represented by a single base 50 digit.

To find the number of binary digits of information represented by this system, we use the logarithm base conversion formula:

$$\log_b N = \frac{\log_K N}{\log_K b}$$

In this case,

$$\log_2 50 = \frac{\log_{50} 50}{\log_{50} 2}$$

1.4 Continued...

Since $\log_{50} 50 = 1$,

$$\log_2 50 = \frac{1}{\log_{50} 2}$$

Calculating $\log_{50} 2$ yields 0.1772 approximately.

$$\log_2 50 = \frac{1}{0.1772}$$

$$\boxed{\log_2 50 = 5.64}$$

So, 5.64 binary digits (base 2) are needed to represent all of the discrete states in the system.

exercises

1.5 A classroom has an old clock on the wall whose minute hand broke off.

a) If you can read the hour hand to the nearest 15 minutes, how many bits of information does the clock convey about the time?

b) If you know whether it is before or after noon, how many additional bits of information do you know about the time?

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a) There are 4 discrete states per hour. There are 12 hours on the clock. Therefore, there are 48 (4 x 12) discrete states in the system.

This may be represented by a single ~~48-bit~~ base 48 digit.
To find the number of binary digits of information represented by this system, use the logarithm base conversion formula:

$$\log_b N = \frac{\log_K N}{\log_K b}$$

In this case,

$$\log_2 48 = \frac{\log_{48} 48}{\log_{48} 2}$$

Since $\log_{48} 48 = 1$,

$$\log_2 48 = \frac{1}{\log_{48} 2}$$

exercises

1.5 continued...

Calculating $\log_{48} 2$ yields 0.1791 approximately.

$$\log_2 48 = \frac{1}{0.1791}$$

$$\boxed{\log_2 48 = 5.583}$$

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b) If we know whether the time is before noon or after noon, the number of discrete states in the system is doubled.

Expressed mathematically, doubling the number of discrete states can be written:

$$\log_2 (N \cdot 2)$$

Which is equivalent to:

$$\log_2 N + \log_2 2$$

But we have already calculated $\log_2 N$.
And $\log_2 2$ is equal to 1.

So, the answer is that $\log_2 2 = 1$ binary digit is added.

Going further, we can write this as follows:

$$\log_2 (N \cdot 2) = \log_2 N + \log_2 2$$

$$\log_2 (48 \cdot 2) = 5.583 + \underline{1}$$

exercises

1.6 The Babylonians developed the sexagesimal (base 60) number system about 4,000 years ago. How many bits of information is conveyed with one sexagesimal digit? How do you write the number 4000_{10} in sexagesimal?

The base of the number system represents the number of discrete states per digit.

To find the number of binary digits of information in a single sexagesimal digit, we apply the logarithm base conversion formula:

$$\log_b N = \frac{\log_k N}{\log_k b}$$

In this case,

$$\log_2 60 = \frac{\log_{60} 60}{\log_{60} 2}$$

Since $\log_{60} 60 = 1$,

$$\log_2 60 = \frac{1}{\log_{60} 2}$$

Calculating $\log_{60} 2$ yields 0.1693 approximately.

$$\log_2 60 = \frac{1}{0.1693}$$

$$\boxed{\log_2 60 = 5.907}$$

exercises

1.6 continued...

To convert 4000_{10} to sexagesimal, work from the right and repeatedly divide by the base (60) as in Example 1.5 from the text:

$$4000 / 60 = 66 \text{ r } 40$$

So the 1's column is (40_{60}).

Now, divide the quotient by 60:

$$66 / 60 = 1 \text{ r } 6$$

So the 60's column (or 10's column depending on your point of reference) is 6_{60}.

Now divide the last quotient by 60:

$$1 / 60 = 0 \text{ r } 1$$

So the 3600's (100's) column is 1_{60}.

Combining the 1's, 10's, and 100's columns gives the answer:

$$4000_{10} = 1\ 6\ 40_{60}$$

exercises

1.7 How many different numbers can be represented with 16 bits?

A bit is a binary digit. The number system is base 2 since each digit can have only two discrete states.

The range of a 16-bit number is 2^{16} or 65,536 — 0, 1, ..., 65,535.

1. You are choosing a password on a system that limits password length to 8 characters. Each character is further limited to 0-9, A-Z (lower case characters not permitted.) What is the maximum number of bits of entropy (information) in the password you choose? Repeat the calculation with lower case characters allowed.

The number of characters in the password represents the number of digits of our number system.

Each character (digit) represents 36 discrete states. Therefore, the base of our number system is 36.

Use the logarithm base conversion formula to calculate the number of binary digits needed to represent the information (states) in 1 base 36 digit.

$$\log_b N = \frac{\log_K N}{\log_K b}$$

In this case,

$$\log_2 36 = \frac{\log_{36} 36}{\log_{36} 2}$$

1. continued...

Since $\log_{36} 36 = 1$,

$$\log_2 36 = \frac{1}{\log_{36} 2}$$

Calculating $\log_{36} 2$ yields 0.1934 approximately.

$$\log_2 36 = \frac{1}{0.1934}$$

$$\log_2 36 = 5.171$$

Since there are 8 characters (digits) in our password, we find the result by multiplying by 8,

$$\boxed{8 \times 5.171 = 41.368 \text{ bits}}$$

So, 41.368 binary digits are needed to represent the maximum information contained in the password as specified.

Permitting lower case characters adds 26 discrete states to each character (digit.) So now, each password character represents 36 + 26 = 62 discrete states. Therefore, the base of our number system is 62.

So now, we simply repeat the previous calculation using the new base.

$$\log_2 62 = \frac{\cancel{\log_{62} 62}\ 1}{\log_{62} 2}$$

Calculating $\log_{62} 2$ yields 0.1679 approximately.

$$\log_2 62 = \frac{1}{0.1679}$$

$$\log_2 62 = 5.96$$

Now multiply the number of bits by 8 for the number of characters in our password.

$$\boxed{8 \times 5.96 = 47.68}$$

With the addition of lower case, the maximum number of binary digits required to represent the password is 47.68.

2. A decimal digit contains approximately 3.32 bits of information. Explain what this means. As you do so, there are some things you should be thinking about. Try changing the word "bit" to "binary digit." What is the meaning of a fractional digit? Think about the number of distinct states and the number of binary digits needed to represent these distinct states. Please be detailed. Include tables of numbers to illustrate as you feel appropriate.

The fractional binary digit simply means that the decimal number system is not an even multiple of two. Octal evenly maps onto a 3-bit number and hexadecimal evenly maps onto a 4-bit number. A decimal digit with 10 discrete states falls somewhere between 3 and 4 bits.

We know that the ratio of decimal states to octal states is $10/8 = 1.25$. Also, recall from Exercise 1.5 that

$$\log_2(N \times X) = \log_2 N + \log_2 X$$

Since we already know some of these quantities, we can rewrite this relationship.

First though, let's state what we know...

2. continued...

We know the quantity $\log_2 (N * x) = 3.32$ because it was a given, and we are already familiar with the logarithm base conversion formula.

We also know that $\log_2 8 = 3$ and since we are concerned with the fractional portion, it is logical to make that connection. So, we can rewrite the formula (relationship):

$$\cancel{\log_2}(N * x) = \log_2 8 + \log_2 X$$

And doing more substitution,

$$3.3219 = 3 + 0.3219$$

But we still need to solve for X.

$$\log_2 X = 0.3219$$

Exponentiate both sides:

$$X = 2^{0.3219}$$

$$X = 1.25$$

So, the fractional binary digit simply represents the ratio of the number of decimal states to the number of octal states.