Electrical and Electronics Engineering
Study Material

Unit I: Electrical Circuits

1.1 Basic Circuit Concepts

1.1.1 Circuit elements

1.1.2 Kirchhoff's Laws

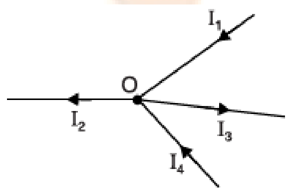
Kirchhoff gave two laws to solve complex circuits, namely ;

1. Kirchhoff's Current Law (KCL) 2. Kirchhoff's Voltage Law (KVL)

1. KIRCHHOFF'S CURRENT LAW (KCL)

This law relates to the currents at the *junctions of an electric circuit and may be stated as under :
The algebraic sum of the currents meeting at a junction in an electrical circuit is zero.

An algebraic sum is one in which the sign of the quantity is taken into account. For example, consider four conductors carrying currents I1, I2, I3 and I4 and meeting at point O as shown in below figure.



If we take the signs of currents flowing towards point O as positive, then currents flowing away from point O will be assigned negative sign. Thus, applying Kirchhoff's current law to the junction O in Figure, we have,

$$(I_1) + (I_4) + (-I_2) + (-I_3) = 0$$

or $$I_1 + I_4 = I_2 + I_3$$

i.e., Sum of incoming currents = Sum of outgoing currents

Hence, Kirchhoff's current law may also be stated as under :

The sum of currents flowing towards any junction in an electrical circuit is equal to the sum of currents flowing away from that junction. Kirchhoff's current law is also called junction rule.

Kirchhoff's current law is true because electric current is merely the flow of free electrons and they cannot accumulate at any point in the circuit. This is in accordance with the law of conservation of charge. Hence, Kirchhoff's current law is based on the law of conservation of charge.

2. KIRCHHOFF'S VOLTAGE LAW (KVL)

This law relates to e.m.fs and voltage drops in a closed circuit or loop and may be stated as under :

In any closed electrical circuit or mesh, the algebraic sum of all the electromotive forces (e.m.fs) and voltage drops in resistors is equal to zero, i.e.,

In any closed circuit or mesh,

Algebraic sum of e.m.fs + Algebraic sum of voltage drops = 0

The validity of Kirchhoff's voltage law can be easily established by referring to the closed loop ABCDA shown in Fig. 2.61. If we start from any point (say point A) in this closed circuit and go back to this point (i.e., point A) after going around the circuit, then there is no increase or decrease in potential. This means that algebraic sum of the e.m.fs of all the sources (here only one e.m.f. source is considered) met on the way plus the algebraic sum of the voltage drops in the resistances must be zero. Kirchhoff's voltage law is based on the law of *conservation of energy, i.e., net change in the energy of a charge after completing the closed path is zero.

Note. Kirchhoff's voltage law is also called loop rule.

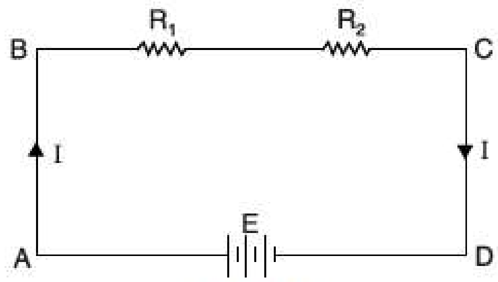


Fig. 2.61

Sign Convention:

While applying Kirchhoff's voltage law to a closed circuit, algebraic sums are considered. Therefore, it is very important to assign proper signs to e.m.fs and voltage drops in the closed circuit. The following convention may be followed :

A **rise in potential should be considered positive and fall in potential should be considered negative.

(i) Thus if we go from the positive terminal of the battery to the negative terminal, there is fall of potential and the e.m.f. should be assigned a negative sign. Thus in Fig. 2.62 (i), as we go from A to B, there is a fall in potential and the e.m.f. of the cell will be assigned a negative sign. On the other hand, if we go from the negative terminal to the positive terminal of the battery or source, there is a rise in potential and the e.m.f should be assigned a positive sign. Thus in Fig. 2.62 (ii) as we go from A to B, there is a rise in potential and the e.m.f. of the cell will be assigned a positive sign. It may be noted that the sign of e.m.f. is independent of the direction of current through the branch under consideration.

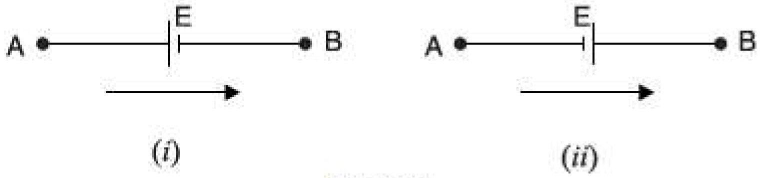


Fig. 2.62

(ii) When current flows through a resistor, there is a voltage drop across it. If we go through the resistor in the same direction as the current, there is a fall in potential because current flows from higher potential to lower potential. Hence this voltage drop should be assigned a negative sign. In Fig.2.63 (i), as we go from A to B, there is a fall in potential and the voltage drop across the resistor will be assigned a negative sign. On the other hand, if we go through the resistor against the current flow, there is a rise in potential and the voltage drop should be given a positive sign. Thus referring to Fig. 2.63 (ii), as we go from A to B, there is a rise in potential and this voltage drop will be given a positive sign. It may be noted that sign of voltage drop depends on the direction of current and is independent of the polarity of the e.m.f. of source in the circuit under consideration.

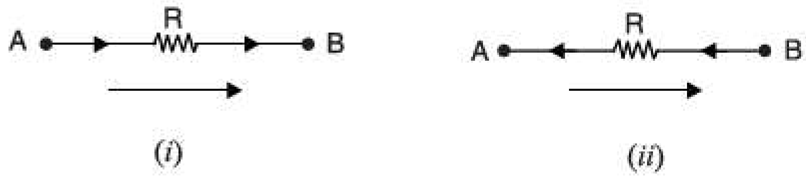


Fig. 2.63

Illustration of Kirchhoff's Laws:

Kirchhoff's Laws can be beautifully explained by referring to Fig. 2.64. Mark the directions of currents as indicated. The direction in which currents are assumed to flow is unimportant, since if wrong direction is chosen, it will be indicated by a negative sign in the result.

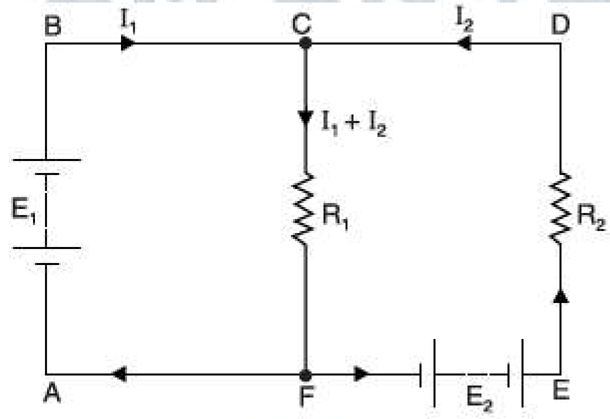


Fig. 2.64

(i) The magnitude of current in any branch of the circuit can be found by applying Kirchhoff's current law. Thus at junction C in Fig. 2.64, the incoming currents to the junction are I1 and I2. Obviously, the current in branch CF will be I1+I2.

(ii) There are three closed circuits in Fig 2.64 viz. ABCFA, CDEFC and ABCDEFA. Kirchhoff's voltage law can be applied to these closed circuits to get the desired equations.

Loop ABCFA. In this loop, e.m.f. E1 will be given positive sign. It is because as we consider the loop in the order ABCFA, we go from −ve terminal to the positive terminal of the battery in the branch AB and hence there is a rise in potential. The voltage drop in branch CF is (I1 + I2) R1 and shall bear negative sign. It is because as we consider the loop in the order ABCFA, we go with current in branch CF and there is a fall in potential. Applying Kirchhoff's voltage law to the loop ABCFA,

$$-(I_1+I_2)R_1+E_1 = 0$$

or $$E_1 = (I_1+I_2)R_1 \quad ...(i)$$

Loop CDEFC. As we go around the loop in the order CDEFC, drop I2R2 is positive, e.m.f. E2 is negative and drop (I1 + I2) R1 is positive. Therefore, applying Kirchhoff's voltage law to this loop, we get,

$$I_2R_2+(I_1+I_2)R_1-E_2 = 0$$

or $$I_2R_2+(I_1+I_2)R_1 = E_2 \quad ...(ii)$$

Since E1, E2, R1 and R2 are known, we can find the values of I1 and I2 from the above two equations. Hence currents in all branches can be determined.

Method to Solve Circuits by Kirchhoff's Laws:

(i) Assume unknown currents in the given circuit and show their direction by arrows.
(ii) Choose any closed circuit and find the algebraic sum of voltage drops plus the algebraic sum of e.m.fs in that loop.
(iii) Put the algebraic sum of voltage drops plus the algebraic sum of e.m.fs equal to zero.
(iv) Write equations for as many closed circuits as the number of unknown quantities. Solve equations to find unknown currents.
(v) If the value of the assumed current comes out to be negative, it means that actual direction of current is opposite to that of assumed direction.
Note. It may be noted that Kirchhoff's laws are also applicable to a.c. circuits. The only thing to be done is that I, V and Z are substituted for I, V and R. Here I, V and Z are phasor quantities.
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1.1.3 Mesh and Nodal Analysis

5.5. NODEL AND MESH ANALYSIS

Nodel Voltage Network Analysis. This method consists in finding the voltags at all the principal modes with reference to zero potential reference mode.

Mesh/Loop Analysis. Here we select closed loops of currents and with loop currents I_1, I_2, I_3 etc. assigned, we apply *KVL* to each current loop.

5.6. TIME AND FREQUENCY DOMAIN RESPONSES

Transient (Time) Response. When a circuit is switched from one condition (OFF) to another (ON), either by a change in applied voltage or a change in one of the circuit elements, there is a transitional period called "transient" during which the branch currents and voltage drops change their former values to new ones.

For *RL* transient $v_R + v_L$

$$v = Ri + L\frac{di}{dt}.$$

For v = constant direct current

$$i = \frac{V}{R} \qquad \left(\because \frac{di}{dt} = 0\right).$$

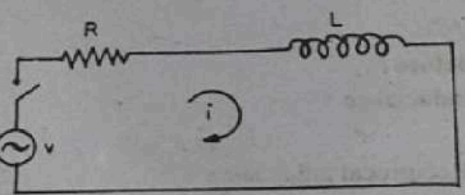


Fig. 3. (a)

but if the conditions are changed, the new steady state is attained through a transient condition.

The instantaneous current for the initial conditions $t = 0$ $i(0) = 0$.

For *RC* transient :

$$i = \frac{V}{R}\left[1 - e^{\left[-\frac{R}{L}\right]t}\right]$$

$$i = \frac{V}{R}e^{-t/RC}.$$

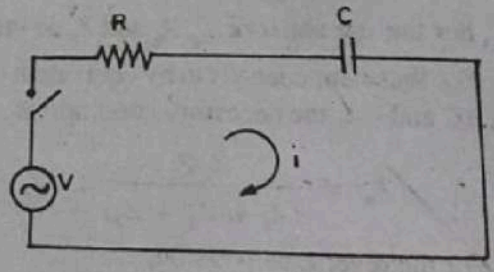


Fig. 3 (b).

Steady State Response (Frequency Domain Response).

It is the response of the circuit after the transient state.

For the sine wave :

(1) $I_{(RMS)} = \frac{I_{max}}{\sqrt{2}}$.

(2) Average value of current over half a cycle $= \frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n}$.

(3) Average value of current $= \frac{\text{Area enclosed over half a cycle}}{\text{Length of base over half a cycle}}$.

(4) Ratio of RMS value to average value = 1.11.

(5) In a resistance R,

$$v = V_m \sin \omega t$$

$$I_m = \frac{V_m}{R}$$

$$V_{RMS} = I_{RMS} \times R.$$

Average power over the whole cycle, $P = V_{RMS} \times I_{RMS}$.

(6) In an inductance $I_{RMS} = \frac{V_{RMS}}{\omega L}$

Instantaneous power $= - = \frac{1}{2} V_m I_m \sin 2\omega t$

(7) In an capacitance, $i = I_m \cos \omega t$

$$p = \frac{1}{2} V_m I_m \sin 2\omega t.$$

1.2 Network Theorems and Applications

1.2.1 Thevenin's Theorem

Fig. 3.82 (i) shows a network enclosed in a box with two terminals A and B brought out. The network in the box may consist of any number of resistors and e.m.f. sources connected in any manner. But according to Thevenin, the entire circuit behind terminals A and B can be replaced by a single source of e.m.f. VTh (called Thevenin voltage) in series with a single resistance RTh (called Thevenin resistance) as shown in Fig. 3.82 (ii). The values of VTh and RTh are determined as mentioned in Thevenin's theorem. Once Thevenin's equivalent circuit is obtained [See Fig. 3.82 (ii)], then current I through any load resistance RL connected across AB is given by ;

$$I = \frac{V_{TH}}{R_{TH} + R_L}$$

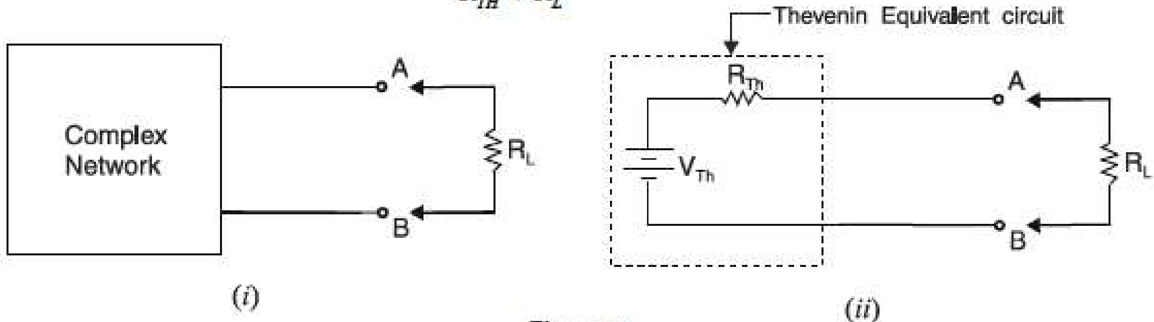


Fig. 3.82

Thevenin's theorem as applied to d.c. circuits is stated below :

Any linear, bilateral network having terminals A and B can be replaced by a single source of e.m.f. VTh in series with a single resistance RTh.

(i) The e.m.f. VTh is the voltage obtained across terminals A and B with load, if any removed i.e. it is open-circuited voltage between terminals A and B.

(ii) The resistance RTh is the resistance of the network measured between terminals A and B with load removed and sources of e.m.f. replaced by their internal resistances. Ideal voltage sources are replaced with short circuits and ideal current sources are replaced with open circuits.

Note how truly remarkable the implications of this theorem are. No matter how complex the circuit and no matter how many voltage and / or current sources it contains, it is equivalent to a single voltage source in series with a single resistance (i.e. equivalent to a single real voltage source). Although Thevenin equivalent circuit is not the same as its original circuit, it acts the same in terms of output voltage and current.

Explanation. Consider the circuit shown in Fig. 3.83 (i). As far as the circuit behind terminals AB is concerned, it can be replaced by a single source of e.m.f. VTh in series with a single resistance RTh as shown in Fig. 3.84 (ii).

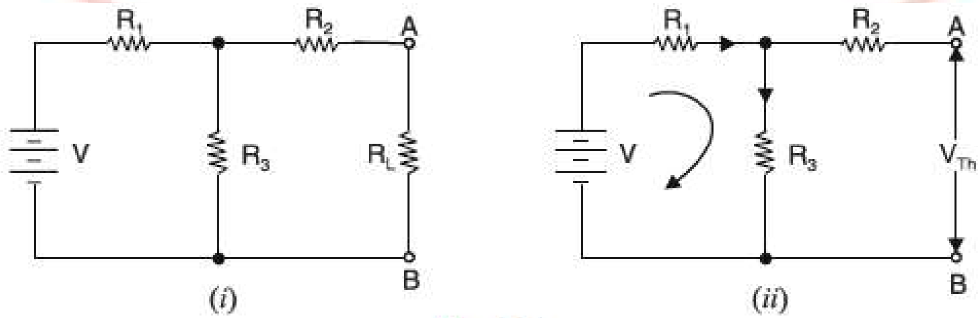
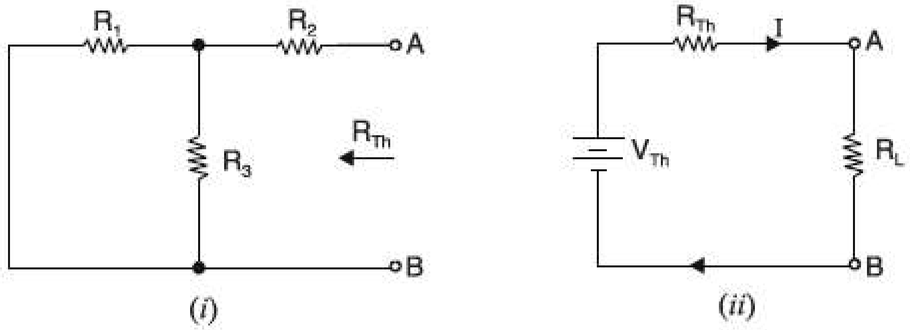


Fig. 3.83

(i) Finding VTh. The e.m.f. VTh is the voltage across terminals AB with load (i.e. RL) removed as shown in Fig. 3.83 (ii). With RL disconnected, there is no current in R2 and VTh is the voltage appearing across R3.

$$\therefore \quad V_{Th} = \text{Voltage across } R_3 = \frac{V}{R_1 + R_3} \times R_3$$



(ii) Finding RTh. To find RTh, remove the load RL and replace the battery by a short-circuit because its internal resistance is assumed zero. Then resistance between terminals A and B is equal to RTh as shown in Fig. 3.84 (i). Obviously, at the terminals AB in Fig. 3.84 (i), R1 and R3 are in parallel and this parallel combination is in series with R2.

$$\therefore \quad R_{Th} = R_2 + \frac{R_1 R_3}{R_1 + R_3}$$

When load R_L is connected between terminals A and B [See Fig. 3.84 (*ii*)], then current in R_L is given by ;

$$I = \frac{V_{Th}}{R_{Th} + R_L}$$

Procedure for Finding Thevenin Equivalent Circuit

(i) Open the two terminals (i.e., remove any load) between which you want to find Thevenin equivalent circuit.

(ii) Find the open-circuit voltage between the two open terminals. It is called Thevenin voltage VTh.

(iii) Determine the resistance between the two open terminals with all ideal voltage sources shorted and all ideal current sources opened (a non-ideal source is replaced by its internal resistance). It is called Thevenin resistance RTh.

(iv) Connect VTh and RTh in series to produce Thevenin equivalent circuit between the two terminals under consideration.

(v) Place the load resistor removed in step (i) across the terminals of the Thevenin equivalent circuit. The load current can now be calculated using only Ohm's law and it has the same value as the load current in the original circuit.

Note. Thevenin's theorem is sometimes called Helmholtz's theorem.

Thevenin Equivalent Circuit:

(Circuits containing both independent and dependent sources)

Sometimes we come across circuits which contain both independent and dependent sources. One such example is shown in Fig. 3.135. The procedure for finding Thevenin equivalent circuit (i.e.finding vTh and RTh) in such cases is as under :

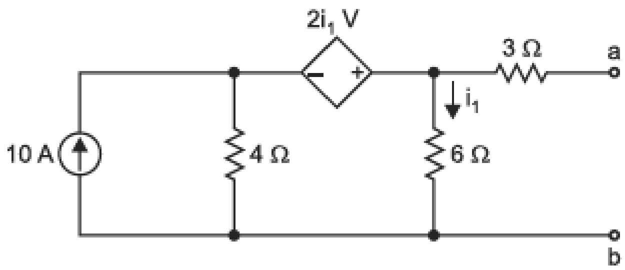


Fig. 3.135

(i) The open-circuit voltage voc (= vTh) at terminals ab is determined as usual with sources present.

(ii) We cannot find RTh at terminals ab simply by calculating equivalent resistance because of the presence of the dependent source. Instead, we place a short circuit across the terminals ab and find the value of short-circuit current isc at terminals ab.

(iii) Therefore, Thevenin resistance *RTh = voc/isc(= vTh/isc). It is the same procedure as adopted for Norton's theorem.

Note. In case the circuit contains dependent sources only, the procedure of finding voc(= vTh) and RTh is as under :

(a) In this case, voc = 0 and isc = 0 because no independent source is present.

(b) We cannot use the relation RTh = voc/isc as we do in case the circuit contains both independent and dependent sources.

(c) In order to find RTh, we excite the circuit at terminals ab by connecting 1A source to the terminals a and b and calculate the value vab. Then RTh = vab/1Ω.

Advantages of Thevenin's Theorem:

The Thevenin equivalent circuit is always an equivalent voltage source (VTh) in series with an equivalent resistance (RTh) regardless of the original circuit that it replaces. Although the Thevenin equivalent is not the same as its original circuit, it acts the same in terms of output voltage and current. It is worthwhile to give the advantages of Thevenin's theorem.

(i) It reduces a complex circuit to a simple circuit viz. a single source of e.m.f. VTh in series with a single resistance RTh.

(ii) It greatly simplifies the portion of the circuit of lesser interest and enables us to view the action of the output part directly.

(iii) This theorem is particularly useful to find current in a particular branch of a network as the resistance of that branch is varied while all other resistances and sources remain constant.

(iv) Thevenin's theorem can be applied in successive steps. Any two points in a circuit can be chosen and all the components to one side of these points can be reduced to Thevenin's equivalent circuit.

1.2.2 Norton's Theorem

Fig. 3.155 (*i*) shows a network enclosed in a box with two terminals *A* and *B* brought out. The network in the box may contain any number of resistors and e.m.f. sources connected in any manner. But according to Norton, the entire circuit behind *AB* can be replaced by a current source I_N in parallel with a resistance R_N as shown in Fig. 3.155 (*ii*). The resistance R_N is the same as Thevenin resistance R_{Th}. The value of I_N is determined as mentioned in Norton's theorem. Once *Norton's equivalent circuit* is determined [See Fig. 3.155 (*ii*)], then current in any load R_L connected across *AB* can be readily obtained.

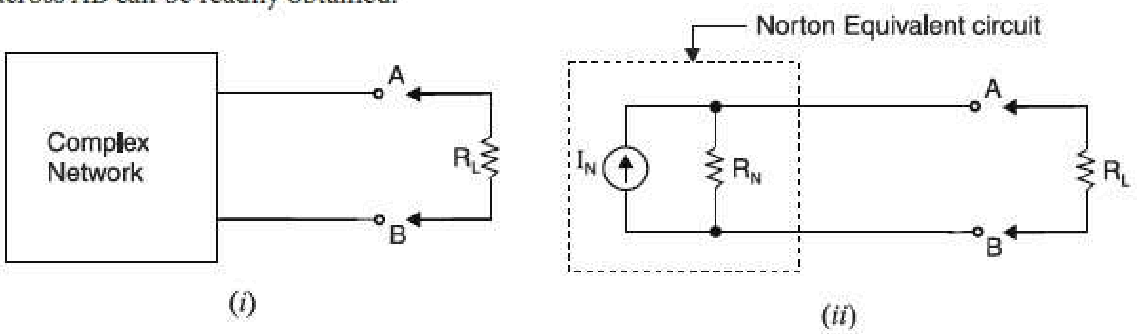


Fig. 3.155

Hence Norton's theorem as applied to d.c. circuits may be stated as under :

Any linear, bilateral network having two terminals A and B can be replaced by a current source of current output I_N in parallel with a resistance R_N.

(*i*) *The output I_N of the current source is equal to the current that would flow through AB when A and B are short-circuited.*

(*ii*) *The resistance R_N is the resistance of the network measured between A and B with load removed and the sources of e.m.f. replaced by their internal resistances. Ideal voltage sources are replaced with short circuits and ideal current sources are replaced with open circuits.*

Norton's Theorem is *converse* of Thevenin's theorem in that Norton equivalent circuit uses a current generator instead of voltage generator and the resistance R_N (which is the same as R_{Th}) in parallel with the generator instead of being in series with it. *Thus the use of either of these theorems enables us to replace the entire circuit seen at a pair of terminals by an equivalent circuit made up of a single source and a single resistor.*

Illustration. Fig. 3.156 illustrates the application of Norton's theorem. As far as the circuit behind terminals AB is concerned [See Fig. 3.156 (*i*)], it can be replaced by a current source I_N in parallel with a resistance R_N as shown in Fig. 3.156 (*iv*). The output I_N of the current generator is equal to the current that would flow through AB when terminals A and B are short-circuited as shown in Fig. 3.156 (*ii*). The load on the source when terminals AB are short-circuited is given by ;

$$R' = R_1 + \frac{R_2 R_3}{R_2 + R_3} = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2 + R_3}$$

$$\text{Source current, } I' = \frac{V}{R'} = \frac{V(R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$\text{Short-circuit current, } I_N = \text{Current in } R_2 \text{ in Fig. 3.156 } (ii)$$

$$= I' \times \frac{R_3}{R_2 + R_3} = \frac{VR_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

To find R_N, remove the load R_L and replace battery by a short because its internal resistance is assumed zero [See Fig. 3.156 (*iii*)].

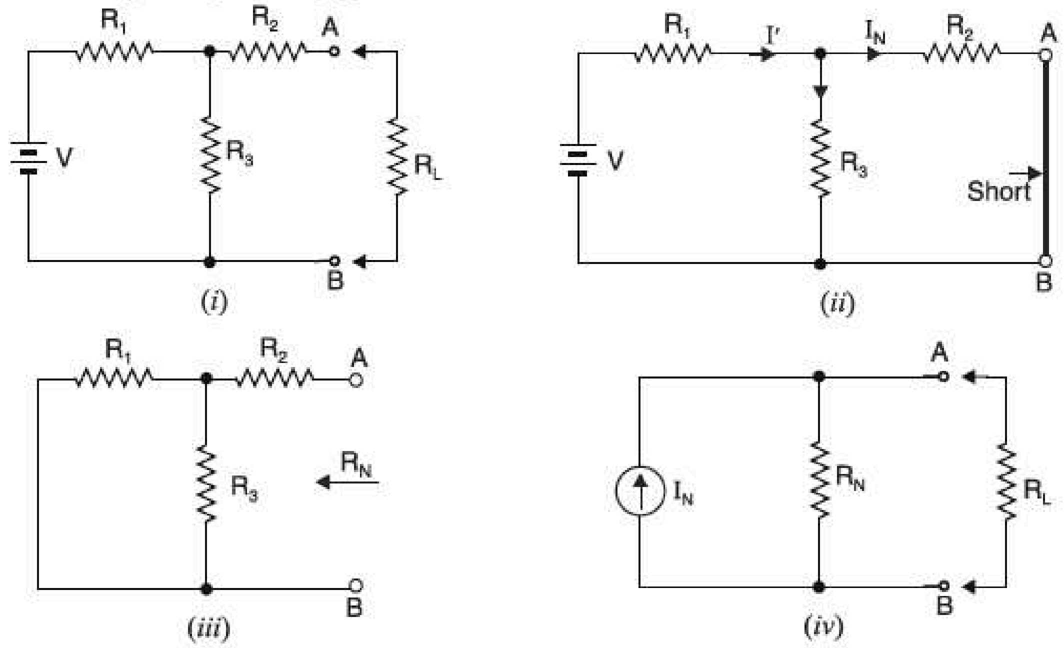


Fig. 3.156

$$\therefore \quad R_N = \text{Resistance at terminals } AB \text{ in Fig. 3.156 } (iii).$$

$$= R_2 + \frac{R_1 R_3}{R_1 + R_3}$$

Thus the values of I_N and R_N are known. The Norton equivalent circuit will be as shown in Fig. 3.156 (*iv*).

Procedure for Finding Norton Equivalent Circuit:

(i) Open the two terminals (i.e. remove any load) between which we want to find Norton equivalent circuit.

(ii) Put a short-circuit across the terminals under consideration. Find the short-circuit current flowing in the short circuit. It is called Norton current IN.

(iii) Determine the resistance between the two open terminals with all ideal voltage sources shorted and all ideal current sources opened (a non-ideal source is replaced by its internal resistance). It is called Norton's resistance RN. It is easy to see that RN = RTh.

(iv) Connect IN and RN in parallel to produce Norton equivalent circuit between the two terminals under consideration.

(v) Place the load resistor removed in step (i) across the terminals of the Norton equivalent circuit. The load current can now be calculated by using current-divider rule. This load current will be the same as the load current in the original circuit.

Norton Equivalent Circuit

(Circuits containing both independent and dependent sources)

Sometimes we come across circuits which contain both independent and dependent sources.

One such example is shown in Fig. 3.190. The procedure for finding Norton equivalent circuit (i.e. finding iN and RN) in such cases is as under :

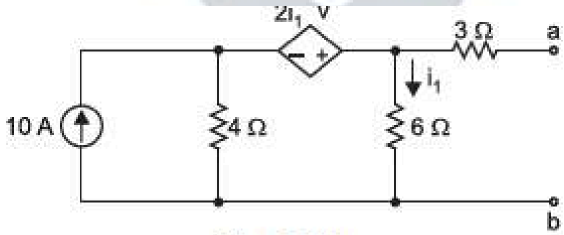


Fig. 3.190

(*i*) The open-circuited voltage $v_{oc}(= v_{Th})$ at terminals *ab* is determined as usual with sources present.

(*ii*) We cannot find R_N $(= R_{Th})$ at terminals *ab* simply by calculating equivalent resistance because of the presence of the dependent source. Instead, we place a short circuit across the terminals *ab* and find the value of short-circuit current i_{sc} $(= i_N)$ at terminals *ab*.

(*iii*) Norton resistance, $R_N = v_{oc}/i_{sc}$ $(= v_{Th}/i_{sc})$.

Note. In case the circuit contains dependent sources *only*, the procedure for finding v_{oc} $(= v_{Th})$ and $R_N (= R_{Th})$ is as under :

(*a*) In this case, $v_{oc} = 0$ and $i_{sc} = 0$ because no independent source is present.

(*b*) We cannot use the relation $R_N = v_{oc}/i_{sc}$ as we do in case the circuit contains both independent and dependent sources.

(*c*) In order to find R_N, we excite the circuit at terminals *ab* by connecting 1A source to the terminals *a* and *b* and calculate the value of v_{ab}. Then $R_N (= R_{Th}) = v_{ab}/1\Omega$.

1.2.3 Superposition Theorem

Superposition is a general principle that allows us to determine the effect of several energy sources (voltage and current sources) acting simultaneously in a circuit by considering the effect of each source acting alone, and then combining (superposing) these effects. This theorem as applied to d.c. circuits may be stated as under :

In a linear, bilateral d.c. network containing more than one energy source, the resultant potential difference across or current through any element is equal to the algebraic sum of potential differences or currents for that element produced by each source acting alone with all other independent ideal voltage sources replaced by short circuits and all other independent ideal current sources replaced by open circuits (non-ideal sources are replaced by their internal resistances).

Procedure. The procedure for using this theorem to solve d.c. networks is as under :

(i) Select one source in the circuit and replace all other ideal voltage sources by short circuits and ideal current sources by open circuits.

(ii) Determine the voltage across or current through the desired element/branch due to single source selected in step (i).

(iii) Repeat the above two steps for each of the remaining sources.

(iv) Algebraically add all the voltages across or currents through the element/branch under consideration. The sum is the actual voltage across or current through that element/branch when all the sources are acting simultaneously.

Note. This theorem is called superposition because we superpose or algebraically add the components (currents or voltages) due to each independent source acting alone to obtain the total current in or voltage across a circuit element.

1.2.4 Maximum Power Transfer Theorem

This theorem deals with transfer of maximum power from a source to load and may be stated as under :

In d.c. circuits, maximum power is transferred from a source to load when the load resistance is made equal to the internal resistance of the source as viewed from the load terminals with load removed and all e.m.f. sources replaced by their internal resistances.

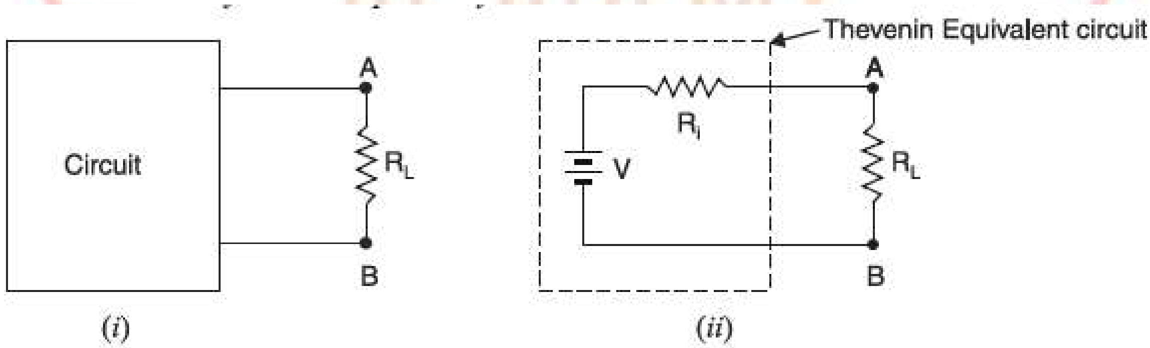


Fig. 3.200

Fig. 3.200 (i) shows a circuit supplying power to a load RL. The circuit enclosed in the box can be replaced by Thevenin's equivalent circuit consisting of Thevenin voltage V = VTh in series with Thevenin resistance Ri(=RTh) as shown in Fig. 3.200 (ii). Clearly, resistance Ri is the resistance measured between terminals AB with RL removed and e.m.f. sources replaced by their internal resistances. According to maximum power transfer theorem, maximum power will be transferred from the circuit to the load when RL is made equal to Ri, the Thevenin resistance at terminals AB.

Proof of Maximum Power Transfer Theorem

Consider a voltage source V of internal resistance Ri delivering power to a load RL. We shall prove that when RL = Ri, the power delivered to RL is maximum. Referring to Fig. 3.201 (i), we have,

$$\text{Circuit current, } I = \frac{V}{R_L + R_i}$$

$$\text{Power delivered to load, } P = I^2 R_L$$

$$= \left(\frac{V}{R_L + R_i}\right)^2 R_L \qquad \ldots(i)$$

For a given source, generated voltage V and internal resistance R_i are constant. Therefore, power delivered to the load depends upon R_L. In order to find the value of R_L for which the value of P is maximum, differentiate eq. (*i*) w.r.t. R_L and set the result equal to zero.

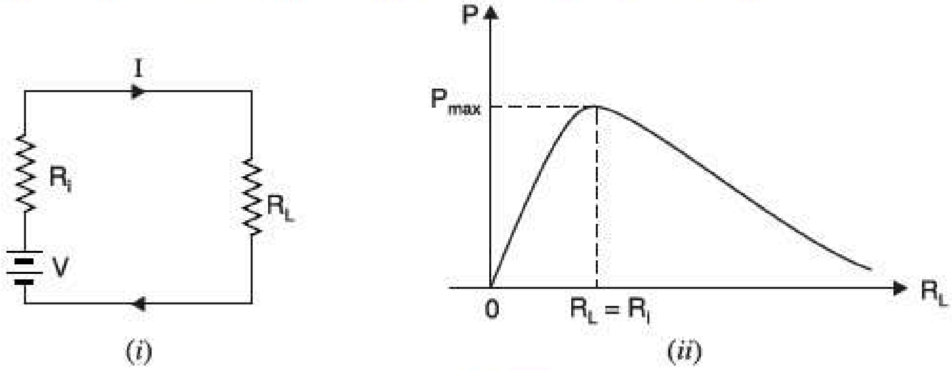


Fig. 3.201

Thus, $$\frac{dP}{dR_L} = V^2\left[\frac{(R_L + R_i)^2 - 2R_L(R_L + R_i)}{(R_L + R_i)^4}\right] = 0$$

or $$(R_L + R_i)^2 - 2R_L(R_L + R_i) = 0$$

or $$(R_L + R_i)(R_L + R_i - 2R_L) = 0$$

or $$(R_L + R_i)(R_i - R_L) = 0$$

Since $R_L + R_i$ cannot be zero,

$\therefore$ $$R_i - R_L = 0$$

or $$R_L = R_i$$

or **Load resistance = Internal resistance of the source**

Thus, for maximum power transfer, load resistance R_L must be equal to the internal resistance R_i of the source. Fig. 3.201 (*ii*) shows the graph between power delivered (P) and R_L. We may extend the maximum power transfer theorem to a linear circuit rather than a single source by means of Thevenin's theorem as under :

The maximum power is obtained from a linear circuit at a given pair of terminals when terminals are loaded by Thevenin's resistance (R_{Th}) of the circuit.

The above statement is obviously true because by Thevenin's theorem, the circuit is equivalent to a voltage source in series with internal resistance (R_{Th}) of the circuit.

Important Points. The following points are worth noting about maximum power transfer theorem :

(*i*) *The circuit efficiency at maximum power transfer is only 50%* as one-half of the total power generated is dissipated in the internal resistance R_i of the source.

$$\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}} = \frac{I^2 R_L}{I^2(R_L + R_i)}$$

$$= \frac{R_L}{2R_L} = \frac{1}{2} = 50\% \qquad (\because R_L = R_i)$$

(*ii*) *Under the conditions of maximum power transfer, the load voltage is one-half of the open-circuited voltage at the load terminals.*

$$\text{Load voltage} = I R_L = \left(\frac{V}{R_L + R_i}\right) R_L = \frac{V R_L}{2 R_L} = \frac{V}{2}$$

(iii) Max. power transferred $= \left(\dfrac{V}{R_L + R_i}\right)^2 R_L = \left(\dfrac{V}{2 R_L}\right)^2 R_L = \dfrac{V^2}{4 R_L}$

Note. In case of a practical current source, the maximum power delivered is given by ;

$$P_{max} = \frac{I_N^2 R_N}{4}$$

where I_N = Norton current

R_N = Norton resistance $(= R_{Th} = R_i)$

Applications of Maximum Power Transfer Theorem

This theorem is very useful in situations where transfer of maximum power is desirable. Two important applications are listed below :

(i) In communication circuits, maximum power transfer is usually desirable. For instance, in a public address system, the circuit is adjusted for maximum power transfer by making load (i.e. speaker) resistance equal to source (i.e. amplifier) resistance. When source and load have the same resistance, they are said to be matched. In most practical situations, the internal resistance of the source is fixed. Also, the device that acts as a load has fixed resistance. In order to make RL = Ri, we use a transformer. We can use the reflected-resistance characteristic of the transformer to make the load resistance appear to have the same value as the source resistance, thereby “fooling” the source into “thinking” that there is a match (i.e. RL = Ri). This technique is called impedance matching.

(ii) Another example of maximum power transfer is found in starting of a car engine. The power delivered to the starter motor of the car will depend upon the effective resistance of the motor and internal resistance of the battery. If the two resistances are equal (as is the case when battery is fully charged), maximum power will be transferred to the motor to turn on the engine. This is particularly desirable in winter when every watt that can be extracted from the battery is needed by the starter motor to turn on the cold engine. If the battery is weak, its internal resistance is high and the car does not start.

Note. Electric power systems are never operated for maximum power transfer because the efficiency under this condition is only 50%. This means that 50% of the generated power will be lost in the power lines. This situation cannot be tolerated because power lines must operate at much higher than 50% efficiency.

1.3 AC and Transient Circuit Analysis

1.3.1 Sinusoidal Steady State Analysis of RL-RC-RLC Circuits

A sinusoid is a signal that has the form of sine or cosine function:

$$\text{Sinusoidal voltage, } V(t) = V_m \sin \omega t$$

Sinusoidal waveforms produce minimum disturbance in electrical circuits during operation. It produces electromagnetic torque which is free of noise and oscillations. It causes less interferences to nearby communication lines.

- **Waveform:** Alternating quantities represented graphically. The shape of the curve obtained by plotting the values of the function at different instants is known as waveform.
- **Period:** The time taken by an alternating quantity to complete one cycle is called time period.
- **Frequency:** The number of cycles completed per second by an alternating quantity is called frequency.

$$f = \frac{1}{T}$$

- **Instantaneous value:** The value of an alternating quantity at any instant of time is known as instantaneous value.
- **Peak value (Crest value):** The maximum value of an alternating quantity attained in each cycle is called peak value.
- **RMS Value:** The RMS (Root Mean Square) also known as the quadratic mean, is a statistical measure of the magnitude of a varying quantity. It is especially valid for positive and negative sinusoids. It can be calculated for a series of discrete values or for a continuously varying function.

$$x_{rms} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2} = \sqrt{\frac{x_1^2 + x_2^2 + \ldots + x_n^2}{n}}$$

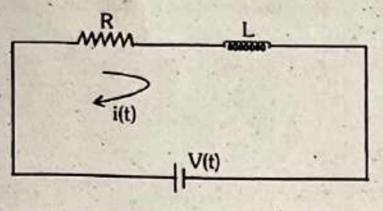
- **Form factor:** It is the ratio of RMS value to average value for an alternating wave.

$$\text{Form factor} = \frac{\text{RMS value}}{\text{Average value}} = 1.11$$

- **Peak factor:** It is the ratio of peak value to the RMS value for an alternating wave.

$$\text{Peak factor} = \frac{\text{Maximum Peak Value}}{\text{RMS Value}}$$

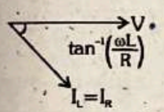
Phasor analysis of RL Series Circuit :



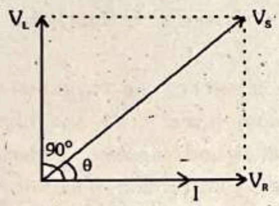
$$V = V_R + V_L$$

$$I = \frac{V_m}{\sqrt{R^2+\omega^2 L^2}} \angle -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

- The current flow is same for both the resistor and inductor.
- The resistor voltage V_R is in phase with the current. The inductor voltage V_L leads the current by 90°. There is a phase difference of 90° between the resistor voltage V_R and the inductor voltave V_L.



(Current Phasor diagram)



$$\text{Impedance, } Z = \sqrt{R^2+X_L^2}$$

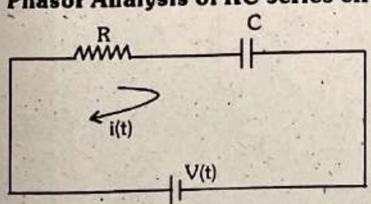
$$\text{Phase angle, } \theta = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$\text{Real power, } P = VI\cos\theta$$

$$\text{Reqactive Power, } Q = VI\sin\theta$$

$$\text{Apparent power, } S = VI$$

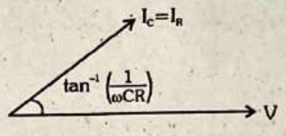
- **Phasor Analysis of RC series circuit :**



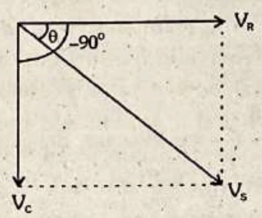
$$V = V_R + V_C$$

$$I = \frac{V}{R+\frac{1}{j\omega c}} \angle \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

The current flow is same for both the resistor and the capacitor.



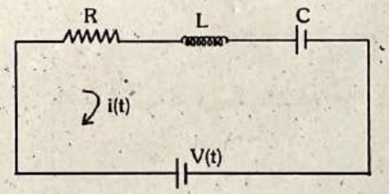
- The resistor voltage V_R is in phase with the current. The capacitor voltage V_C lags the current by 90°. There is a phase difference of 90° between the resistor voltage V_R and capacitor voltave V_C.



$$\text{Impedance, } Z = \sqrt{R^2+X_c^2}$$

$$\text{Phase angle, } \theta = \tan^{-1}\left(\frac{X_c}{R}\right)$$

- **Phasor Analysis of RLC series circuit :**



$$\text{Impedance } Z = \sqrt{R^2+(X_L-X_c)^2}$$

$$X_L = 2\pi fL, \quad X_C = \frac{1}{2\pi fc}$$

The impedance Z is minimum at resonance.

- ✦ At frequency below resonance frequency $X_C > X_L$ and the circuit is capacitive circuit. The current leads the supply voltage.
- ✦ At equal to resonance frequency $X_C = X_L$ and the circuit is purely resistive circuit.
- ✦ At frequency above resonance frequency $X_C < X_L$ and the circuit is inductive circuit. The current lags the supply voltage.

1.3.2 Resonant Circuits

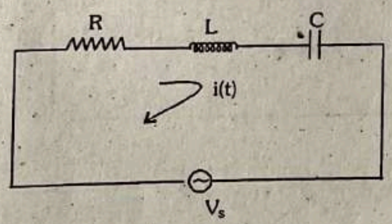
and the circuit is capacitive circuit. The current leads the supply voltage.

- At equal to resonance frequency $X_C = X_L$ and the circuit is purely resistive circuit.
- At frequency above resonance frequency $X_C < X_L$ and the circuit is inductive circuit. The current lags the supply voltage.

RESONANCE

In an electrical circuit, resonance occurs when the inductive reactance and capacitive reactance are of equal magnitude, causing electrical energy to oscillate between the magnetic field of the inductor and the electric field of the capacitor. The circuit under resonance is purely resistive in nature. The resonance circuit also called tuned circuit. The resonance can be determine the operation of communication circuits by causing unwanted sustained and transient oscillations that may cause noise, signal distortion and damage to circuit elements. Resonance occurs when the current flow in an inductor, energy gets stored in magnetic field and when a capacitor is charged, energy gets stored in static electric field.

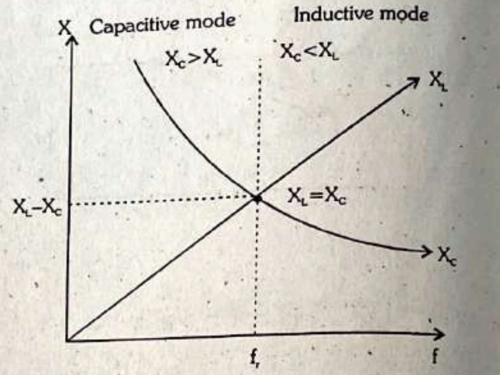
- **The electrical resonance broadly classified in to two categories:**
  - Series Resonance
  - Parallel Resonance
- **Series Resonance :**



R → Resistor

L → Inductor, gets energy stored in magnetic field.

C → Capacitor, gets energy stored in static electric field.



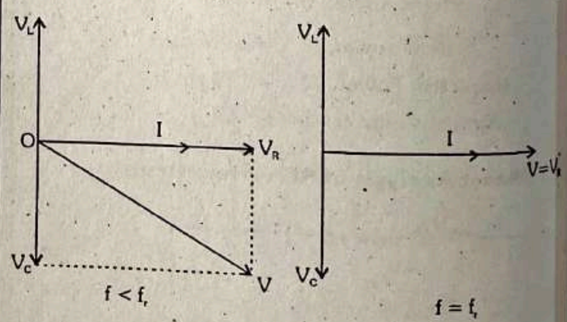
(Inductive Reactance and Capacitive Reactance vs Frequency)

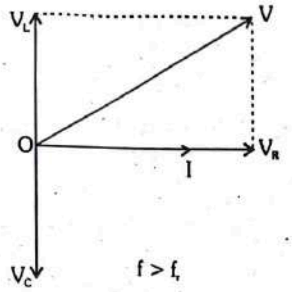
Resonance frequency, $f_r = \dfrac{1}{2\pi\sqrt{LC}}$

Inductive Reactance, $X_L = 2\pi fL$

Capacitive Reactance, $X_C = \dfrac{1}{2\pi fc}$

At resonance, the inductive reactance is equal to capacitive reactance and hence the voltage across inductor and capacitor cancel each other. The total impedance of circuit is resistance only. The phase angle between voltage and current is zero and the power factor is unity.





* When $f<f_r$; the circuit is capacitive circuit and power factor is leading power factor.
* When $f>f_r$; the circuit is inductive circuit and power factor is lagging power factor.
* When $f=f_r$; the circuit is resistive circuit and power factor is unity power factor.
* Frequency at which V_C is maximum

$$f_c = \frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{2L^2}}$$

* Frequency at which V_L is maximum

$$f_L = \frac{1}{2\pi\sqrt{LC-(R^2C^2/2)}}$$

* Resonance frequency

$$f_r = \sqrt{f_1 f_2}$$

* Quality factor,

$$Q = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{\omega_0}{B.W}$$

* Bandwidth,

$$BW = f_2 - f_1 = \frac{f_r}{Q} = \frac{R}{2\pi L}$$

* Lower cutoff frequency,

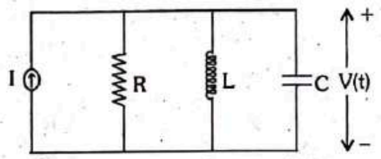
$$f_1 = f_r - \frac{R}{4\pi L}$$

* Upper cutoff frequency,

$$f_2 = f_r + \frac{R}{4\pi L}$$

* The frequencies f_1 and f_2 at which the current reduces 0.707 times the maximum value are called lower and upper cutoff frequencies.

* **Parallel Resonance :**

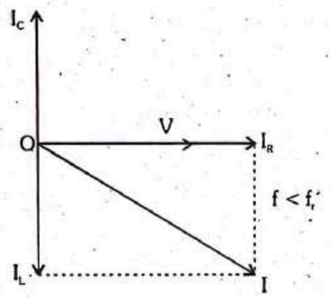


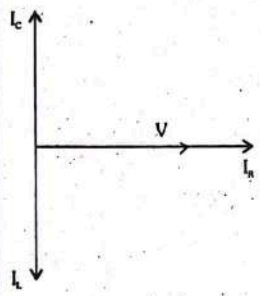
Resonance frequency $f_r = \frac{1}{2\pi\sqrt{LC}}$ at $X_L = X_C$

Inductive reactance, $X_L = 2\pi fL$

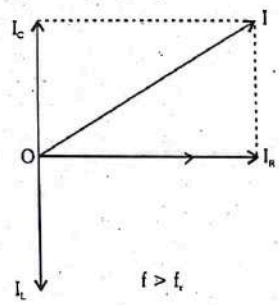
Capacitive Reactance, $X_C = \frac{1}{2\pi fc}$

* At parallel resonance, circuit have a coil and a capacitor in parallel. It is said to resonate when the reactive component of line current I reduces to zero. Here the impedance is maximum and the line current is minimum. The parallel resonance circuit is also called tank circuit.





$f = f_r$



* When $f<f_r$; the circuit is inductive and power factor is lagging.

* When $f > f_r$; the circuit is capacitive and power factor is leading.
* When $f = f_0$; the circuit is resistive and power factor is unity.

Impedance at resonance, $Z = \frac{L}{CR}$

Quality factor, $Q = \frac{\omega_0 L}{R}$

Parameter	Series Resonance	Parallel Resonance
Impedance	minimum	maximum
Current	maximum	minimum
Power factor	unity	unity
Magnitude	voltage	current
Q factor	$\frac{\omega L}{R}$ or $\frac{1}{R}\sqrt{\frac{L}{C}}$	$Q = R\sqrt{\frac{C}{L}}$ or $\frac{R}{\omega L}$

TWO PORT NETWORKS

+ A pair of terminals which an electrical signal may enter or leave a network is called port.
+ A network having only one pair of terminals called one port network.
+ A network with two input terminals and two output terminals called four terminal or two port network.
+ Two port network is used in the construction of filters, attenuators, equalizers and impedance matching network in cascade, series and parallel connection.
+ Two port network parameters,
  - z – parameters or Impedance parameter
  - y – parameter or Admittance parameter
  - h – parameter or Hybrid parameter
  - ABCD parameter
+ Z parameter

$V_1 = Z_{11}I_1 + Z_{12}I_2$ $\quad$ $V_2 = Z_{21}I_1 + Z_{22}I_2$

$Z_{11} = \left|\frac{V_1}{I_1}\right|_{I_2=0}$; $Z_{22} = \left|\frac{V_2}{I_2}\right|_{I_1=0}$

$Z_{12} = \left|\frac{V_1}{I_2}\right|_{I_1=0}$; $Z_{21} = \left|\frac{V_2}{I_1}\right|_{I_2=0}$

+ Y parameter :

$I_1 = Y_{11}V_1 + Y_{12}V_2$

$I_2 = Y_{21}V_1 + Y_{22}V_2$

$Y_{11} = \left|\frac{I_1}{V_1}\right|_{V_2=0}$; $Y_{22} = \left|\frac{I_2}{V_2}\right|_{V_1=0}$

$Y_{12} = \left|\frac{I_1}{V_2}\right|_{V_1=0}$; $Y_{21} = \left|\frac{I_2}{V_1}\right|_{V_2=0}$

+ Hybrid parameter :

$V_1 = h_{11}I_1 + h_{12}V_2$

$I_2 = h_{21}I_1 + h_{22}V_2$

$h_{11} = \left|\frac{V_1}{I_1}\right|_{V_2=0}$; $h_{21} = \left|\frac{I_2}{I_1}\right|_{V_2=0}$

$h_{22} = \left|\frac{I_2}{V_2}\right|_{I_1=0}$; $h_{12} = \left|\frac{V_1}{V_2}\right|_{I_1=0}$

+ ABCD parameter :

$V_1 = AV_2 + B(-I_2)$

$I_1 = CV_2 + D(-I_2)$

$A = \frac{V_1}{V_2}$; $C = \frac{I_1}{V_2}$

$B = \frac{V_1}{-I_2}$; $D = \frac{I_1}{-I_2}$

+ Condition for symmetry and reciprocity

z parameter:

$Z_{11} = Z_{22} \rightarrow$ Symmetry

$Z_{12} = Z_{21} \rightarrow$ Reciprocity

y parameter :

$Y_{11} = Y_{22} \rightarrow$ Symmetry

$Y_{12} = Z_{21} \rightarrow$ Reciprocity

h parameter :

$h_{11}h_{22} - h_{12}h_{21} = 1 \rightarrow$ Symmetry

$h_{12} = -h_{21} \rightarrow$ Reciprocity

ABCD parameter :

$A = D \rightarrow$ Symmetry

$AD - BC = 1 \rightarrow$ Reciprocity

1.3.3 Natural and Forced Response

Source Free Response of RC circuit

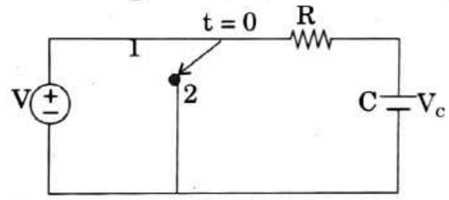


Fig.110

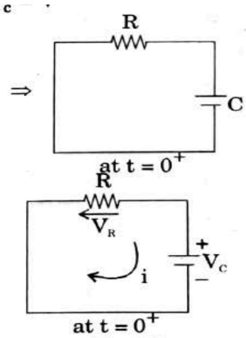


Fig.111

$t = \bar{0}$, $V_c = V$

$t = 0^+$, $V_c = V$

Applying KVL

$V_R + V_C = 0$

$i_R + V_C = 0$

$i = C\frac{dV_C}{dt}$

$R\left[C\frac{dV_C}{dt}\right] + V_C = 0$

$\frac{dV_C}{dt} + \frac{1}{R_C}V_C = 0\left(\frac{di}{dt} + P_i = 0\right)$

- This circuit contains. Source free response, where P.I = 0 and C.F part is there. (C.F → complimentary function)

C.F or source free Response = A. e^{-Pt}

$\frac{dV_C}{dt} + \frac{1}{R_C}V_C = 0$

$V_C(t) = Ae^{-t/RC}$

$V_c(0^+) = A\,e^{-0}$

$V_c(0^+) = A$

$\boxed{V = A}$

$\boxed{V_C(t) = V\,e^{\frac{-t}{RC}}} \Rightarrow V_C(t) = Ve^{-\frac{t}{\tau}}$

RC → Time constant

RC = τ

Let, τ = T,

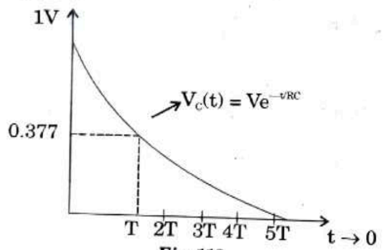


Fig.112

If, τ = 1 → 0.366 V

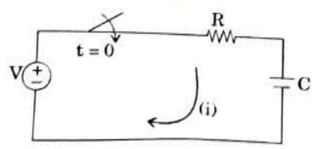
T = 2 → 0.135 V

T = 3 → 0.005 V

T = 4 → 0.001 V

T = 5 → 0.0007 V

Forced Response of RC circuit :



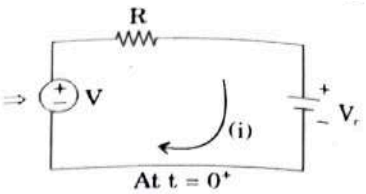


Fig.113

At $t = \bar{0}$; $V_C = 0$

(Capacitors opposes instantaneous changes of voltage)

$t = 0^+$; $V_c = 0$

$V = Ri + V_c = R\left[C\frac{dV_C}{dt}\right] + V_C \quad \because i = c\frac{dV_c}{dt}$

$$\boxed{\frac{dV_C}{dt} + \frac{1}{RC}V_C = \frac{V}{RC}}$$

Now compare this equation with $\frac{di}{dt} + Pi = Q$

Here, $P = \frac{1}{RC}$, $Q = \frac{V}{RC}$

- This is forced excitation circuit so we have two terms :-

$\boxed{P.I + C.F}$

C.F is same for source free circuit

$C.F = A\,e^{-Pt} \qquad \left(\because P = \frac{1}{RC}\right)$

$$\boxed{V_C(t) = A\,e^{-\frac{t}{RC}}}$$

P.I for $t \to \infty$

P.I is the voltage across the V_c at $t \to \infty$

$P.I \to V_o(t \to \infty)$

- At $t \to \infty$ this capacitor acts as a open circuit, the value of V_c at time = ∞ is V

$\boxed{P.I = V}$

$\Rightarrow P.I + C.F$

$\Rightarrow V + A\,e^{-\frac{1}{RC}t}$

$V_c(t) = V + A\,e^{-\frac{t}{RC}}$

Lets apply initial condition.

$V_c(0^+) = V + A\,e^{-0}$

$V_c(0^+) = V + A$

$V_c(0^+) = 0$ (Capacitor voltage is zero at $t = 0$)

$\boxed{A = -V}$

$\boxed{V_C(t) = V - V e^{-\frac{t}{RC}}}$

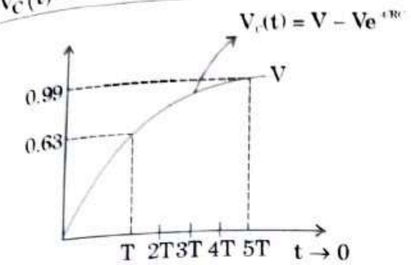


Fig.114

RC circuit :

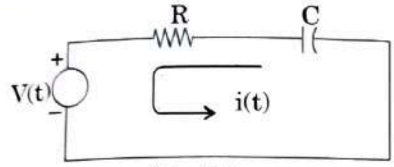


Fig.115

- To find i (t) through the RC circuit at t > 0, assume zero initial condition.

  Let, u (t) = A u(t) − A u(t−1)

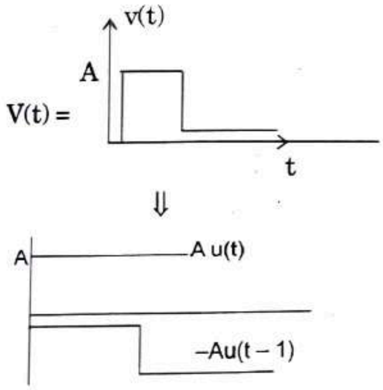


Fig.116

$$u(t) = A\,u(t) - A\,u(t-1)$$

$$V(S) = \frac{A}{S} - \frac{A}{S}e^{-S}$$

$$V(S) = \frac{A}{S}\left(1 - e^{-S}\right)$$

Apply KVL :

$$R\,i(t) + \frac{1}{C}\int_0^t i(t)\,dt = U(t)$$

Assume, $\boxed{R = i\,\Omega, C = 1\,\mu F}$

Laplace of function

$$\int_0^t f(t)\,dt = \frac{f(S)}{S} + \frac{f^{-1}(0^+)}{S}$$

Here, f (t) = i (t)

$$\int_0^t i(t)\,dt = \frac{i(S)}{S} + \frac{i^{-1}(0^+)}{S}$$

Now, $R\,i(t) + \frac{1}{C}\int_0^t i(t)\,dt = U(t)$

take Laplace transform of U (t)

$$I(S) + \frac{I(s)}{s} + \frac{i^{-i}(0^+)}{s} = U(s)$$

$$U(S) = \frac{A}{s}\left(1 - e^{-s}\right)$$

$$I(S) + \frac{I(s)}{s} + \frac{i^{-1}(0^+)}{s} = \frac{A}{s}\left(1 - e^{-s}\right)$$

$$I(S) + \frac{I(s)}{s} + \frac{i^{-1}(0^+)}{s} = \frac{A}{s+1} - \frac{A}{s+1}e^{-s}$$

$$I(S) = \frac{A}{s+1}\left(1 - e^{-s}\right)$$

$i(t) = A\,e^{-t}\,U(t) - A\,e^{-(t-1)}\,.\,u(t\text{-}1)$

initially at i = 0 capacitor acts a short circuit.

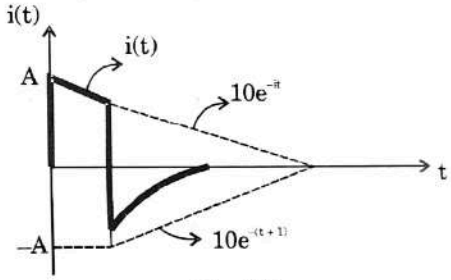


Fig.117

- In case of RC network there is reversal of current.
- If $u(t) = V_o\,u(t)$

  then $\boxed{i(t) = \frac{V_o}{R}e^{-\frac{t}{T}}u(t)}$

  T = RC (time constant)

Significance of time constant :

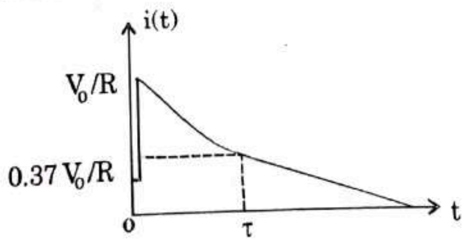


Fig.118

at, t = τ

$$i(t) = \frac{V_0}{R} e^{-1}$$

$$i(t) = 0.377\, V_0 / R$$

- Initially at t = 0, any capacitor acts as a short circuit so that the current through RC network is maximum. At $t = \infty$ or the steady state condition the capacitor acts as an open circuit so that the current through the RC network becomes zero.
- For pulse excitation reversal of current takes place in the RC network.
- For step excitation the current through the RC network decreases exponentially with time constant τ. This time constant controls the rate at which the current through RC network decreases and depends only upon the numerical values of R and C component values.
- The time constant of RC network represent the time at which the current through RC network is decreased to 33.7% its initial value.

Source Free Response of RL Circuits :

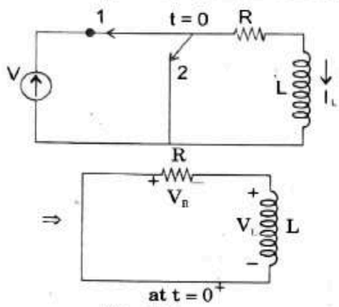


Fig.119

$t = \bar{0}\,;\, I_L = I$

$t = 0^+\,;\, I_L = I$

(Inductor opposed sudden change in current)

Applying KVL :

$V_R + V_L = 0$

$$iR + \frac{L\,di}{dt} = 0$$

$$\frac{di}{dt} + \frac{R}{L} i = 0 \left(\frac{di}{dt} + Pi = Q \right)$$

For a source free response Q = 0 ; $\frac{di}{dt} + Pi = 0$

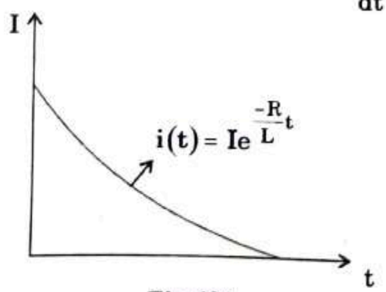


Fig.120

P.I = 0

$$i(t) = A\, e^{-\frac{R}{L}t}$$

$I_L = I$

$i(0^+) = A\, e^{-0}$, $\boxed{A = I}$

$$i(t) = I\, e^{-\frac{R}{L}t},\; i(t) = I\, e^{-\frac{t}{\tau}}$$

τ = time constant

$$\tau = \frac{L}{R}$$

Inductor Voltage

$$V_L = L\frac{di}{dt}$$

$$= L\frac{d}{dt}\left[Ie^{-\frac{R}{L}t} \right] \left(\because i(t) = Ie^{-\frac{R}{L}t} \right)$$

$$= L \times I \left[e^{-\frac{Rt}{L}} \right] \left(\frac{-R}{L} \right)$$

$$\boxed{V_L = -IRe^{-\frac{R}{L}t}}$$

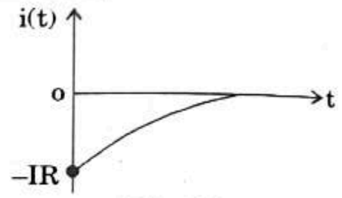


Fig.121

when t = 0

$\boxed{V_L = -IR}$

Forced Response of RL circuit

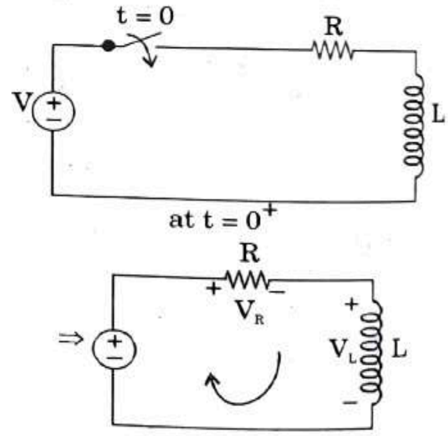


Fig.122

Applying KVL :

$V = V_R + V_L$

$$= iR + L\frac{di}{dt}$$

$$\frac{di}{dt}+\frac{R}{L}i=\frac{V}{L}$$

compare this equation from

$$\frac{di}{dt}+Pi=Q$$

$$P=\frac{R}{L},\ Q=\frac{V}{L}$$

The solution have two parts

P.I + CF

$t=\infty$ $Q=0$ (source free response)

P.I $\rightarrow$ steady state response at $t\rightarrow\infty$

I_L (at $=\infty$)

at $t=\infty$ inductor behaves as a short circuit.

$$\text{So, } I_L=\frac{V}{R}(\text{P.I})$$

$$\text{P.I}=\frac{V}{R}$$

$$\text{C.F}=Ae^{-Pt}\left(P=\frac{R}{L}\right)$$

$$\text{C.F}=Ae^{-\frac{R}{L}t}$$

$$i(t)=PI+CF$$

$$i(t)=\frac{V}{R}+Ae^{-\frac{R}{L}t}$$

$$i(0^+)=\frac{V}{R}+A$$

$$i(0^+)=0,\ \frac{V}{R}+A=0$$

$$\boxed{A=-\frac{V}{R}}$$

$$\boxed{i(t)=\frac{V}{R}-\frac{V}{R}e^{-\frac{R}{L}t}}$$

$$\boxed{i(t)=\frac{V}{R}\left(1-e^{-\frac{R}{L}t}\right)}$$

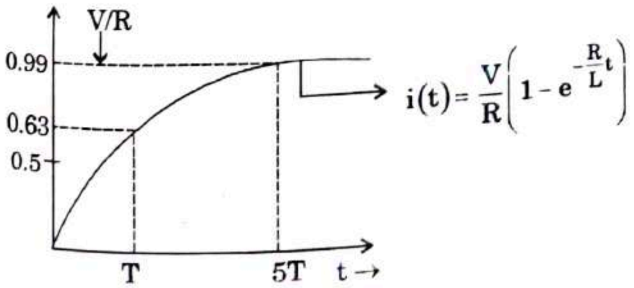


Fig.123

RL circuit

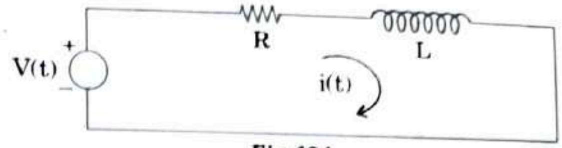
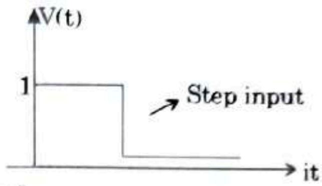


Fig.124

$$u(t)=A\,u(t)-A\,u(t-1)$$



Put A = 1

$$\boxed{u(t)=u(t)-u(t-1)}$$

Applying KVL :

$$R\,i(t)+\frac{L\,di}{dt}=u(t)$$

$$R=1\Omega\ ,\ L=1H$$

$$i(t)+\frac{di}{dt}=u(t)-u(t-1)$$

$$I(s)+s\,I(s)-i(0^+)=\frac{1}{s}\left(1-e^{-3}\right)$$

$$I(s)=\frac{1}{s(s+1)}\left(1-e^{-3}\right)$$

$$I(s)=\left[\frac{1}{s}\ \frac{1}{s+1}\right]\left[1-e^{-3}\right]$$

$$I(S)=\left(\frac{1}{s}-\frac{1}{s+1}\right)-\left(\frac{1}{s}-\frac{1}{s+1}\right)e^{-3}$$

$$i(t)=(1-e^{-t})\,u(t)-[1-e^{-(t-1)}]\,u(t)$$

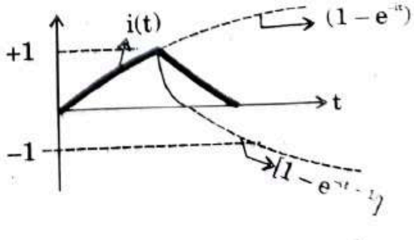


Fig.125

If $u(t)=V_o\,u(t)$ then.

$$i(t)=\frac{V_0}{R}\left(1-e^{-\frac{t}{\tau}}\right)u(t)$$

$$\tau=\frac{L}{R}\text{(Time constant)}$$

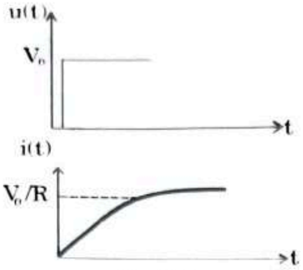
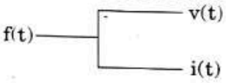


Fig. 126

- Initially at $t = 0$ the inductor acts as a open circuit So that current through RL network is zero.
- For step excitation the current through the inductor increasing exponentially with the same time constant this time constant depends only upon the numerical values of R and L components and controls the rate at which current through the network increases exponentially.
- For pulse excitation a reversal of current takes places in a RC network where as no such reversal takes place in a RL network.

Initial and Final values of function :



Initial Value

$$f(0^+) = f(t)|_{t=0^+} \text{ (initial condition)}$$

$$\boxed{f(0^+) = \lim_{s \to \infty}[s\,F(s)]}$$

- According to initial value theorem :

$$f(0^+) = \lim_{s \to \infty}[s\,F(s)]$$

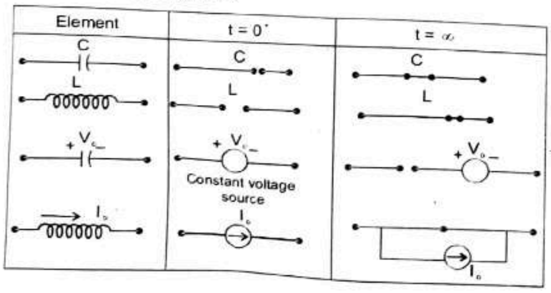
- According to final value or steady state value :-

$$f(\infty) = f(t)|_{t=\infty}$$

$$\boxed{f(\infty) = \lim_{s \to 0}[s\,F(s)]}$$

- Transient Response of an electrical network explains the behavior of any circuit at $t = 0$ to $t = \infty$.

Behavior of L and C



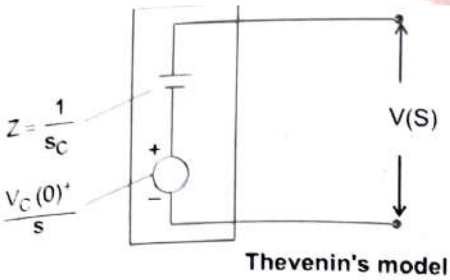


Fig. 128 (b)

Case - 1 : In time domain

- Initially at $t = 0$ any capacitor acts as a short circuit where as any inductor acts as an open circuit
- At $t = \infty$ or in the steady state condition any capacitor acts as open circuit where as any inductor acts as a short circuit.
- Initially at $t = 0$ any capacitor with initial voltage acts as a constant voltage source where as any inductor with initial current acts as constant current source.

Case - 2 : In S domain

$$u(t) = L\frac{d\,i(t)}{dt}$$

$$V(s) = L\left[s\,I(s) - i_2(0^+)\right]$$

$$\boxed{V(s) = L\,sI(s) - L\,i_L(0^+)}$$

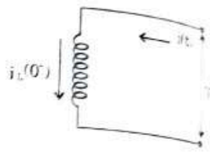


Fig. 127 (a)

Representation of inductor in Laplace domain at initial condition.

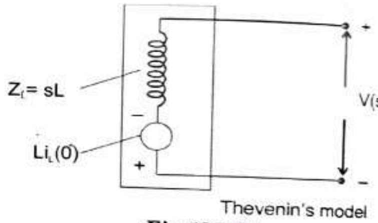


Fig. 127 (b)

Capacitor :

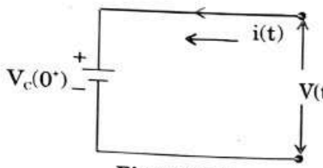


Fig. 128 (a)

$$i(t) = C\frac{d\,V(t)}{dt}$$

$$i(t) = C\frac{d\,V(t)}{dt} \text{ (in time domain)}$$

$I(s) = C\,[s\,V(s) - V_c(0^+)]$ (in s- domain)

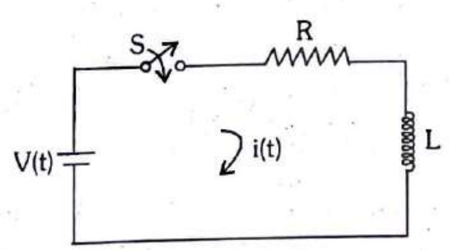
$$\boxed{V(s) = \frac{1}{sC}I(s) + \frac{V_C(0^+)}{s}} \text{ (KVL series circuit)}$$

1.3.4 Transient Response of RL-RC-RLC Circuits

In general, transient occur due to a circuit is suddenly connected or disconnected from the supply, there is a sudden change in applied voltage from one finite value to another and a circuit is short circuited. We consider the transient analysis for the following circuits,

- RL series circuit
- RC series circuit
- RLC series circuit
- RLC parallel circuit

✸ RL Series Circuit



Current equation for step input

$$i(t) = \frac{V}{R}\left(1-e^{-t/\tau}\right)$$

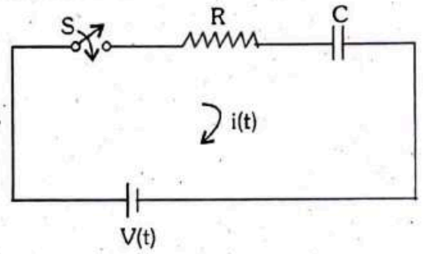
where

$$\tau \rightarrow \text{Time constant} = \frac{L}{R}$$

Current equation for impulse input

$$i(t) = \frac{V}{L}e^{-t/\tau}$$

✸ RC series circuit



Current equation for step input,

$$i(t) = \frac{V}{R}e^{-t/\tau}$$

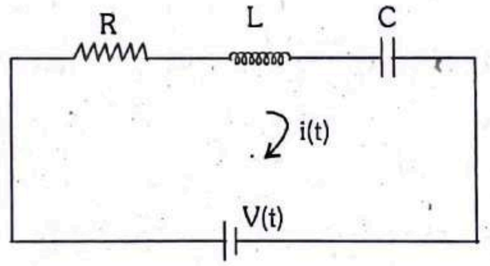
where,

$$\tau \rightarrow \text{Time constant} = RC$$

Current equation for impulse input,

$$i(t) = \frac{V}{R}\left[\delta(t) - \frac{1}{RC}e^{-t/\tau}\right]$$

✸ **RLC series circuit**



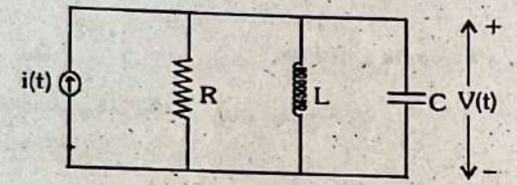
Current equation,

$$i(t) = \frac{V}{2\omega_o L\sqrt{\delta^2 - 1}} e^{-\delta\omega_o t}\left[e^{\left(\omega_o\sqrt{\delta^2-1}\right)t} - e^{-\left(\omega_o\sqrt{\delta^2-1}\right)t}\right]$$

Depending upon the value of R, L and C the current equation will change,

- $\frac{R}{2L} > \frac{1}{\sqrt{LC}}$ → Overdamped condition.
- $\frac{R}{2L} < \frac{1}{\sqrt{LC}}$ → Under damped condition.
- $\frac{R}{2L} = \frac{1}{\sqrt{LC}}$ → Critically damped condition.

✸ **RLC Parallel Circuit**

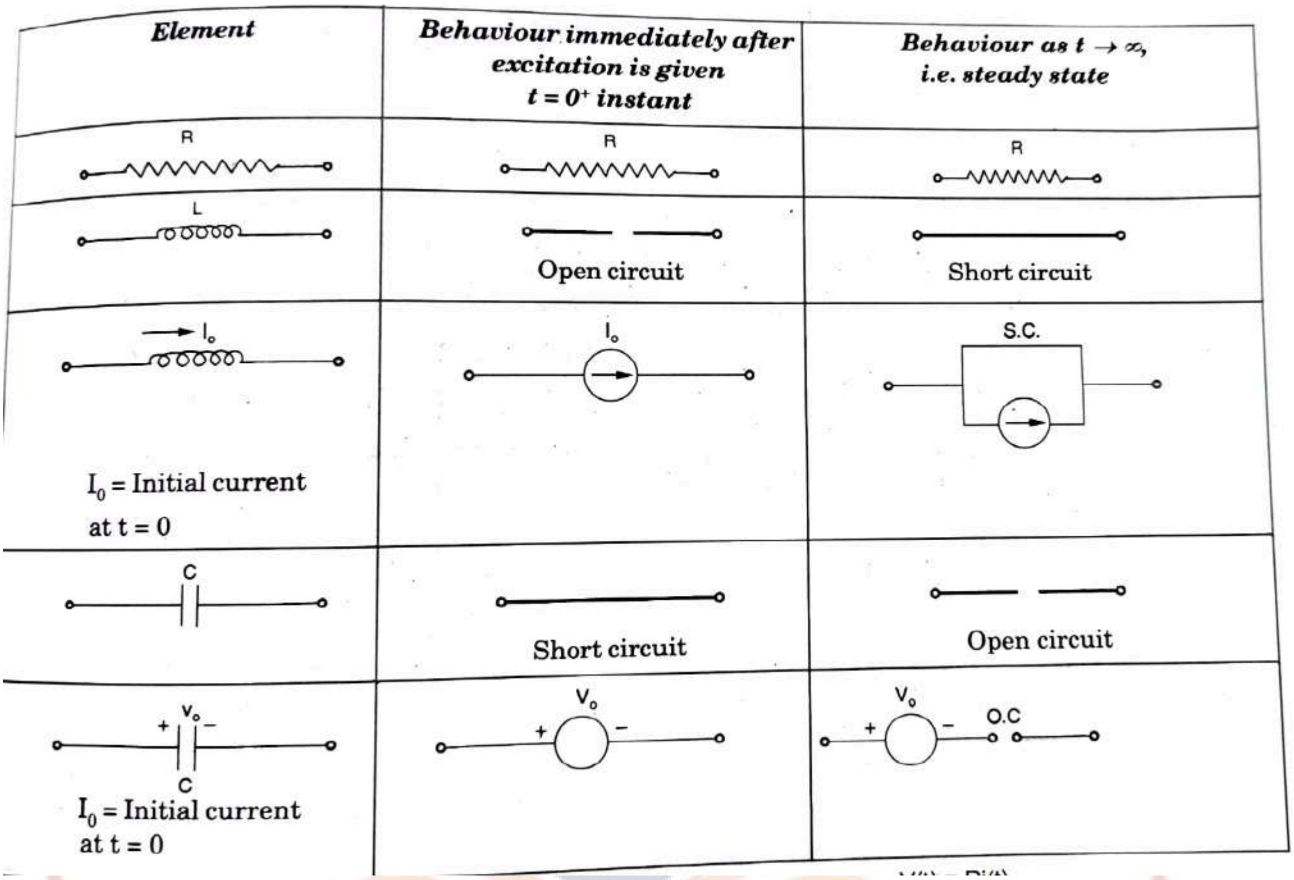


Voltage equation,

$$V(t) = \frac{1}{2\omega_0 C\sqrt{\delta^2 - 1}} e^{-\delta\omega_o t}\left[e^{\left(\omega_o\sqrt{\delta^2-1}\right)t} - e^{-\left(\omega_o\sqrt{\delta^2-1}\right)t}\right]$$

Depending upon the value of R, L and C the voltage equation may change,

- $\frac{1}{2RC} > \frac{1}{\sqrt{LC}}$ → Overdamped condition.
- $\frac{1}{2RC} < \frac{1}{\sqrt{LC}}$ → Under damped condition.
- $\frac{1}{2RC} = \frac{1}{\sqrt{LC}}$ → Critically damped condition.



Element	Behaviour immediately after excitation is given $t = 0^+$ instant	Behaviour as $t \to \infty$, i.e. steady state
R	R	R
L	Open circuit	Short circuit
I_o I_0 = Initial current at t = 0	I_o	S.C.
C	Short circuit	Open circuit
V_o + − C I_0 = Initial current at t = 0	V_o + −	V_o + − O.C

1.4 Network Configurations

1.4.1 Two-port networks

- A pair of terminals which an electrical signal may enter or leave a network is called a port.
- A network having only one pair of terminals called one port network.
- A network with two input terminals and two output terminals is called four terminal or two port network.
- Two port network is used in the construction of filters, attenuators, equalizers and impedance matching network in cascade, series and parallel connection.
- Two port network parameters,
    - Z-parameters or Impedance parameter
    - y-parameters or Admittance parameter
    - h-parameters or Hybrid parameter
    - ABCD Parameter

Some two part networks :

(1) *T-network* (2) *π-network*

(3) *Lattice network* (4) *Lader network combination of T and π networks connected in series.*

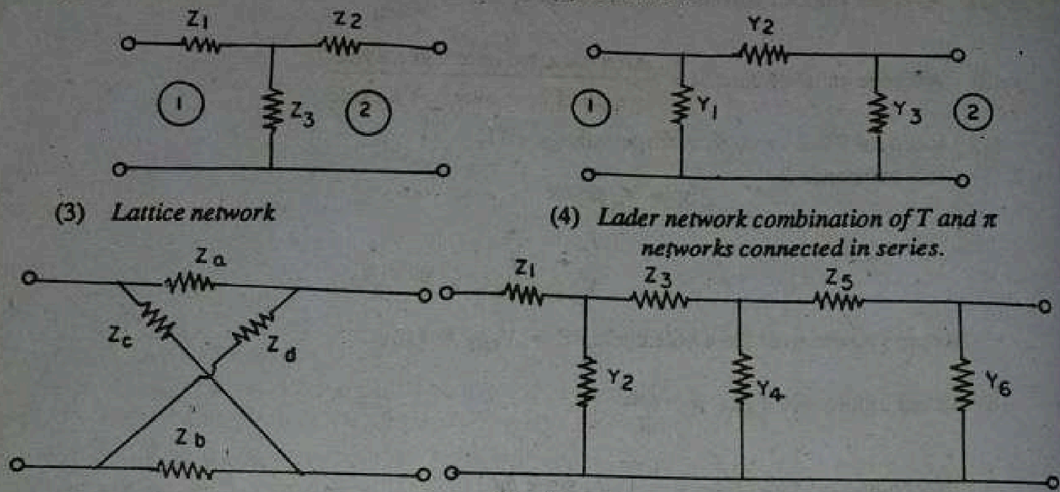


Fig. 4.

5.8.2. Z (Impedance) parameters or open circuit parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Fig. 5.

where, $Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0}$ = input impedance

$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_2 = 0}$ = Reverse transfer impedance

$Z_{21} = \left.\frac{V_1}{Ia_2}\right|_{I_2 = 0}$ = Reverse transfer impedance

$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1 = 0}$ = output impedance.

For equivalent Π network :

$$Y_{11} = Y_A + Y_C$$
$$Y_{12} = -Y_C$$
$$Y_{21} = -Y_C$$
$$Y_{22} = Y_B + Y_C$$

Z_{11} and Z_{22} are called driving point functions

$$Z_{11} = Z_A + Z_C, \; Z_{22} = Z_B + Z_C,$$

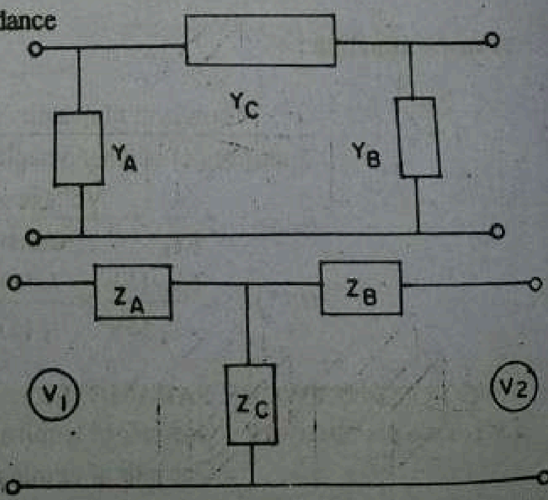


Fig. 6.

Z_{21} and Z_{12} are called transfer functions

$$Z_{21} = Z_C = Z_{12}$$

For **reciprocal** networks $\boxed{Z_{12} = Z_{21}}$

For **symmetrical** networks $\boxed{Z_A = Z_B}$

For 2 port networks connected in series over all Z parameter matrix = sum of individual Z parameters

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{11'} & Z_{12} + & Z_{12'} \\ Z_{21} & Z_{21'} & Z_{22} + & Z_{22'} \end{bmatrix}$$

5.8.3. Y (admittance) parameters or Short circuit parameters

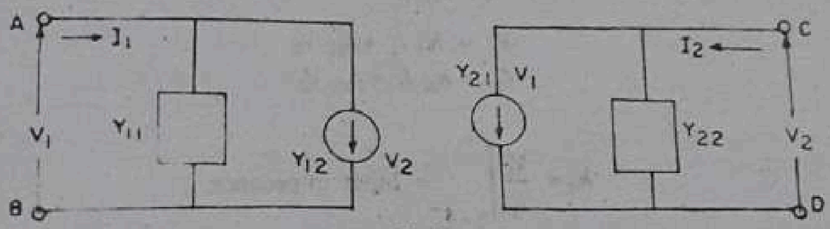


Fig. 7.

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

where, $Y_{11} = \left.\dfrac{I_1}{V_1}\right|_{V_2 = 0}$ = Short - circuit input admittance,

$Y_{21} = \left.\dfrac{I_2}{V_1}\right|_{V_2 = 0}$ = Short - circuit forward admittance,

$Y_{12} = \left.\dfrac{I_1}{V_2}\right|_{V_1 = 0}$ = Short - circuit reverse admittance,

$Y_{22} = \left.\dfrac{I_2}{V_2}\right|_{V_1 = 0}$ = Short - circuit output admittance,

For 2 ports networks in parallel overall Y parameter matrix = sum of individual networks parameters

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} + & Y_{11'} & Y_{12} + & Y_{12'} \\ Y_{21} + & Y_{21'} & Y_{22} + & Y_{22'} \end{bmatrix}$$

For reciprocal networks

$$\boxed{Y_{12} = Y_{21}}$$

For symmetric networks

$$\boxed{Y_{11} = Y_{22}}$$

h (hybrid) parameters :

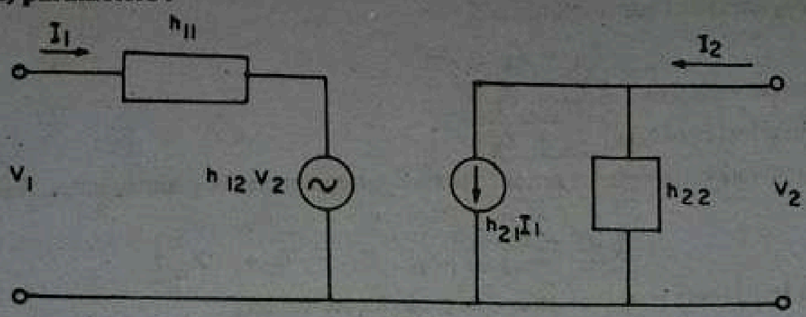


Fig. 8.

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

where,

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \text{Input impedance,}$$

$$h_{21} = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \text{Forward current gain } A_i \text{ ,}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \text{Reverse voltage gain } A_v\text{,}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \text{Output admittance.}$$

5.8.4. ABCD (Transmission) Parameters. Here the output current flows into the transmission line and not into the out port as in other parameters

$$V_1 = A V_2 - B I_2$$
$$I_1 = C V_2 - D I_2$$

where, $A = \left.\dfrac{V_1}{V_2}\right|_{I_2=0}$ = Inverse open circuit voltage gain,

$C = \left.\dfrac{I_1}{V_2}\right|_{I_2=0}$ = Open circuit forward transfer admittance,

$B = \left.\dfrac{V_1}{I_2}\right|_{V_2=0}$ = Open circuit reverse transfer impedance,

$D = \left.\dfrac{I_1}{I_2}\right|_{V_2=0}$ = Inverse short circuit current gain.

For reciprocal network $AD - BC = 0$.

For lossless network $\left[\begin{array}{l} A \text{ and } D \text{ are real.} \\ B \text{ and } C \text{ are imaginary.} \end{array}\right.$

For symmetrical network $A = D$.

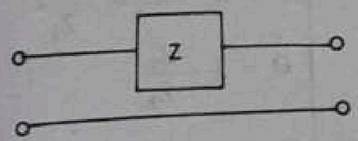
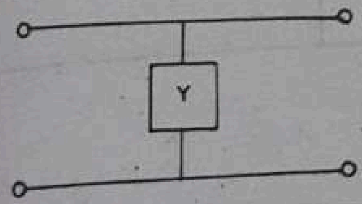
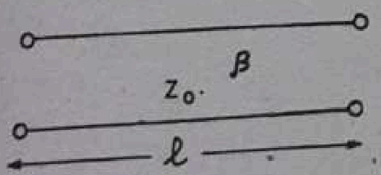
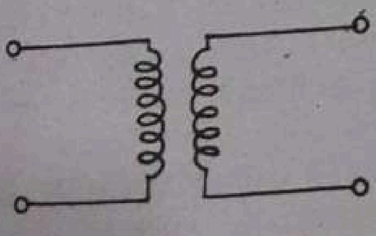
Image impedance $Z_0 = \sqrt{\frac{B}{C}}$.

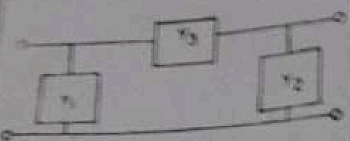
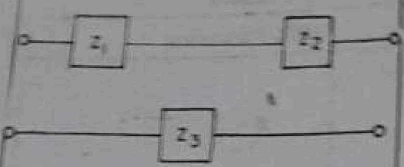
If two networks are connected in series.

Overall transmission matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$

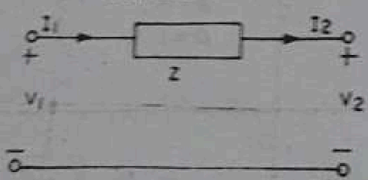
i.e. we multiply matrices of individual networks.

The ABCD Parameters of Some Useful Two-Port Circuits

Circuit		*ABCD Parameters*
	$A = 1$ $C = 0$	$B = Z$ $D = 1$
	$A = 1$ $C = Y$	$B = 0$ $D = 1$
	$A = \cos \beta l$ $C = j Y_0 \sin \beta l$	$B = j Z_0 \sin \beta l$ $D = \cos \beta l$
	$A = N$ $C = 0$	$B = 0$ $D = \frac{1}{N}$

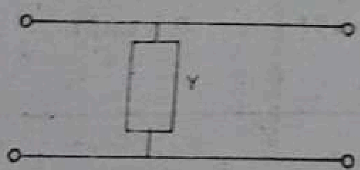
Circuit		ABCD Parameters
	$A = 1 + \frac{Y_2}{Y_3}$ $C = Y_1 + Y_2 + \frac{Y_1 Y_2}{Y_3}$	$B = \frac{1}{Y_3}$ $D = 1 + \frac{Y_1}{Y_3}$
	$A = 1 + \frac{Z_1}{Z_3}$ $B = \frac{1}{Z_3}$	$B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$ $D = 1 + \frac{Z_2}{Z_3}$

Network

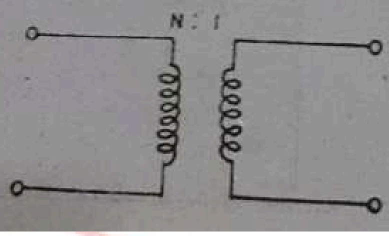
A 

$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$

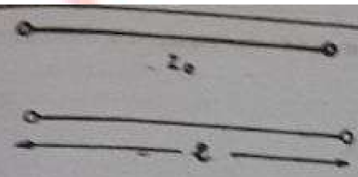
$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$

B 

$\begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$

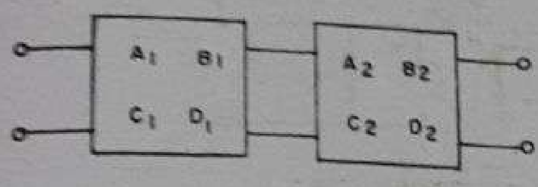
C 

$\begin{bmatrix} n & 0 \\ 0 & 1/n \end{bmatrix}$

D 

$\begin{bmatrix} \cos \beta l & j Z_0 \sin \beta l \\ j Y_0 \sin \beta l & \cos \beta l \end{bmatrix}$ lossless case

$\begin{bmatrix} \cos h \gamma l & Z_0 \sinh \gamma l \\ j Y_0 \sin h \gamma l & \cosh \gamma l \end{bmatrix}$ lossy line

E 

$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}$

Fig. 9.

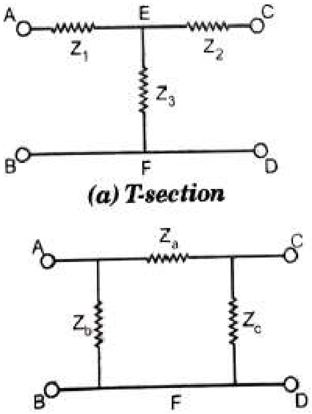
1.4.2 Three Phase Circuits

There are some networks in which the resistances are neither in series nor in parallel. A familiar case is a three terminal network e.g. delta network or star network. In such situations, it is not possible to simplify the network by series and parallel circuit rules. However, converting delta network into star and vice-versa often simplifies the network and makes it possible to apply series parallel circuit techniques.

1.4.3 Star-delta transformation

WYE-DELTA TRANSFORMATION

Three elements of a network may be arranged either as T-section or a π–section as shown in Fig.56 (a) and (b) respectively.



(a) T-section

(b) π-section

Fig.56

Again, T-section may be redrawn as a star or Wye section Fig.57 (a) and π-section as mesh or a delta-section Fig.57 (b)

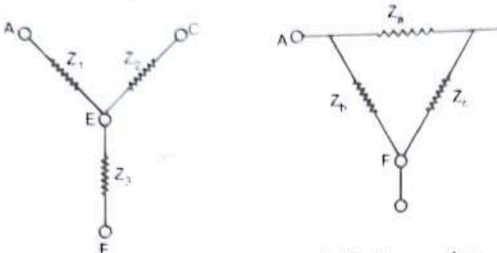
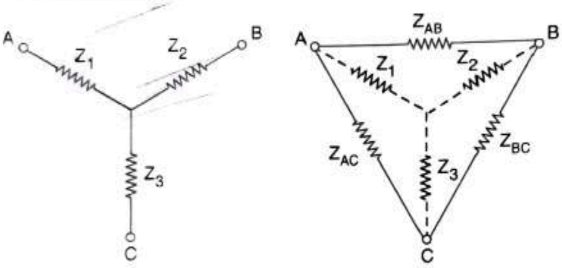


Fig.57

According to delta star conversion theorem, T-section or star section can be interchange to a π-section or delta section and vice versa at any frequency provided certain relations are maintained between elements of two sections.



(a) Wye-Network ***(b) Delta-Network***

Fig.58

Wye-to-Delta Conversion

$$Z_{AB} = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3}$$

$$= \Sigma\frac{Z_1Z_2}{Z_3};$$

$$Z_{BC} = \Sigma\frac{Z_1Z_2}{Z_1};$$

$$Z_{AC} = \Sigma\frac{Z_1Z_2}{Z_2}$$

Delta-to-star conversion

$$Z_1 = \frac{Z_{AB}Z_{AC}}{Z_{AB} + Z_{AC} + Z_{BC}} = \frac{Z_{AB}Z_{AC}}{\Sigma Z_{AB}};$$

$$Z_2 = \frac{Z_{AB}Z_{BC}}{\Sigma Z_{AB}}, \; Z_3 = \frac{Z_{BC}Z_{AC}}{\Sigma Z_{AB}}$$

Conversion of delta to star Analysis:

Delta → $[Z_{12}, Z_{13}, Z_{23}]$ → (given)

Star → $[Z_1, Z_2, Z_3]$ → (find out)

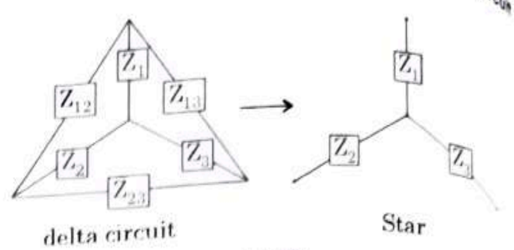


Fig.59

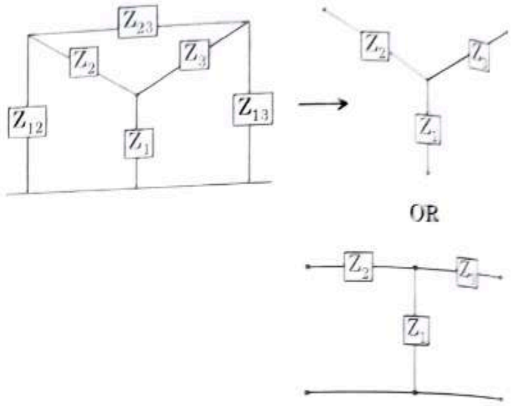


Fig.60

(i) $Z_1 = \dfrac{Z_{12} \cdot Z_{13}}{Z_{12} + Z_{13} + Z_{23}}$

(ii) $Z_2 = \dfrac{Z_{12} \cdot Z_{23}}{Z_{12} + Z_{13} + Z_{23}}$

(iii) $Z_3 = \dfrac{Z_{13} \cdot Z_{23}}{Z_{12} + Z_{13} + Z_{23}}$

Case : 1 → If $Z_{12} = Z_{13} = Z_{23}$

(i) $Z_1 = \dfrac{Z \cdot Z}{Z + Z + Z} = \dfrac{Z}{3}$

(ii) $Z_2 = \dfrac{Z \cdot Z}{Z + Z + Z} = \dfrac{Z}{3}$

(iii) $Z_3 = \dfrac{Z \cdot Z}{Z + Z + Z} = \dfrac{Z}{3}$

$\because \boxed{Z_{eq} = \frac{2}{3}}$

Note:

1. Generally Δ to star conversion decreases the impedance.
2. If all impedance are same then it decreases the impedance by a factor of 3.

   Z(R) → R

   Z(L) → jωL

   $Z(C) \rightarrow \dfrac{1}{j\omega C}$

Case : 1.1 → For Resistor :-

$$Z_{eq} = \frac{Z}{3}$$

$$R_{eq} = \frac{R}{3}$$

Case : 1.2 → For inductor :-

$$Z_{eq} = \frac{Z}{3}$$

$$j\omega L_{eq} = \frac{j\omega L}{3}$$

$$\boxed{L_{eq} = \frac{L}{3}}$$

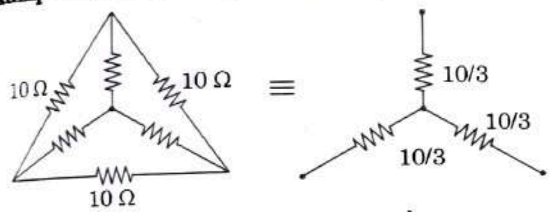
Case : 1.3 → For Capacitor :-

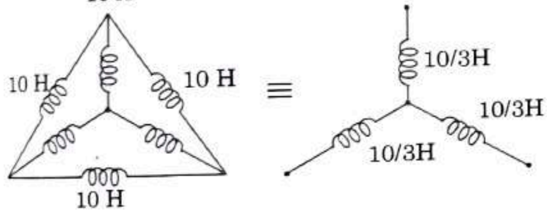
$$Z_{eq} = \frac{Z}{3}$$

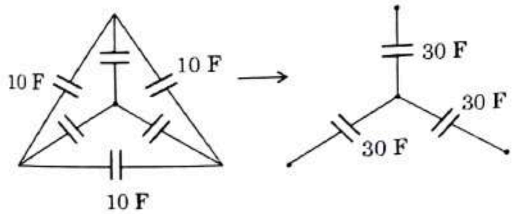
$$\frac{1}{j\omega C_{eq}} = \frac{1}{j\omega C} \cdot \frac{1}{3}$$

$$\boxed{C_{eq} = 3C}$$

Example 1 : Find star equivalent of given delta circuit.







Case : 2 If $Z_{12} \neq Z_{13} Z_{23}$ then use formula →

Case : 2.1 → For register

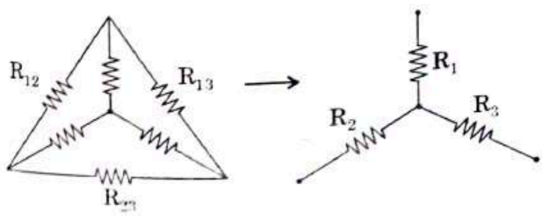


Fig.61

(i) $R_1 = \dfrac{R_{12} \cdot R_{13}}{R_{12} + R_{13} + R_{23}}$

(ii) $R_2 = \dfrac{R_{12} \cdot R_{23}}{R_{12} + R_{13} + R_{23}}$

(iii) $R_3 = \dfrac{R_{13} \cdot R_{23}}{R_{12} + R_{13} + R_{23}}$

Case : 2.2 → For inductor

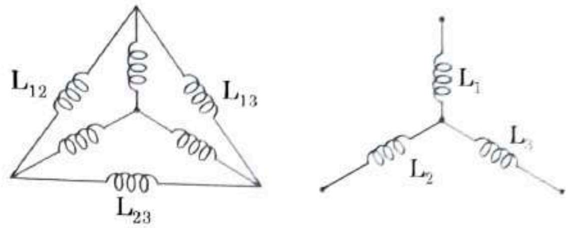


Fig.62

(i) $L_1 = \dfrac{L_{12} \cdot L_{13}}{L_{12} + L_{13} + L_{23}}$

(ii) $L_2 = \dfrac{L_{12} \cdot L_{23}}{L_{12} + L_{13} + L_{23}}$

(iii) $L_3 = \dfrac{L_{13} \cdot L_{23}}{L_{12} + L_{13} + L_{23}}$

Case : 2.3 → Capacitor

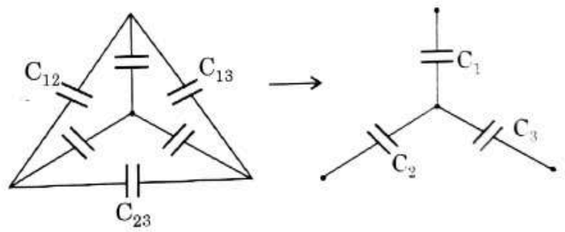


Fig.63

(i) $\dfrac{1}{C_1} = \dfrac{\dfrac{1}{C_{12}} \cdot \dfrac{1}{C_{13}}}{\dfrac{1}{C_{12}} + \dfrac{1}{C_{13}} + \dfrac{1}{C_{23}}}$

(ii) $\dfrac{1}{C_2} = \dfrac{\dfrac{1}{C_{12}} \cdot \dfrac{1}{C_{23}}}{\dfrac{1}{C_{12}} + \dfrac{1}{C_{13}} + \dfrac{1}{C_{23}}}$

(iii) $\dfrac{1}{C_3} = \dfrac{\dfrac{1}{C_{13}} \cdot \dfrac{1}{C_{23}}}{\dfrac{1}{C_{12}} + \dfrac{1}{C_{13}} + \dfrac{1}{C_{23}}}$

1.5 Power in Electrical Circuits

The instantaneous power of an a.c. circuit is defined as the product of the instantaneous e.m.f. and the instantaneous current in it.

Let the instantaneous e.m.f. in an a.c. circuit is given by

$$E = E_0 \cos \omega t$$

and the current at that instant be given by

$$I = I_0 \cos (\omega t - \phi),$$

where ϕ is the phase angle by which e.m.f. leads the current in the a.c. circuit. Here, I_0 and E_0 are peak values of current and e.m.f. respectively and $\omega = 2\pi/T$, where T is period of a.c.

If we assume that the values of e.m.f. and current in the a.c. circuit remain constant for a small time dt, then small amount of electrical energy consumed is given by

$$\begin{aligned} dW &= EI\,dt = E_0 \cos \omega t\, I_0 \cos (\omega t - \phi)\, dt \\ &= E_0 I_0 \cos \omega t (\cos \omega t \cos \phi + \sin \omega t \sin \phi)\, dt \\ &= E_0 I_0 (\cos^2 \omega t \cos \phi + \sin \omega t \cos \omega t \sin \phi)\, dt. \end{aligned}$$

Now, $\quad \cos 2\omega t = 2\cos^2 \omega t - 1 \quad$ or $\quad \cos^2 \omega t = \dfrac{1 + \cos 2\omega t}{2}$

Also, $\quad \sin 2\omega t = 2 \sin \omega t \cos \omega t \quad$ or $\sin \omega t \cos \omega t = \dfrac{\sin 2\omega t}{2}$

Substituting for $\cos^2 \omega t$ and $\sin \omega t \cos \omega t$, we have

$$\begin{aligned} dW &= E_0 I_0 \left[\frac{1 + \cos 2\omega t}{2} \cos \phi + \frac{\sin 2\omega t}{2} \sin \phi \right] dt \\ &= \frac{E_0 I_0}{2} [\cos \phi + \cos \phi \cos 2\omega t + \sin \phi \sin 2\omega t]\, dt. \end{aligned}$$

The electrical energy consumed in the circuit in time T (period of a.c.) can be obtained by integrating the R.H.S. of the above equation between $t = 0$ to $t = T$ *i.e.,*

$$W = \int_0^T \frac{E_0 I_0}{2} dt [\cos \phi + \cos \phi \cos 2\omega t + \sin \phi \sin 2\omega t]\, dt$$

or $\quad W = \dfrac{E_0 I_0}{2} \left[\cos \phi \int_0^T dt + \cos \phi \int_0^T \cos 2\omega t\, dt + \sin \phi \int_0^T \sin 2\omega t\, dt\right]$

Now, $\quad \int_0^T dt = |t|_0^T = T - 0 = T$

It can be shown that $\quad \int_0^T \cos 2\omega t\, dt = \int_0^T \cos 2\omega t\, dt = 0$

Therefore, we have,

$$W = \frac{E_0 I_0}{2} [\cos \phi (T) + \cos \phi (0) + \sin \phi (0)]$$

or $\quad W = \dfrac{E_0 I_0 T}{2} \cos \phi.$

The average power of the a.c. circuit is

$$P_{av} = \frac{W}{T} = \frac{E_0 I_0 T}{2} \cos \phi \times \frac{1}{T} = \frac{E_0 I_0}{2} \cos \phi = \frac{E_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cos \phi$$

or $\quad P_{av} = E_v I_v \cos \phi.$

1.5.1 Real and Reactive Power

3.3.1. Real power. In ac circuits, reactance (inductive or capacitive) is invariably present resulting in phase difference between voltage and currents. Under these circumstances voltage may have a high value when current is near zero or vice versa. Hence real power is less than the apparent power and is given by

$$\text{real power} = \text{apparent power} \times \cos\phi = EI\cos\phi$$

where ϕ is the angle between voltage and current.

3.3.2. Reactive power. The power taken by pure reactance (inductive or capacitive) in a circuit is called reactive power. Thus

$$\text{Reactive power} = \text{Apparent power} \times \sin\phi = EI\sin\phi.$$

The unit of reactive power is volt ampere reactive (VAR).

1.5.2 Power Factor

3.3.3. Power factor. Power factor may be defined as :

(a) Cosine of angle between voltage and current.

(b) The ratio $\dfrac{\text{Resistance}}{\text{Impedance}}$.

(C) The ratio $\dfrac{\text{Real power}}{\text{Apparent power}}$.

The value of power factor ($\cos\phi$) can never be more than unity.

(d) When power factor is lagging, current lags the voltage which is possible in case of inductive circuits.

(e) When power factor is leading, current leads the voltage, which occurs in capacitive circuits.

(f) Sometimes power factor is expressed as percentage. Thus 0.8 lagging power factor can be expressed as 80% lagging.

3.3.3.1. *Effects of low power factor :*

(1) In order to obtain a certain power, with a low power factor, total current must be increased resulting in increase in resistive losses so that efficiency decreases.

(2) The low power factor limits the output of the both–the generators and transformers. This happens because of high currents drawn out of the generators and transformers, resulting in temperature rise.

(3) Low power factors cause greater fall in the terminal voltage, hence make the voltage regulation greater than at unity.

3.3.3.2. Special Cases :

1. *A.C. circuit having R only.* For such a circuit, Therefore, $P_{av} = E_v I_v \cos 0 = E_v I_v (1)$.

$$= E_v I_v = \frac{E_v E_v}{R} = \frac{E_v^2}{R}.$$

2. *A.C. circuit having L only.* For such a circuit, $\phi = \frac{\pi}{2}$.

Therefore, $$P_{av} = E_v I_v \cos\frac{\pi}{2} = E_v I_v (0) = 0.$$

3. *A.C. circuit containing C only.* For such a circuit, $\phi = \frac{\pi}{2}$.

$1 + \tan^2\theta = \sec^2\theta$

Therefore, $P_{av} = E_v I_v \cos\left(-\frac{\pi}{2}\right) = E_v I_v (0) = 0.$

4. *A.C. circuit containing L and R.* For such a circuit, $\tan\phi = \frac{\omega L}{R}$ so that $\cos\phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$.

$$\therefore \quad P_{av} = E_v I_v \frac{R}{\sqrt{R^2 + \omega^2 L^2}} = E_v \cdot \frac{E_v}{\sqrt{R^2 + \omega^2 L^2}} \times \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$?

or $$P_{av} = \frac{E_v^2 \cdot R}{R^2 + \omega^2 L^2}.$$

5. *A.C. circuit containing C and R.* For such a circuit, $\tan\phi = \frac{1/\omega C}{R}$ so that

$$\cos\phi = \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 L^2}}}.$$

$$\therefore \quad P_{av} = E_v I_v \cdot \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} = E_v \cdot \frac{E_v}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \times \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

or $$P_{av} = \frac{E_v^2 \cdot R}{R^2 + \frac{1}{\omega^2 C^2}}.$$

6. ***A.C. circuit containing L, C and R.*** For such a circuit, $\tan\phi = \dfrac{\omega L - \dfrac{1}{\omega C}}{R}$ so that

$$\cos\phi = \frac{R}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}.$$

Therefore,

$$P_{av} = E_v I_v . \frac{R}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$$

$$= E_v . \frac{E_v}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}} . \frac{R}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$$

or

$$P_{av} = \frac{E_v^2 R}{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}.$$

I AM AN IAS ACADEMY

Previous Year Questions (2024) with Answers:

1.

When a load is connected to $230\angle 10^\circ V$, it draw a current of $10\angle -50^\circ A$, what is the real and reactive power of the load?

(A) 1992 W, 1150 VAR
(B) 1150 W, 1150 VAR
(C) 1626 W, 2086 VAR
(D) 1150 W, 1992 VAR
(E) Answer not known

2.

A three phase balanced inductive load draw a current of 10 A and consumed 6 KW when connected to 400 V supply. What is the power factor of the load?

(A) 0.866 lag
(B) 0.5 lag
(C) 0.866 lead
(D) 0.5 lead
(E) Answer not known

3.

The reading of the wattmeter connected to measure the reactive power in a 3ϕ circuit is given by zero, the line voltage is 400 V and line current 15 A; then the power factor of the circuit is

(A) Zero
(B) Unity
(C) 0.8
(D) 0.5
(E) Answer not known

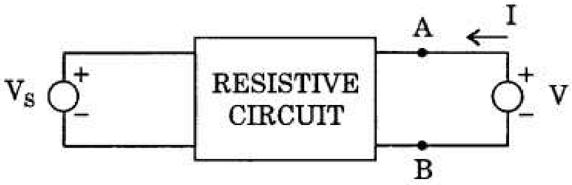
4.

A balanced star – connected load of $(4 + j3)\,\Omega$ per phase is connected to a balanced 3 – phase 400 V supply. The phase current is 12 A. The value of real power is

(A) 5650.24 watts
(B) 6650.88 watts
(C) 7650.78 watts
(D) 8750.67 watts
(E) Answer not known

5.

For the network shown in the figure if $V_s = 1V$ and $V = 0$, then $I = -5\,A$ and if $V_5 = 0$ and $V = 1V$, then $I = \frac{1}{2}A$. The values of I_{SC} and R_N of the Norton's equivalent across AB would be respectively



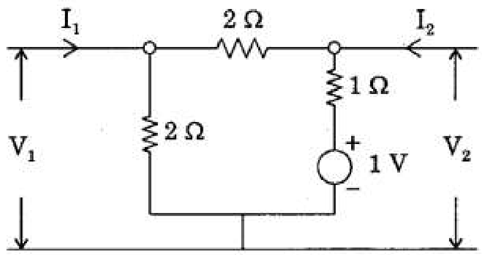
(A) $-5A$ and $2\,\Omega$ (B) $10A$ and $0.5\,\Omega$

(C) $5A$ and $2\,\Omega$ (D) $2.5\,\Omega$ and $5\,\Omega$

(E) Answer not known

6.

The φ parameters of the circuit shown below are



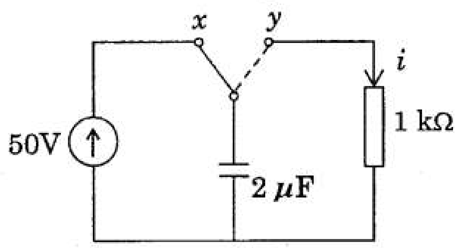
(A) $1, \frac{-1}{2}, \frac{-1}{2}$ and $\frac{3}{2}$ (B) 2,–2,–2 and 1

(C) 1,–1,–1 and 3 (D) 2,–1,–1 and 3

(E) Answer not known

7.

The expression for current i in $1k\,\Omega$ resistor in the circuit below when the switch is moved from x to y is



(A) $0.05\,e^{-500\,t}A$ (B) $5\,e^{-500\,t}A$

(C) $0.5\,e^{-1000\,t}A$ (D) $50\,e^{-1000\,t}A$

(E) Answer not known

8.

In an a.c. circuit, $V = 100\sin(wt + 30^\circ)V$, $i = 5\sin(wt - 30^\circ)A$. Find apparent power and reactive power

(A) 500 VA, 433 VAr
(B) 354 VA, 30 6.5 VAr
(C) 250 VA, 217 VAr
(D) 354 VA, 177 VAr
(E) Answer not known

9.

The parameters widely used in transmission line theory

(A) Z – Parameters
(B) Y – Parameters
(C) ABCD – Parameters
(D) H – Parameters
(E) Answer not known

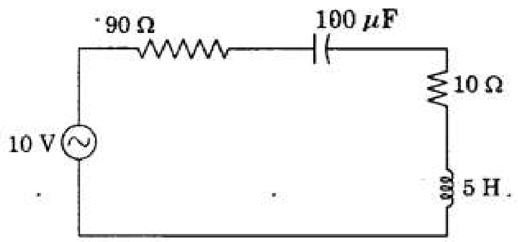
10.

A RL series circuit draws a current of 1 A, when connected across 10 V, 1 rad/sec a.c supply. Assuming the resistance to be $5\,\Omega$, find the inductance of the circuit

(A) $\sqrt{75}H$
(B) $100\ mH$
(C) $\sqrt{10}\ mH$
(D) $0.1H$
(E) Answer not known

11.

For the circuit shown in fig. determine the bandwidth. The resonant frequency of the circuit is 7.12 Hz.



(A) 2.17 Hz
(B) 3.23 Hz
(C) 2.57 Hz
(D) 3.178 Hz
(E) Answer not known