

3D Format

Baofeng Shi

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1 Orignal functions

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \quad (1)$$

$$\nabla \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} - en_e \vec{v}_e \quad (2)$$

$$\frac{\partial \vec{v}_e}{\partial t} = -\frac{e\vec{E}}{m_e} - \nu_m \vec{v}_e \quad (3)$$

Expand to seperated equations:

$$\frac{\partial H_x}{\partial t} = \frac{1}{\mu_0} \left[\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} \right] \quad (4)$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu_0} \left[\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \right] \quad (5)$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu_0} \left[\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \right] \quad (6)$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\epsilon_0} \left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} + en_e v_{ex} \right] \quad (7)$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\epsilon_0} \left[\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} + en_e v_{ey} \right] \quad (8)$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\epsilon_0} \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} + en_e v_{ez} \right] \quad (9)$$

$$\frac{\partial v_{ex}}{\partial t} = -\frac{eE_x}{m_e} - \nu_m v_{ex} \quad (10)$$

$$\frac{\partial v_{ey}}{\partial t} = -\frac{eE_y}{m_e} - \nu_m v_{ey} \quad (11)$$

$$\frac{\partial v_{ez}}{\partial t} = -\frac{eE_z}{m_e} - \nu_m v_{ez} \quad (12)$$

2 Approximations

First solve velocity equations:

$$\frac{v_e^{n+1} - v_e^n}{\Delta t} = -\frac{e}{2m_e} [E^{n+1} + E^n] - \frac{\nu_m}{2} [v_e^{n+1} + v_e^n] \quad (13)$$

after adjusting we get

$$v_e^{n+1} = \alpha v_e^n - \frac{e\Delta t}{2m_e\gamma} [E^{n+1} + E^n] \quad (14)$$

$$\alpha = \frac{1-a}{1+a}, \beta = \frac{\omega_p^2 \Delta t^2}{4\gamma}, \gamma = 1+a, a = \frac{\nu_m \Delta t}{2} \quad (15)$$

Solving E-equations.

$$\frac{\epsilon_0}{\Delta t} [E^{n+1} - E^n] - \frac{en_e}{2} [v_e^{n+1} + v_e^n] = \nabla H \quad (16)$$

put (14) into (16) we get

$$E^{n+1} = E^n \frac{1-\beta}{1+\beta} + \frac{en_e \Delta t}{2\epsilon_0} \frac{1+\alpha}{1+\beta} v_e^n + \frac{\Delta t}{(1+\beta)\epsilon_0} \nabla H \quad (17)$$

3 Full iteration equations

H-equations

$$H_x^{n+1/2}(i, j+1/2, k+1/2) = H_x^{n-1/2}(i, j+1/2, k+1/2) + C_{hxy} [E_y^n(i, j+1/2, k+1) - E_y^n(i, j+1/2, k+1)] + C_{hxe} [E_x^n(i+1/2, j+1/2, k+1) - E_x^n(i+1/2, j+1/2, k+1)] \quad (18)$$

$$\text{where } C_{hxy} = \frac{\Delta t}{\mu_0 \Delta z}, C_{hxe} = -\frac{\Delta t}{\mu_0 \Delta y}$$

$$H_y^{n+1/2}(i+1/2, j, k+1/2) = H_y^{n-1/2}(i+1/2, j, k+1/2) + C_{hyez} [E_z^n(i+1, j, k+1/2) - E_z^n(i, j, k+1/2)] + C_{hyex} [E_x^n(i+1/2, j+1/2, k+1) - E_x^n(i+1/2, j+1/2, k+1)] \quad (19)$$

$$\text{where } C_{hyez} = \frac{\Delta t}{\mu_0 \Delta x}, C_{hyex} = -\frac{\Delta t}{\mu_0 \Delta z}$$

$$H_z^{n+1/2}(i+1/2, j+1/2, k) = H_z^{n-1/2}(i+1/2, j+1/2, k) + C_{hzez} [E_x^n(i+1/2, j+1, k) - E_x^n(i+1/2, j, k)] + C_{hzey} [E_y^n(i+1/2, j+1/2, k+1) - E_y^n(i+1/2, j+1/2, k+1)] \quad (20)$$

$$\text{where } C_{hzez} = \frac{\Delta t}{\mu_0 \Delta x}, C_{hzey} = -\frac{\Delta t}{\mu_0 \Delta z}$$

E-equations

$$\begin{aligned} E_x^{n+1}(i+1/2, j, k) &= C_{exex} E_x^n(i+1/2, j, k) + C_{exvx} V_{ex}^n(i+1/2, j, k) \\ &+ C_{exhy} [H_y^{n+1/2}(i+1/2, j, k+1/2) - H_y^{n+1/2}(i+1/2, j, k-1/2)] \\ &+ C_{exhz} [H_z^{n+1/2}(i+1/2, j+1/2, k) - H_z^{n+1/2}(i+1/2, j-1/2, k)] \end{aligned} \quad (21)$$

$$\begin{aligned} E_y^{n+1}(i, j+1/2, k) &= C_{eyey} E_y^n(i, j+1/2, k) + C_{eyvy} V_{ey}^n(i, j+1/2, k) \\ &+ C_{eyhz} [H_z^{n+1/2}(i+1/2, j+1/2, k) - H_z^{n+1/2}(i-1/2, j+1/2, k)] \\ &+ C_{eyhx} [H_x^{n+1/2}(i, j+1/2, k+1/2) - H_x^{n+1/2}(i, j+1/2, k-1/2)] \end{aligned} \quad (22)$$

$$\begin{aligned}
E_z^{n+1}(i, j, k + 1/2) &= C_{ezez} E_z^n(i, j, k + 1/2) + C_{ezvz} V_{ez}^n(i, j, k + 1/2) \\
&+ C_{ezhx} \left[H_x^{n+1/2}(i, j + 1/2, k + 1/2) - H_x^{n+1/2}(i, j - 1/2, k + 1/2) \right] \\
&+ C_{ezhy} \left[H_y^{n+1/2}(i + 1/2, j, k + 1/2) - H_y^{n+1/2}(i - 1/2, j, k + 1/2) \right] \quad (23)
\end{aligned}$$

Velocity Equations

$$V_{ex}^{n+1}(i+1/2, j, k) = \alpha V_{ex}^n(i+1/2, j, k) - C_{vzex} \left[E_{ex}^{n+1}(i + 1/2, j, k) + E_{ex}^n(i + 1/2, j, k) \right] \quad (24)$$

$$V_{ey}^{n+1}(i, j+1/2, k) = \alpha V_{ey}^n(i, j+1/2, k) - C_{vzey} \left[E_{ey}^{n+1}(i, j + 1/2, k) + E_{ey}^n(i, j + 1/2, k) \right] \quad (25)$$

$$V_{ez}^{n+1}(i, j, k+1/2) = \alpha V_{ez}^n(i, j, k+1/2) - C_{vzez} \left[E_{ez}^{n+1}(i, j, k + 1/2) + E_{ez}^n(i, j, k + 1/2) \right] \quad (26)$$