hw1

September 11, 2017

1 CSCI 5992 - HW1 - Kelly/Milne

1.1 Part 1

1.1.1 Solution

1a) w1 = -2.04427331 w2 = 3.99683416 b = -0.92427055
1b) Levenberg–Marquardt as per implementation below

In [1]:
```
#%%
import os
import numpy as np
import scipy.io
import scipy.optimize as optimization
#from IPython.core.debugger import set_trace

# Data
filedir = os.path.dirname(os.path.realpath('__file__'))
datapath = os.path.join(filedir,'assign1_data.mat')
data = scipy.io.loadmat(datapath)

# fitter wants shape (k,M) (k number of predictors)
# data['x'] has shape (M,k) so we fix that
x1 = data['x'][:,0]
x2 = data['x'][:,1]
X = np.array([x1,x2])
# argg - shape (100,1) - gives obstuse error in the fitter
# we fix that too (flatten)
y = np.array(data['y']).flatten()

# Initial guess
w0 = np.array([1, 1, 1])

# Objective function
# y = w1 * x1 + w2 * x2 + b.
def func(X, b, w1, w2):
  # unpack independent vars
  rval = w1*X[0] + w2*X[1] + b
```

```
    return rval

result = optimization.curve_fit(func, X, y, w0)
print(result[0])
```

```
[-0.92427055 -2.04427331  3.99683416]
```

1.2 Part 2

1.2.1 Solution

2a) w1 = -2.0427036 w2 = 3.99155586 b = -0.92233149

2b) Batch processed. A step size of 0.05 diverged so used 0.01. Approx N=100 epochs to recover the results from the LM method. Termination was based on plot below of error vs epoch, which amounts to a % change in the error function of ~10^-4 between epochs.

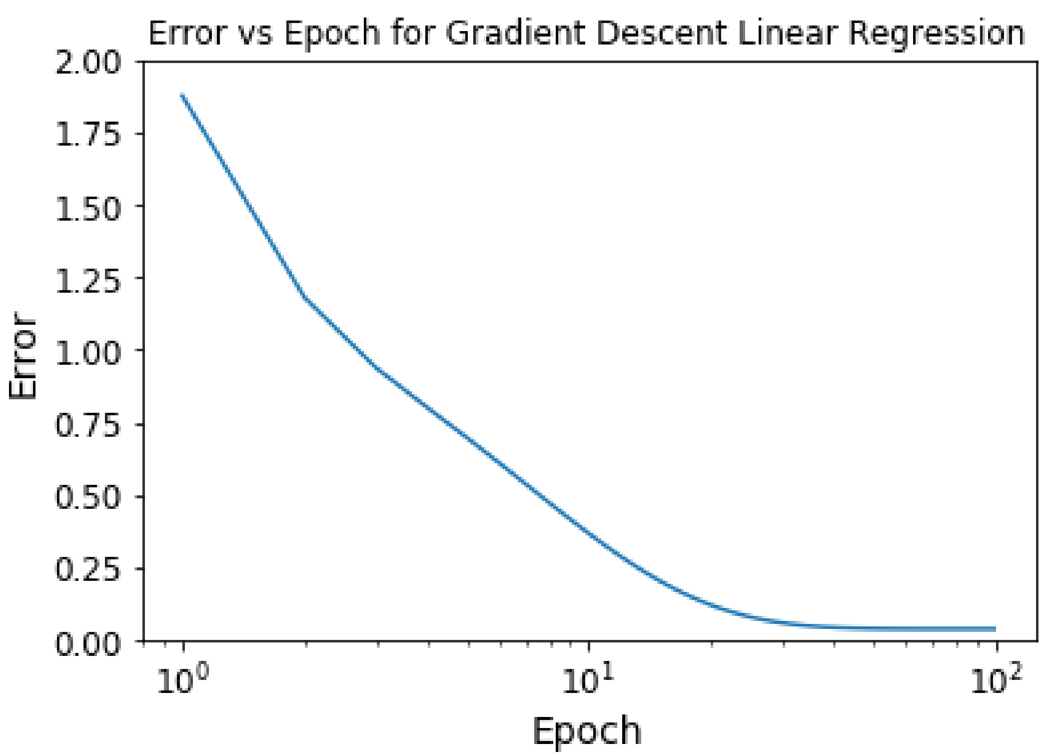
In [2]:
```
import os
import numpy as np
import scipy.io
import scipy.optimize as optimization
#from IPython.core.debugger import set_trace

EPS = 0.01
N_EPOCH = 100
err = []
def dw(w,d,X):
    # using numpy einstein summation to vectorize the computation
    # vector of coefficients (d-w.x) for each pattern (alpha)
    C = d - np.einsum('i,ij->j', w, X) # shape is (100,)
    e = 1.0/len(C)*np.sum([ c**2 for c in C])
    err.append(e)
    # C_i * X_ji or C * transpose(X)
    dw = EPS*np.einsum('i,ji->j', C, X) # shape is (3,)
    return dw

def main():
    # data
    filedir = os.path.dirname(os.path.realpath('__file__'))
    datapath = os.path.join(filedir,'assign1_data.mat')
    data = scipy.io.loadmat(datapath)
    x1 = data['x'][:,0]
    x2 = data['x'][:,1]
    # inputs - with bias input tied high, shape is (3,100)
    X = np.array([np.full(len(x1),1),x1,x2])
    # output
    d = np.array(data['y']).flatten()
    # starting vector of weights
    w = [1,1,1]
```

```
        for _ in range(N_EPOCH):
            w = w + dw(w,d,X)
        print(w)

    # invoke main
    main()
```

```
[-0.92263116 -2.04308226  3.99249859]
```

In [3]:
```
# To plot pretty figures
%matplotlib inline
import matplotlib
import matplotlib.pyplot as plt
plt.rcParams['axes.labelsize'] = 14
plt.rcParams['xtick.labelsize'] = 12
plt.rcParams['ytick.labelsize'] = 12
ax = plt.subplot(111)
plt.semilogx(err)
ax.set_ylim([0, 2])
plt.ylabel('Error')
plt.xlabel('Epoch')
plt.title('Error vs Epoch for Gradient Descent Linear Regression')
```

Out[3]:
```
<matplotlib.text.Text at 0x115214278>
```



1.3 Part 3

1.3.1 Solution

3a) b = -16. w1=-50.43661604 w2 = 81.86640442
3b) See plot in 'Visualization of Perceptron Results'

In [4]:
```
import os as os
import numpy as np
import scipy.io
import scipy.optimize as optimization
from IPython.core.debugger import set_trace

N_EPOCH = 30

n_mis = []
def dwp(w,d,X):
    # using numpy einstein summation to vectorize the computation
    # calculating w.x.d -- w.x.d < 0 => incorrect classification
    C = d*np.einsum('i,ij->j', w, X) # shape is (100,)
    XT = X.transpose()
    # use enumerate for the equivalent to each_with_index (Ruby)
    xd = [ d[i]*XT[i] for i,c in enumerate(C) if c < 0]
    # for N vs EPOCH plot
    n_mis.append(len(xd))
    # sum xd element-wise
    # dw is x.t summed over misclassified teachers
    dw =  np.einsum('ij->j',xd) if(len(xd) > 0) else 0
    return dw

def main():
    # data
    filedir = os.path.dirname(os.path.realpath('__file__'))
    datapath = os.path.join(filedir,'assign1_data.mat')
    data = scipy.io.loadmat(datapath)
    x1 = data['x'][:,0]
    x2 = data['x'][:,1]
    # inputs - with bias input tied high, shape is (3,100)
    X = np.array([np.full(len(x1),1),x1,x2])
    # output
    d = np.array(data['z']).flatten()
    # use domain -1,1 for teachers so we can use x.w.d<0 for the classification test
    d = [z if z == 1 else -1 for z in d]
    # starting vector of weights
    w = [1,1,1]
    # sweep through the data N_EPOCH times
```

```
    for _ in range(N_EPOCH):
        w = w + dwp(w,d,X)

    return w

# invoke main
w_perceptron = main()
print(w_perceptron)
```

```
[-16.         -50.43661604  81.86640442]
```

2 Visualization of perceptron results

Read the data into a panda dataframe for visualization

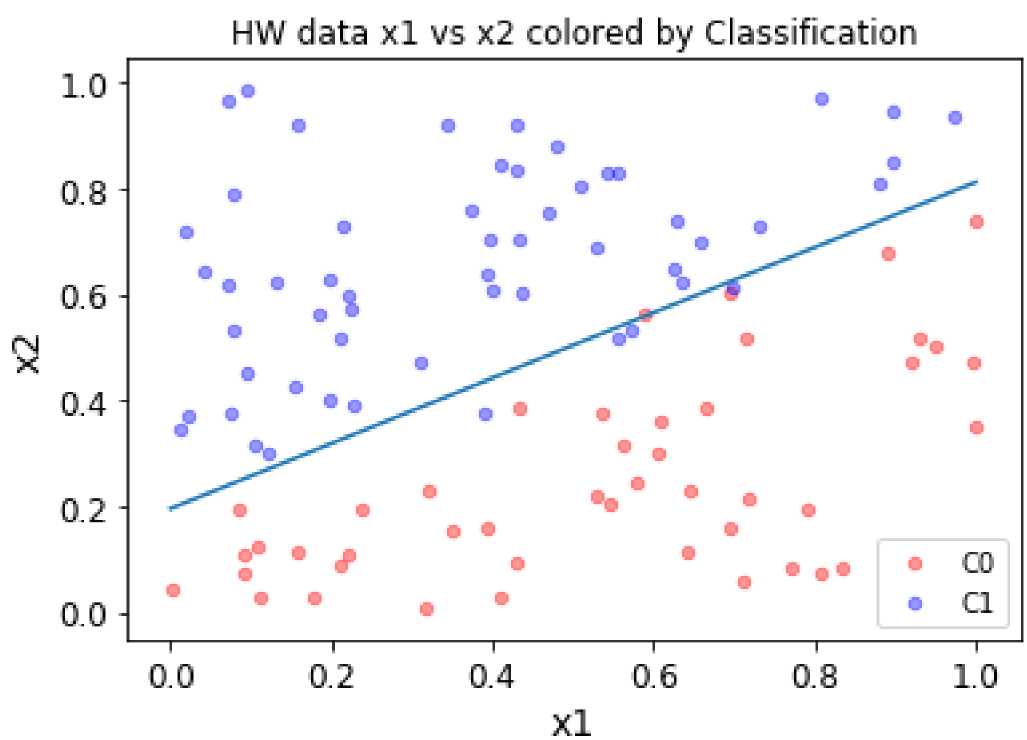
In [5]:
```
# Common imports
import numpy as np
import os

# to make this notebook's output stable across runs
np.random.seed(42)

# To plot pretty figures
%matplotlib inline
import matplotlib
import matplotlib.pyplot as plt
plt.rcParams['axes.labelsize'] = 14
plt.rcParams['xtick.labelsize'] = 12
plt.rcParams['ytick.labelsize'] = 12

# Where to save the figures
PROJECT_ROOT_DIR = "."

import pandas as pd

def load_data():
    csv_path = os.path.join(".", "assign1_data.txt")
    return pd.read_csv(csv_path, sep="\s+")

def load_data():
    csv_path = os.path.join(".", "assign1_data.txt")
    return pd.read_csv(csv_path, sep="\s+")

data = load_data()
data.head()
c0 = data[(data.z==0)]
c1 = data[(data.z==1)]
```

2.0.1 HW data in parameter space with classification indicated by color

The plotted line is the surface of separation given by x2 = -1/w2(w1x1 + w0) where w is the weight vector calculated using perceptron algorithm. As is indicated in the figure the data is not linearly seperable so the perceptron algorithm does not converge, instead it jitters around near the minimum.

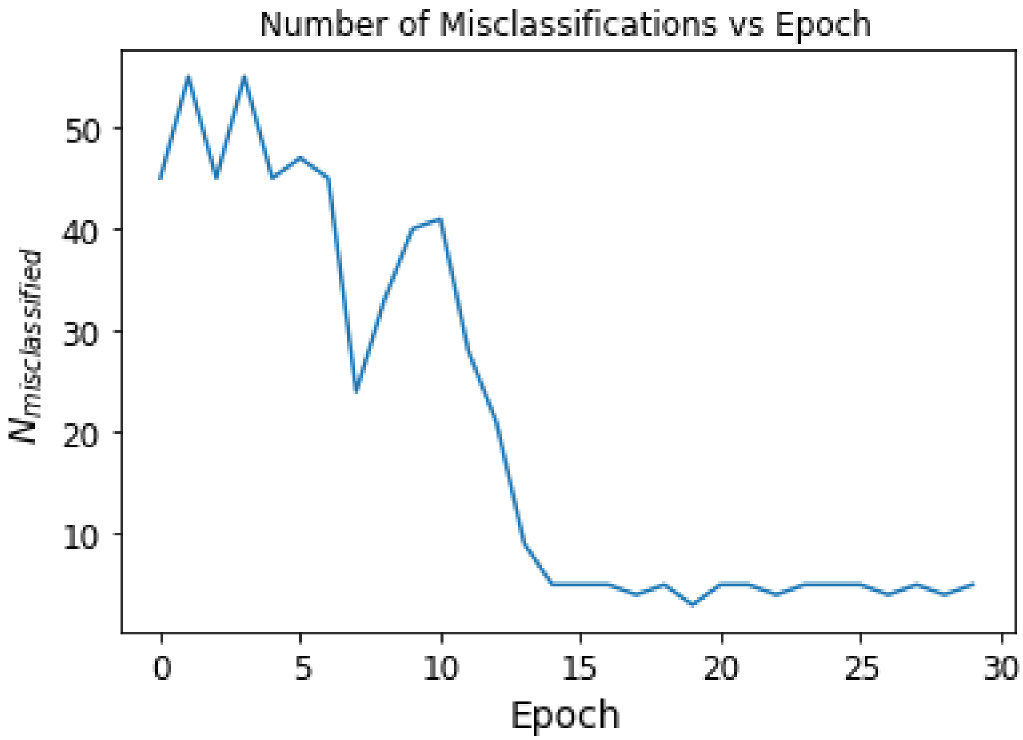
In [6]:
```
X = np.linspace(0, 1, 128, endpoint=True)
W = w_perceptron
L = -(1/W[2])*(W[1]*X + W[0])

ax = c0.plot.scatter(x="x1", y="x2", color='Red', alpha=0.4, label='C0')
c1.plot.scatter( x="x1", y="x2", color='Blue', alpha=0.4, label='C1', ax=ax)
ax.plot(X,L)
plt.title('HW data x1 vs x2 colored by Classification')
```

Out[6]: <matplotlib.text.Text at 0x1169c1a58>



In [7]:
```
plt.ylabel(r'$N_{misclassified}$')
plt.xlabel('Epoch')
plt.title('Number of Misclassifications vs Epoch')
plt.plot(n_mis)
```

Out[7]: [<matplotlib.lines.Line2D at 0x116ae3c18>]



3 Part 4

Plot of performance vs number of training samples below.

```
In [8]: import os as os
        import numpy as np
        import scipy.io
        import scipy.optimize as optimization

        N_EPOCH = 20

        n_mis = []
        def dwp(w,d,X):
            # using numpy einstein summation to vectorize the computation
            # calculating w.x.d -- w.x.d < 0 => incorrect classification
            C = d*np.einsum('i,ij->j', w, X) # shape is (100,)
            XT = np.einsum('ij->ji', X)
            # use enumerate for the equivalent to each_with_index (Ruby)
            xd = [ d[i]*XT[i] for i,c in enumerate(C) if c < 0]
            # sum xd element-wise
            # dw is x.t summed over misclassified teachers
            dw =  np.einsum('ij->j',xd) if(len(xd) > 0) else 0 # shape is (3,)
```

```
            return dw

        def train(n, X, d):
            # use n_sample for training
            X = X[:,0:n]
            d = d[0:n]
            # starting vector of weights
            w = [1,1,1]
            # sweep through the data N_EPOCH times
            for _ in range(N_EPOCH):
                w = w + dwp(w,d,X)

            return w

        def  performance(w,X,d):
            X = X[:,75:]
            d = d[75:]
            C = d*np.einsum('i,ij->j', w, X) # shape is (100,)
            XT = np.einsum('ij->ji', X)
            # d.x.t for misclassified
            xd = [ d[i]*XT[i] for i,c in enumerate(C) if c < 0]
            return len(xd)/25.0

        # invoke main
        def main():
            # data
            filedir = os.path.dirname(os.path.realpath('__file__'))
            datapath = os.path.join(filedir,'assign1_data.mat')
            data = scipy.io.loadmat(datapath)
            x1 = data['x'][:,0]
            x2 = data['x'][:,1]
            # inputs - with bias input tied high, shape is (3,100)
            X = np.array([np.full(len(x1),1),x1,x2])
            # output
            d = np.array(data['z']).flatten()
            # use domain -1,1 for teachers so we can use x.w.d<0 for the classification test
            d = [z if z == 1 else -1 for z in d]
            samples = [5, 10, 25, 50, 75]
            perf = []
            for n in samples:
                w = train(n, X, d)
                p = performance(w, X, d)
                perf.append(1-p)
            return samples, perf

        samples, perf = main()
```

```
In [9]: plt.ylabel('% Correct')
```

```
plt.xlabel(r'$N_{training}$')
plt.title(r'Percent Correctly Classified vs Number Training Samples')
plt.plot(samples, perf)
```

```
Out[9]: [<matplotlib.lines.Line2D at 0x116b02c50>]
```