$U_0$ : use average speed of phugoid
$Q = \frac{1}{2}\rho U_0^2$ : Use max climb height from FTD

## Dutch Roll

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{r} \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta}{U_0} & -\left(1 - \frac{Y_r}{U_0}\right) \\ N_\beta & N_r \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta r \end{bmatrix}$$

Need to create a function with just A/C data in.

$$Y_\beta = \frac{QSC_{Y_\beta}}{m} \qquad N_\beta = \frac{C_{n_\beta} QS}{I_z}$$

$$I_z = \iiint (x^2 + y^2)\, \delta m$$
(P100)

$$Y_r = \frac{QSbC_{y_r}}{2mU_0} \qquad N_r = \frac{QSb^2 C_{n_r}}{2I_z U_0}$$

Distance from CG to vertical tail aerodynamic chord.

$$C_{y_r} = -2C_{y_{\beta_{tail}}} \frac{l_v}{b} \qquad C_{n_\beta} \simeq 2C_{y_{\beta_{tail}}} \left(\frac{l_v}{b}\right)^2 \text{ (P120)}$$

Wing Span

(P118)

$$-C_{y_{\beta_{tail}}} = C_{L_{\alpha_v}} \eta_v \frac{S_v}{S} \qquad \simeq -2\eta_v V_v \left(\frac{l_v}{b}\right) C_{L_{\alpha_v}}$$

(P121) $C_{y_\beta} = -\eta \frac{S_v}{S} C_{L_{\alpha_v}} \left(1 + \frac{d\sigma}{d\beta}\right)$ $\qquad C_{n_\beta} = C_{n_{\beta_{wf}}} + \eta_v V_v C_{L_{\alpha_v}} \left(1 + \frac{d\sigma}{d\beta}\right)$

vertical tail area

Efficiency factor of the vertical tail

wing area

??

Change in sidewash angle with change in sideslipe angle

??

Verticial tail volume ratio

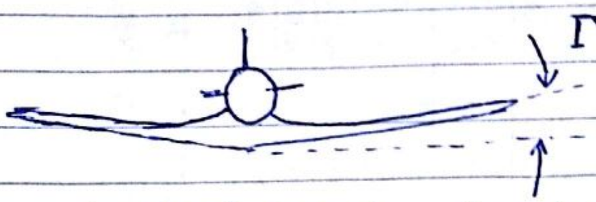
(P122) $\eta = \frac{Y_1}{b_w/2}$

(P111) $C_{D_u} = M\frac{\partial C_D}{\partial M}$ = can be estimated from $C_D$ plot vs Mach No.

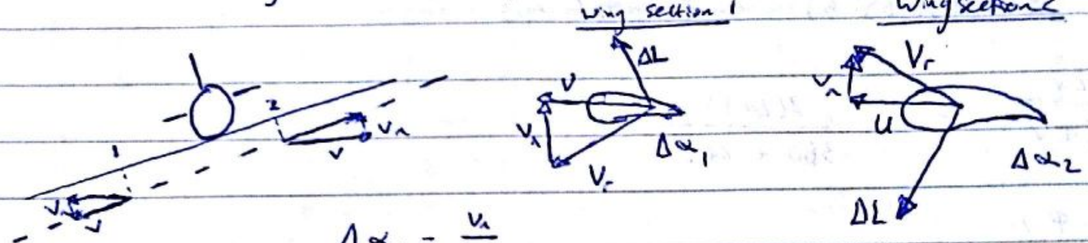
$$C_{L_u} = \frac{M^2}{1-M^2} C_{L_0}$$

## Roll Stability Pg 78

$$\Delta\alpha = \frac{v_n}{u} \rightarrow v_n = V \sin\Gamma$$




$v_n$ = Normal component of side velocity, $V$ = velocity due to sideslip, $U$ = Forward velocity, $V_r$ = Resultant velocity

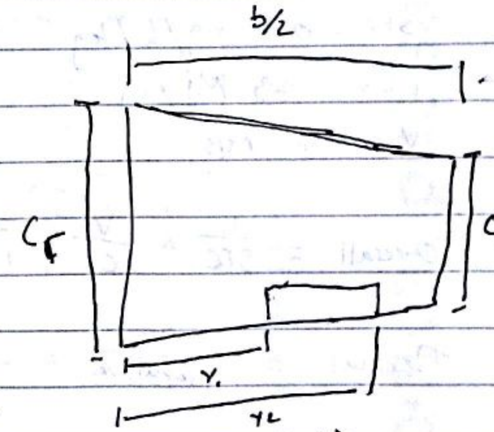


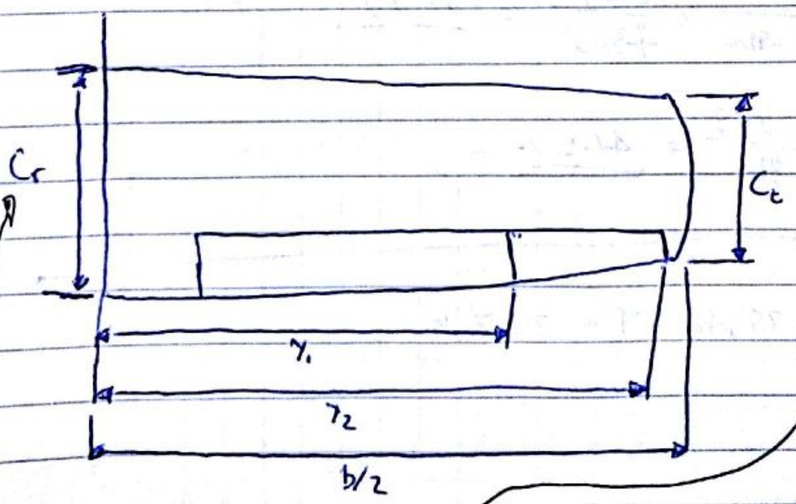

$$\Delta\alpha_1 = \frac{v_n}{u}$$

$\beta = \frac{v}{u}$ but $v_n = v\Gamma$ $\therefore$ $\Delta\alpha_1 = \beta\Gamma$, $\Delta\alpha_2 = \beta\Gamma$

$\therefore \beta = \frac{v}{u}$

Pg 83. $$C_{l_{\delta_a}} = \frac{2 C_{L_{\alpha w}} \tau}{Sb} \int_{y_1}^{y_2} c\,y\,dy$$




$$\tau = \frac{2(W/S)}{C_L \rho U_0 g}$$

$$C_{l_{\delta_a}} = \frac{2 C_{L_{\alpha w}} \tau C_r}{Sb}\left[\frac{y^2}{2} + \left(\frac{\lambda - 1}{b/2}\right)\frac{y^3}{3}\right]_{y_1}^{y_2}$$

$y = y_2$ and $y_1$

$$C_{l_{\delta_a}} = \frac{2 C_{L_{\alpha w}} \tau C_r}{Sb}\left[\left(\frac{y_2^2}{2} + \left(\frac{\lambda-1}{b/2}\right)\frac{y_2^3}{3}\right) - \left(\frac{y_1^2}{2} + \left(\frac{\lambda-1}{b/2}\right)\frac{y_1^3}{3}\right)\right]$$

NACA 63A418

## Phugoid

$$X_u = -(C_{D_u} + 2C_{D_0})QS/(U_0 m)$$

$$Z_u = -(C_{L_u} + 2C_{L_0})QS/(U_0 m)$$

$$\omega_{n_p} = \sqrt{\frac{-Z_u g}{U_0}} \qquad \zeta_p = \frac{-X_u}{2\omega_{n_p}}$$

Period $= \frac{2\pi}{\omega}$

$t_{1/2} = \frac{0.69}{|\eta|}$

$N_{1/2} = 0.110 \frac{\omega_{n_p}}{|\eta|}$

$$\lambda_{1,2} = \underbrace{-\zeta_p \omega_{n_p}}_{\eta} \pm i \underbrace{\omega_{n_p}\sqrt{1-\zeta_p^2}}_{\omega}$$

## Short Period

$$X_w = -(C_{D_\alpha} - C_{L_0})QS/(U_0 m)$$

$$Z_w = -(C_{L_\alpha} + C_{D_0})QS/(U_0 m)$$

$$M_w = C_{M_\alpha} QS\bar{c}/(U_0 I_y)$$

$$M_{\dot{w}} = C_{M_{\dot{\alpha}}} \frac{\bar{c}}{2U_0} QS\bar{c}/(U_0 I_y)$$

(Pg 125)

$$C_{M_\alpha} = C_{L_{\alpha_w}}\left(\frac{x_{cg}}{c} - \frac{x_{ac}}{c}\right) + C_{M_{\alpha_{fus}}} - \eta V_H C_{L_{\alpha_t}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

$$C_{M_{\dot{\alpha}}} = C_{Z_{\dot{\alpha}}} \frac{l_t}{\bar{c}} \qquad \frac{d\varepsilon}{d\alpha} = \frac{2C_{L_{\alpha_w}}}{\pi AR_w}$$

$$C_{Z_{\dot{\alpha}}} = -2C_{L_{\alpha_t}} \eta V_H \frac{d\varepsilon}{d\alpha} \qquad V_H = \frac{l_t S_t}{S\bar{c}}$$

$$M_q = C_{M_q} \frac{\bar{c}}{2U_0} QS\bar{c}/I_y$$

$$C_{M_q} = C_{Z_q} \frac{l_t}{\bar{c}}$$

$$C_{Z_q} = -2C_{L_{\alpha_t}} \eta V_H$$

$Z_\alpha = V_0 Z_w$

$M_\alpha = V_0 M_w$ Pg 159.

$M_{\dot\alpha} = V_0 M_{\dot w}$

$$\omega_{n_{sp}} = \sqrt{\frac{Z_\alpha M_q}{V_0} - M_\alpha} =$$

$$\zeta_{sp} = \left(M_q + M_{\dot\alpha} + \frac{Z_\alpha}{V_0}\right) \Big/ \left[2\omega_{n_{sp}}\right]$$

$$\lambda_{1,2_{sp}} = \underbrace{-\zeta_{sp}\,\omega_{n_{sp}}}_{\eta} \pm \underbrace{I\,\omega_{n_{sp}}\sqrt{1-\zeta_{sp}^2}}_{\omega}$$

$$\text{Period} = \frac{2\pi}{\omega}$$

$$t_{1/2} = \frac{0.69}{|\eta|}$$

$\eta$ = No. of cycles to half amplitude

$$N_{1/2} = 0.110\,\frac{\omega_{n_{sp}}}{|\eta|}$$

$C_{L_0}$ + $C_{D_0}$ the point where they cross

Plot $C_L$ + $C_D$ NO. in Matlab and find the point of intersections

RE No.
Mach
Density

} need to add into initial data

might not need

Dutch roll equation Pg 210

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{r} \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta}{u_0} & -\left(1 - \frac{Y_r}{u_0}\right) \\ N_\beta & N_r \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta r \end{bmatrix} + \begin{bmatrix} \frac{Y_{\delta_r}}{u_0} \\ N_{\delta_r} \end{bmatrix} \Delta\delta_r$$

$$\Delta\delta_r = -k\,\Delta\dot{\psi}$$

$$Y_{\delta_r} = \frac{QS\,C_{y_{\delta_r}}}{m} \qquad N_{\delta_r} = \frac{QSb\,C_{n_{\delta_r}}}{I_z}$$

---

Dutch roll - Characteristic equation

$$\lambda^2 - \left(\frac{Y_\beta + u_0 N_r}{u_0}\right)\lambda + \frac{Y_\beta N_r - N_\beta Y_r + u_0 N_\beta}{u_0} =$$

$$\omega_{n_{DR}} = \sqrt{\frac{Y_\beta N_r - N_\beta Y_r + u_0 N_\beta}{u_0}}$$

$$\zeta_{DR} = -\frac{1}{2\omega_{n_{DR}}}\left(\frac{Y_\beta + u_0 N_r}{u_0}\right)$$

$\lambda_{DR}^2 =$ Number $\pm i$ number

# SIM ~~Calculator~~ equations

(Pg 59)

$$C_{M_{\alpha_t}} = -\eta V_H C_{L_{\alpha t}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

$$C_{n_{\beta_{wf}}} = -k_n k_{Rl} \frac{S_{fs} l_f}{S_w b} \quad \text{(Pg 74)}$$

$k_n$ = ~~empirical~~ empirical wing-body interference factor that is a function of the fuselage geometry

$k_{Rl}$ = empirical correction factor that is a function of the fuselage Reynolds number

$S_{fs}$ = the projected side area of the fuselage

$l_f$ = length of the fuselage

(* Need Graph from Pg 75 in Nelson!!!)

Need for $C_{n_\beta}$ equations which factors into $N_\beta$ equation

$\delta_a$ = aileron deflection

$\delta_s$ = spoiler deflection

$$\tau = \frac{d\alpha}{d\delta_a} \Rightarrow \tau = -\frac{u_0}{Z_\alpha} \quad \text{or} \quad \tau = \frac{2(V_3)}{C_{L_\alpha}\rho u_0 g} \quad \text{(Pg 225)}$$

$$Z_\alpha = \frac{-C_{L_\alpha} QS}{m}$$

$$C_{M_u} = \frac{\partial C_m}{\partial M} M$$

(Pg 122) Empirical factor for $C_{n_{\delta_a}}$ estimate, k.

(Pg 162) look at table explains stability derivatives.

Roll rate Pg 182.

$$V_H = \frac{L_t S_t}{S\bar{c}} \quad \text{or} \quad V_v = \frac{L_t S_t}{S b}$$

$$\varepsilon = \frac{2C_{L_w}}{\pi AR}$$ Down wash term estimate.

$$\varepsilon = \frac{2C_{L_{w_0}}}{\pi AR}$$

ASSUME wing is constant in reality it is thicker at root and thinner at tip.

$$C_{L_\alpha} = \frac{C_{l_\alpha}}{1 + \left( \frac{C_{l_\alpha}}{\pi AR} \right)}$$

$C_{l_\alpha} = \frac{2\pi}{Rad}$ Pg 145.

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