Signal Processing and Speech Communication Lab.
Graz University of Technology

AS

Adaptive Systems—Homework Assignment 3

Name(s) Matr.No(s).

The analytical part of your homework (your calculation sheets) **as well as** the MATLAB simulation protocol have to be delivered as **hard copy** to our mailbox at Inffeldgasse 16c, ground floor, no later than **2012/1/27**. Use a printed version of **this entire document** as the title pages and fill in your **name(s) and matriculation number(s)**. Submitting your homework as a LaTeX document can earn you **up to 3 points!**

Your MATLAB programs (`*.m` files) and the simulation protocol (in pdf format!) have to be submitted via **e-mail** to the address `hw2.spsc@tugraz.at` no later than **2012/1/27**. The subject of the e-mail consists of the assignment number and your matriculation number(s) **"Assignment3, MatrNo1, MatrNo2"**. You have to zip (or tar) all your homework files to one single file with the name `Assignment3_MatrNo1_MatrNo2.zip`, e.g., `Assignment3_9833280_9933281.zip`, which has to be attached to the e-mail.

Please make sure that your approaches, procedures, and results are clearly presented. Justify your answers!

Analytical Problem 3.1 (14 Points)—Predictive Encoding of an AR Process

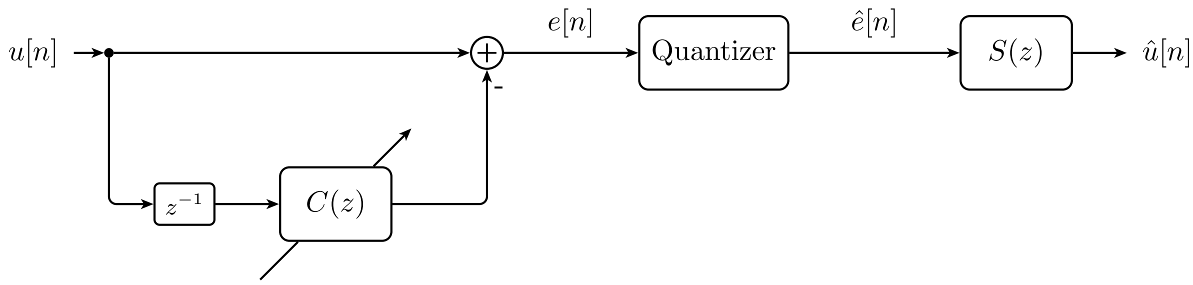


Figure 1: Open-loop prediction prior to quantization.

Let $u[n]$ be samples of an AR process with process generator difference equation

$$u[n] = w[n] + u[n-1] - \frac{1}{8}u[n-2]$$

where $w[n]$ are samples of white, zero-mean, Gaussian noise. The variance of the AR process is known to be $\sigma_u^2 = 1$.

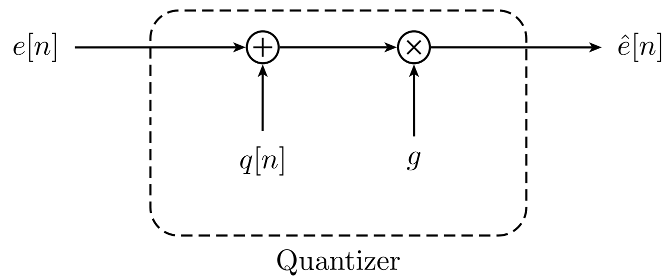
(a) Derive the first three samples of the autocorrelation sequence $r_{uu}[k]$, $k = 0, 1, 2$. Also, compute the variance of the white-noise input, σ_w^2, and the noise gain of the recursive process generator filter, $G_G = \frac{\sigma_u^2}{\sigma_w^2}$.

(b) For above AR process, compute the MSE-optimal linear predictor of zeroth order[1], i.e., $C(z) = c_0$. Also, compute the prediction gain $G_C = \frac{\sigma_u^2}{\sigma_e^2}$. Is the used predictor order optimal for the given source?

(c) For above AR process, compute the MSE-optimal linear predictor of first order, i.e., $C(z) = c_0 + c_1 z^{-1}$. Also, compute the prediction gain $G_C = \frac{\sigma_u^2}{\sigma_e^2}$. Is the used predictor order optimal for the given source?

(d) Specify the transfer function of the synthesis filter $S(z)$ (i.e., the inverse of the prediction-error filter) for both $N = 1$ and $N = 2$. Assume that no quantization was performed. Calculate the noise gain G_S of $S(z)$. Can you use the noise gain of $S(z)$ to compute the variance of $\hat{u}[n]$?

(e) We apply the following model for an ideal[2] quantizer for Gaussian sources:



Here, the quantization noise $q[n]$ is white, uncorrelated with the quantizer input, and has a variance given as

$$\sigma_q^2 = \frac{\sigma_e^2}{2^{2R} - 1}$$

where $R = 1$ is the bit rate. The gain inside the quantizer is given by

$$g = \frac{\sigma_f^2}{\sigma_f^2 + \sigma_{q^\perp}^2}.$$

For both $N = 1$ and $N = 2$ determine the variance σ_r^2 of the reconstruction error $r[n] = \hat{u}[n] - u[n]$ at the synthesis filter output.

(f) Compare above reconstruction error with the distortion achieved by direct encoding of $u[n]$ with 1 bit/sample (i.e., without prediction). Is there any gain by using prediction?

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[1] Note that the overall prediction filter – including the delay element and the direct path depicted in Fig. 1 – is a first-order filter.

[2] according to its rate-distortion performance

MATLAB Problem 3.2 (5 Points)—BONUS: Information vs. Energy

A measure of how much *information* about the original process $u[n]$ is available after quantization is given by the *entropy rate* of the quantized process $\hat{e}[n]$. This entropy rate is given as

$$\bar{H}(\hat{e}) = \lim_{n\to\infty} H(\hat{e}[n]|\hat{e}[n-1], \hat{e}[n-2], \dots)$$

where $H(\hat{e}[n]|\hat{e}[n-1], \hat{e}[n-2], \dots)$ is the conditional entropy of the current sample $\hat{e}[n]$ conditioned on the entire past (it more less tells you how many bits you do not know about the current sample given you know all previous ones). Entropy is non-increasing by conditioning, i.e.,

$$H(\hat{e}[n]) \geq H(\hat{e}[n]|\hat{e}[n-1]) \geq H(\hat{e}[n]|\hat{e}[n-1], \hat{e}[n-2]) \geq \dots$$

(the more samples of the past you know, the more you can guess about the current sample). If further $\hat{e}[n]$ is a white process, i.e., subsequent samples are statistically independent, then $\bar{H}(\hat{e}) = H(\hat{e}[n])$, i.e., knowing previous samples does not tell you anything about the current one.

We now evaluate the effect of linear prediction on the information that can be transported after quantization. To this end, some scripts are made available on the Website:

- `BGsQuantizer.m`: This function quantizes the input data with a B-bit quantizer (input parameter), where it is assumed that the maximum and minimum input values fall together with the maximum and minimum representable numbers. Take a look at the implemented functionality!

- `CondEntropy.m`: This function estimates the entropy $H(\hat{e}[n])$ and the conditional entropies depicted above (which are all upper bounds on the entropy rate).

(a) Implement the system depicted in Fig. 1 *without* the synthesis filter $S(z)$. Use the quantizer function provided on the website and vary the bit count B of the quantizer. Be sure to simulate at least 10^5 samples for quantizers with up to $B = 5$ bits!

(b) Compute (upper bounds) on the entropy rate of the process $\hat{e}[n]$ for first-order ($N = 2$) and zeroth-order ($N = 1$) predictor filters and without any predictor filter ($N = 0$). Plot the bounds for different numbers of bits B.

(c) What can you observe? Does linear prediction have any beneficial effects? What happens to the entropy rate in the case of ideal prediction?

Analytical Problem 3.3 (10 Points)—Least Squares IIR Identification

AR modeling (as in Problem 3.1) can be seen as a blind IIR system identification problem. Blind means that only the system output is observable. For the system input only statistical properties are assumed. On the other hand, in a regular (non-blind) system identification task also the input signal is observable. In this problem we will identify the IIR system

$$u[n] = w[n] + u[n-1] - \frac{1}{8}u[n-2],$$

from given input/output measurements. Assume M pairs of consecutive input/output samples are given.

The task is to determine the parameters of the general linear difference equation

$$u[n] = \sum_{k=0}^{N_b} b_k v[n-k] - \sum_{k=1}^{N_a} a_k u[n-k]. \tag{1}$$

The overall number of parameters is $N = N_a + N_b + 1$. Assume $M \geq N$.

(a) Derive a general LS solution (in matrix/vector notation) for the parameters $\{a_k\}$ and $\{b_k\}$. Hints:

1. Stack the parameters into a single vector $\mathbf{c}^\mathsf{T} = [\mathbf{b}^\mathsf{T}, -\mathbf{a}^\mathsf{T}]$.
2. Rewrite Equ. (1) as an inner product. For this you have to introduce a combined 'tap input/output vector' (i.e., input delay line together with output delay line).
3. Continue in the same way as for FIR identification. Carefully define your vectors and matrices.

(b) The input sequence is $\{v[n]\} = \{1, 0, 0\}$. Compute the corresponding output sequence $\{u[n]\}$ of the given recursive system (evaluate the difference equation for some iterations; the systems are initially at rest). We select $N_a = 1$ and $N_b = 0$ (i.e., we only consider the coefficients b_0 and a_1). Compute the coefficients.

(c) Compare your solution with the synthesis filter of the zeroth-order predictor of Problem 3.1. Note, in both solutions we have undermodeling. but the effect of it is different. Compare the obtained coefficients and the corresponding frequency responses and their approximation quality (you can try different systems and different model orders).

MATLAB Problem 3.4 (10 Points)—Periodic Interference Cancelation

On the course web page you will find a wave file (8 kHz sampling rate) that contains speech with interfering DTMF signals. Implement an interference cancelation *system of your choice* in MATLAB that removes/attenuates the interference. The only requirement the system has to satisfy is a maximum input-to-output delay of 25 ms (i.e., $\Delta \leq 25$ ms). For evaluation, we provide you with the clean speech signal $s[n]$ (which, of course, is NOT allowed to be used for interference cancelation). As a performance metric of your system you shall use the SNR:

$$\mathrm{SNR} = \frac{\sigma_s^2}{\sigma_r^2}$$

where $r[n]$ is any error on the output signal $\hat{s}[n]$ of your system ($r[n] = s[n-\Delta] - \hat{s}[n]$).

The team that delivers the system that achieves the best SNR wins a prize!