hw1

September 12, 2017

1 CSCI 5992 - HW1 - Kelly/Milne

1.1 Part 1

1.1.1 Solution

1a) w1 = -2.04427331 w2 = 3.99683416 b = -0.92427055
1b) Levenberg–Marquardt as per implementation below

In [1]:
```
#%%
import os
import numpy as np
import scipy.io
import scipy.optimize as optimization
#from IPython.core.debugger import set_trace

# Data
filedir = os.path.dirname(os.path.realpath('__file__'))
datapath = os.path.join(filedir,'assign1_data.mat')
data = scipy.io.loadmat(datapath)

# fitter wants shape (k,M) (k number of predictors)
# data['x'] has shape (M,k) so we fix that
x1 = data['x'][:,0]
x2 = data['x'][:,1]
X = np.array([x1,x2])
# argg - shape (100,1) - gives obstuse error in the fitter
# we fix that too (flatten)
y = np.array(data['y']).flatten()

# Initial guess
w0 = np.array([1, 1, 1])

# Objective function
# y = w1 * x1 + w2 * x2 + b.
def func(X, b, w1, w2):
  # unpack independent vars
  rval = w1*X[0] + w2*X[1] + b
```

```
        return rval

    result = optimization.curve_fit(func, X, y, w0)
    print(result[0])
```

```
[-0.92427055 -2.04427331  3.99683416]
```

1.2 Part 2

1.2.1 Solution

2a) w1 = -2.0427036 w2 = 3.99155586 b = -0.92233149

2b) Initially we batch processed using the full 100 samples. A step size of 0.05 diverged so used 0.01. Approx N=100 epochs to recover the results from the LM method. Termination was based on plot below of error vs epoch, which amounts to a % change in the error function of ~10^-4 between epochs.

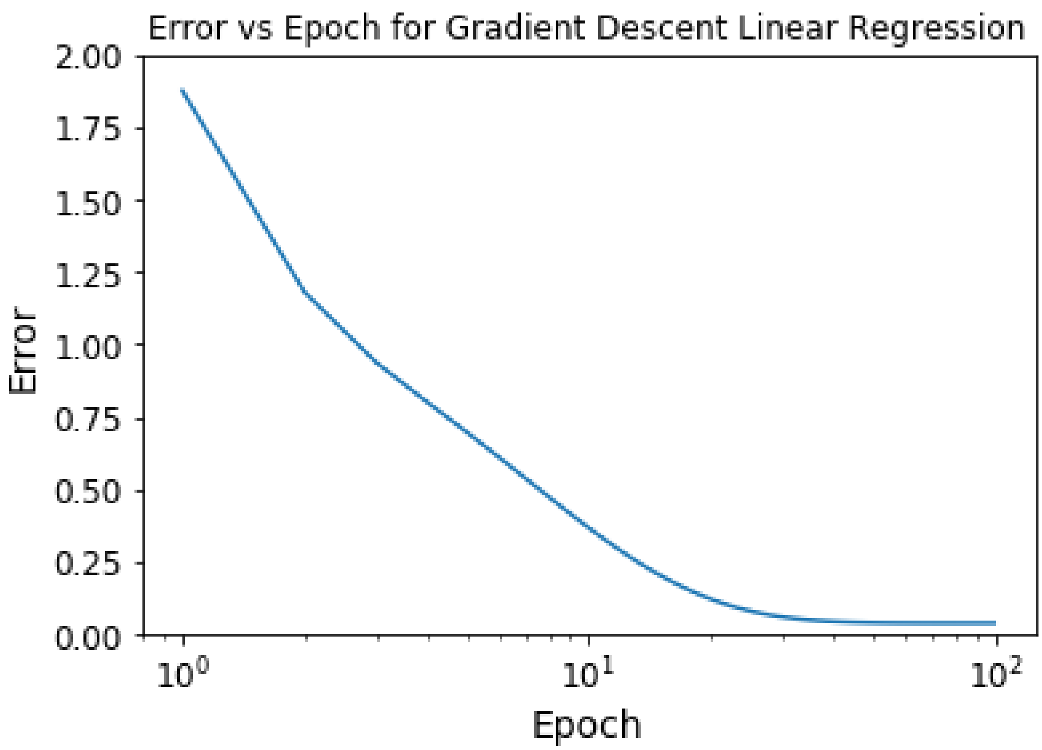
```
In [2]: import os
        import numpy as np
        import scipy.io
        import scipy.optimize as optimization
        #from IPython.core.debugger import set_trace

        EPS = 0.01
        N_EPOCH = 100
        err = []
        def dw(w,d,X):
            # using numpy einstein summation to vectorize the computation
            # vector of coefficients (d-w.x) for each pattern (alpha)
            C = d - np.einsum('i,ij->j', w, X) # shape is (100,)
            e = 1.0/len(C)*np.sum([ c**2 for c in C])
            err.append(e)
            # C_i * X_ji or C * transpose(X)
            dw = EPS*np.einsum('i,ji->j', C, X) # shape is (3,)
            return dw

        def main():
            # data
            filedir = os.path.dirname(os.path.realpath('__file__'))
            datapath = os.path.join(filedir,'assign1_data.mat')
            data = scipy.io.loadmat(datapath)
            x1 = data['x'][:,0]
            x2 = data['x'][:,1]
            # inputs - with bias input tied high, shape is (3,100)
            X = np.array([np.full(len(x1),1),x1,x2])
            # output
            d = np.array(data['y']).flatten()
            # starting vector of weights
```

```
    w = [1,1,1]

    for _ in range(N_EPOCH):
        w = w + dw(w,d,X)
    print(w)

# invoke main
main()
```

```
[-0.92263116 -2.04308226  3.99249859]
```

In [3]:
```
# To plot pretty figures
%matplotlib inline
import matplotlib
import matplotlib.pyplot as plt
plt.rcParams['axes.labelsize'] = 14
plt.rcParams['xtick.labelsize'] = 12
plt.rcParams['ytick.labelsize'] = 12
ax = plt.subplot(111)
plt.semilogx(err)
ax.set_ylim([0, 2])
plt.ylabel('Error')
plt.xlabel('Epoch')
plt.title('Error vs Epoch for Gradient Descent Linear Regression')
```

Out[3]:
```
<matplotlib.text.Text at 0x115214278>
```



We experimented with minibatch size of 25 which converged more smoothly. See attached mini_batch.png

1.3 Part 3

1.3.1 Solution

3a) b = -16. w1=-50.43661604 w2 = 81.86640442

3b) See plot in 'Visualization of Perceptron Results'

In [4]:
```
import os as os
import numpy as np
import scipy.io
import scipy.optimize as optimization
from IPython.core.debugger import set_trace

N_EPOCH = 30

n_mis = []
def dwp(w,d,X):
    # using numpy einstein summation to vectorize the computation
    # calculating w.x.d -- w.x.d < 0 => incorrect classification
    C = d*np.einsum('i,ij->j', w, X) # shape is (100,)
    XT = X.transpose()
    # use enumerate for the equivalent to each_with_index (Ruby)
    xd = [ d[i]*XT[i] for i,c in enumerate(C) if c < 0]
    # for N vs EPOCH plot
    n_mis.append(len(xd))
    # sum xd element-wise
    # dw is x.t summed over misclassified teachers
    dw =  np.einsum('ij->j',xd) if(len(xd) > 0) else 0
    return dw

def main():
    # data
    filedir = os.path.dirname(os.path.realpath('__file__'))
    datapath = os.path.join(filedir,'assign1_data.mat')
    data = scipy.io.loadmat(datapath)
    x1 = data['x'][:,0]
    x2 = data['x'][:,1]
    # inputs - with bias input tied high, shape is (3,100)
    X = np.array([np.full(len(x1),1),x1,x2])
    # output
    d = np.array(data['z']).flatten()
    # use domain -1,1 for teachers so we can use x.w.d<0 for the classification test
    d = [z if z == 1 else -1 for z in d]
    # starting vector of weights
```

```
    w = [1,1,1]
    # sweep through the data N_EPOCH times
    for _ in range(N_EPOCH):
        w = w + dwp(w,d,X)

    return w

# invoke main
w_perceptron = main()
print(w_perceptron)
```

```
[-16.         -50.43661604  81.86640442]
```

2 Visualization of perceptron results

Read the data into a panda dataframe for visualization

In [5]:
```
# Common imports
import numpy as np
import os

# to make this notebook's output stable across runs
np.random.seed(42)

# To plot pretty figures
%matplotlib inline
import matplotlib
import matplotlib.pyplot as plt
plt.rcParams['axes.labelsize'] = 14
plt.rcParams['xtick.labelsize'] = 12
plt.rcParams['ytick.labelsize'] = 12

# Where to save the figures
PROJECT_ROOT_DIR = "."

import pandas as pd

def load_data():
    csv_path = os.path.join(".", "assign1_data.txt")
    return pd.read_csv(csv_path, sep="\s+")

def load_data():
    csv_path = os.path.join(".", "assign1_data.txt")
    return pd.read_csv(csv_path, sep="\s+")

data = load_data()
data.head()
```

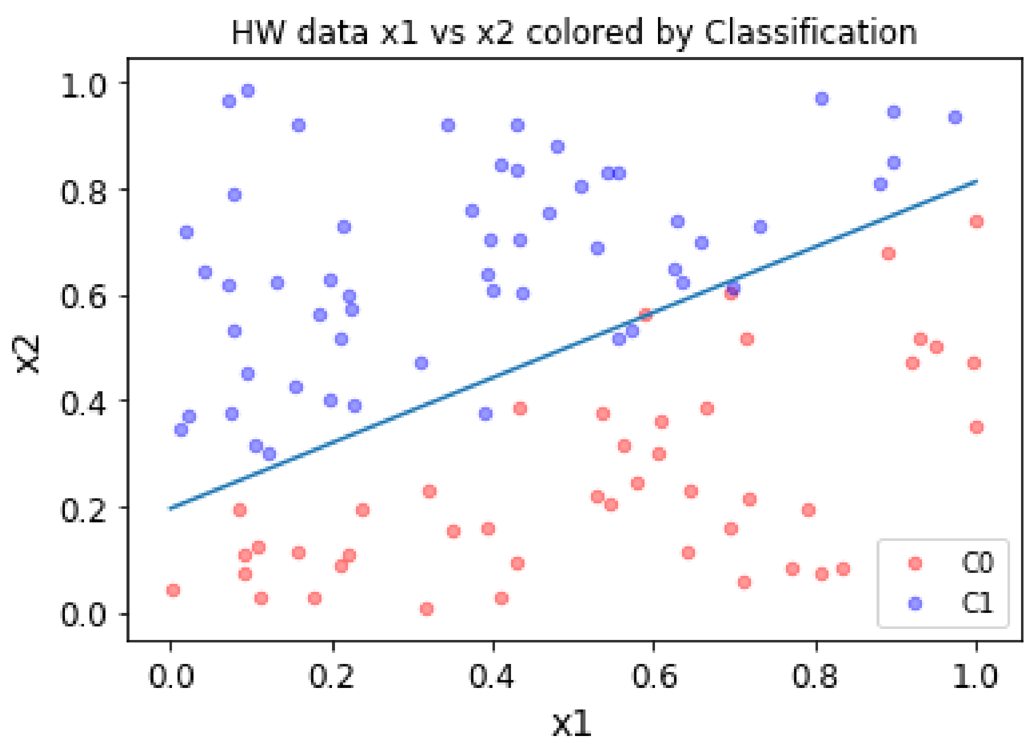
```
c0 = data[(data.z==0)]
c1 = data[(data.z==1)]
```

2.0.1 HW data in parameter space with classification indicated by color

The plotted line is the surface of separation given by x2 = -1/w2(w1x1 + w0) where w is the weight vector calculated using perceptron algorithm. As is indicated in the figure the data is not linearly seperable so the perceptron algorithm does not converge, instead it jitters around near the minimum.

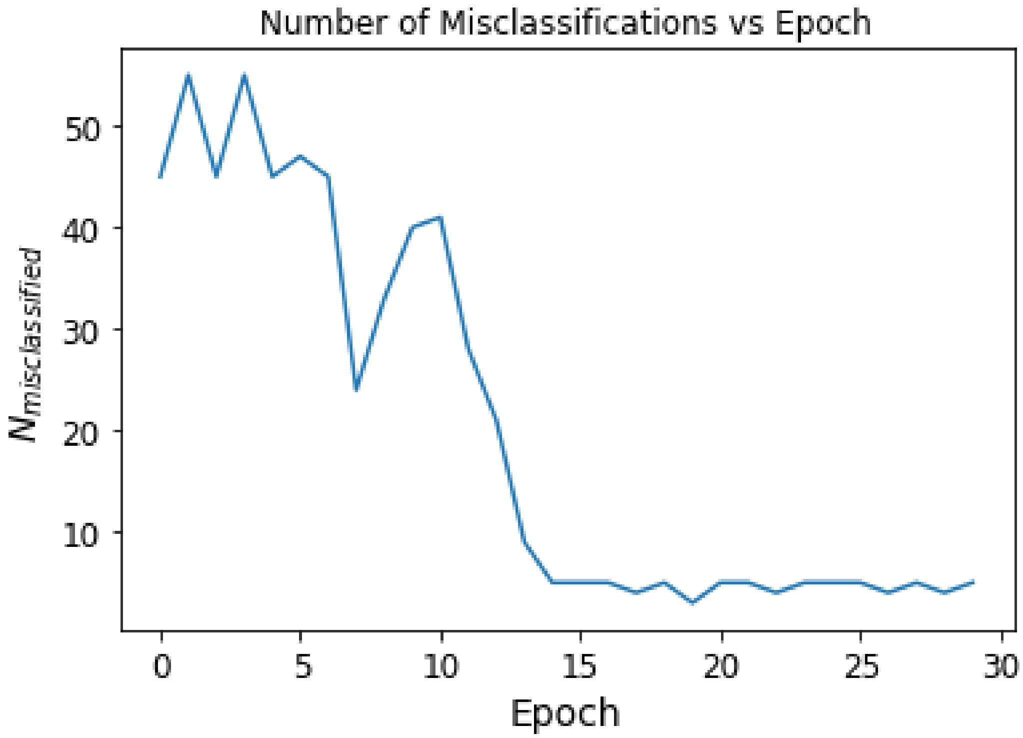
```
In [6]: X = np.linspace(0, 1, 128, endpoint=True)
        W = w_perceptron
        L = -(1/W[2])*(W[1]*X + W[0])

        ax = c0.plot.scatter(x="x1", y="x2", color='Red', alpha=0.4, label='C0')
        c1.plot.scatter( x="x1", y="x2", color='Blue', alpha=0.4, label='C1', ax=ax)
        ax.plot(X,L)
        plt.title('HW data x1 vs x2 colored by Classification')
```

```
Out[6]: <matplotlib.text.Text at 0x1169c1a58>
```




```
In [7]: plt.ylabel(r'$N_{misclassified}$')
        plt.xlabel('Epoch')
        plt.title('Number of Misclassifications vs Epoch')
        plt.plot(n_mis)
```

```
Out[7]: [<matplotlib.lines.Line2D at 0x116ae3c18>]
```



3 Part 4

Plot of performance vs number of training samples below.

```
In [8]: import os as os
        import numpy as np
        import scipy.io
        import scipy.optimize as optimization

        N_EPOCH = 20

        n_mis = []
        def dwp(w,d,X):
            # using numpy einstein summation to vectorize the computation
            # calculating w.x.d -- w.x.d < 0 => incorrect classification
            C = d*np.einsum('i,ij->j', w, X) # shape is (100,)
            XT = np.einsum('ij->ji', X)
            # use enumerate for the equivalent to each_with_index (Ruby)
            xd = [ d[i]*XT[i] for i,c in enumerate(C) if c < 0]
            # sum xd element-wise
            # dw is x.t summed over misclassified teachers
```

```
    dw =  np.einsum('ij->j',xd) if(len(xd) > 0) else 0 # shape is (3,)
    return dw

def train(n, X, d):
    # use n_sample for training
    X = X[:,0:n]
    d = d[0:n]
    # starting vector of weights
    w = [1,1,1]
    # sweep through the data N_EPOCH times
    for _ in range(N_EPOCH):
        w = w + dwp(w,d,X)

    return w

def  performance(w,X,d):
    X = X[:,75:]
    d = d[75:]
    C = d*np.einsum('i,ij->j', w, X) # shape is (100,)
    XT = np.einsum('ij->ji', X)
    # d.x.t for misclassified
    xd = [ d[i]*XT[i] for i,c in enumerate(C) if c < 0]
    return len(xd)/25.0

# invoke main
def main():
    # data
    filedir = os.path.dirname(os.path.realpath('__file__'))
    datapath = os.path.join(filedir,'assign1_data.mat')
    data = scipy.io.loadmat(datapath)
    x1 = data['x'][:,0]
    x2 = data['x'][:,1]
    # inputs - with bias input tied high, shape is (3,100)
    X = np.array([np.full(len(x1),1),x1,x2])
    # output
    d = np.array(data['z']).flatten()
    # use domain -1,1 for teachers so we can use x.w.d<0 for the classification test
    d = [z if z == 1 else -1 for z in d]
    samples = [5, 10, 25, 50, 75]
    perf = []
    for n in samples:
        w = train(n, X, d)
        p = performance(w, X, d)
        perf.append(1-p)
    return samples, perf

samples, perf = main()
```

In [9]:
```
plt.ylabel('% Correct')
plt.xlabel(r'$N_{training}$')
plt.title(r'Percent Correctly Classified vs Number Training Samples')
plt.plot(samples, perf)
```

Out[9]:
```
[<matplotlib.lines.Line2D at 0x116b02c50>]
```