3D Format

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1 Orignal functions

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \tag{1}$$

$$\nabla \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} - en_e\vec{v}_e \tag{2}$$

$$\frac{\partial \vec{v}_e}{\partial t} = -\frac{e\vec{E}}{m_e} - \nu_m \vec{v}_e \tag{3}$$

Expand to seperated equations:

$$\frac{\partial H_x}{\partial t} = \frac{1}{\mu_0}\left[\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y}\right] \tag{4}$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu_0}\left[\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right] \tag{5}$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu_0}\left[\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right] \tag{6}$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\epsilon_0}\left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} + en_e v_{ex}\right] \tag{7}$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\epsilon_0}\left[\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} + en_e v_{ey}\right] \tag{8}$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\epsilon_0}\left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} + en_e v_{ez}\right] \tag{9}$$

$$\frac{\partial v_{ex}}{\partial t} = -\frac{eE_x}{m_e} - \nu_m v_{ex} \tag{10}$$

$$\frac{\partial v_{ey}}{\partial t} = -\frac{eE_y}{m_e} - \nu_m v_{ey} \tag{11}$$

$$\frac{\partial v_{ez}}{\partial t} = -\frac{eE_z}{m_e} - \nu_m v_{ez} \tag{12}$$

2 Approximations

First solve velocity equations:

$$\frac{v_e^{n+1} - v_e^n}{\Delta t} = -\frac{e}{2m_e}\left[E^{n+1} + E^n\right] - \frac{\nu_m}{2}\left[v_e^{n+1} + v_e^n\right] \tag{13}$$

after adjusting we get

$$v_e^{n+1} = \alpha v_e^n - \frac{e\Delta t}{2m_e\gamma}\left[E^{n+1} + E^n\right] \tag{14}$$

$$\alpha = \frac{1-a}{1+a}, \beta = \frac{\omega_p^2 \Delta t^2}{4\gamma}, \gamma = 1 + a, a = \frac{\nu_m \Delta t}{2} \tag{15}$$

Solving E-equations.

$$\frac{\epsilon_0}{\Delta t}\left[E^{n+1} - E^n\right] - \frac{en_e}{2}\left[v_e^{n+1} + v_e^n\right] = \nabla H \tag{16}$$

put (14) into (16) we get

$$E^{n+1} = E^n \frac{1-\beta}{1+\beta} + \frac{en_e \Delta t}{2\epsilon_0}\frac{1+\alpha}{1+\beta} v_e^n + \frac{\Delta t}{(1+\beta)\epsilon_0}\nabla H \tag{17}$$

3 Full iteration equations

H-equations

$$H_x^{n+1/2}(i, j{+}1/2, k{+}1/2) = H_x^{n-1/2}(i, j{+}1/2, k{+}1/2) + C_{hxey}\left[E_y^n(i, j+1/2, k+1) - E_y^n(i, j+1/2, k+1)\right] + C_{hxe} \tag{18}$$

where $C_{hxey} = \frac{\Delta t}{\mu_0 \Delta z}, C_{hxez} = -\frac{\Delta t}{\mu_0 \Delta y}$

$$H_y^{n+1/2}(i{+}1/2, j, k{+}1/2) = H_y^{n-1/2}(i{+}1/2, j, k{+}1/2) + C_{hyez}\left[E_z^n(i+1, j, k+1/2) - E_z^n(i, j, k+1/2)\right] + C_{hyex}\left[E_x^n\right. \tag{19}$$

where $C_{hyez} = \frac{\Delta t}{\mu_0 \Delta x}, C_{hyex} = -\frac{\Delta t}{\mu_0 \Delta z}$

$$H_z^{n+1/2}(i{+}1/2, j{+}1/2, k) = H_z^{n-1/2}(i{+}1/2, j{+}1/2, k) + C_{hzex}\left[E_x^n(i+1/2, j+1, k) - E_x^n(i+1/2, j, k)\right] + C_{hzey}\left[E_z^n\right. \tag{20}$$

where $C_{hzex} = \frac{\Delta t}{\mu_0 \Delta x}, C_{hzey} = -\frac{\Delta t}{\mu_0 \Delta z}$

E-equations

$$\begin{aligned}
&E_x^{n+1}(i+1/2, j, k) = C_{exex} E_x^n(i+1/2, j, k) + C_{exvx} V_{ex}^n(i+1/2, j, k) \\
&+C_{exhy}\left[H_y^{n+1/2}(i+1/2, j, k+1/2) - H_y^{n+1/2}(i+1/2, j, k-1/2)\right] \\
&+C_{exhz}\left[H_z^{n+1/2}(i+1/2, j+1/2, k) - H_z^{n+1/2}(i+1/2, j-1/2, k)\right]
\end{aligned} \tag{21}$$

$$\begin{aligned}
&E_y^{n+1}(i, j+1/2, k) = C_{eyey} E_y^n(i, j+1/2, k) + C_{eyvy} V_{ey}^n(i, j+1/2, k) \\
&+C_{eyhz}\left[H_z^{n+1/2}(i+1/2, j+1/2, k) - H_z^{n+1/2}(i-1/2, j+1/2, k)\right] \\
&+C_{eyhx}\left[H_x^{n+1/2}(i, j+1/2, k+1/2) - H_x^{n+1/2}(i, j+1/2, k-1/2)\right]
\end{aligned} \tag{22}$$

$$E_z^{n+1}(i,j,k+1/2) = C_{ezez}E_z^n(i,j,k+1/2) + C_{ezvz}V_{ez}^n(i,j,k+1/2)$$
$$+C_{ezhx}\left[H_x^{n+1/2}(i,j+1/2,k+1/2) - H_x^{n+1/2}(i,j-1/2,k+1/2)\right]$$
$$+C_{ezhy}\left[H_y^{n+1/2}(i+1/2,j,k+1/2) - H_y^{n+1/2}(i-1/2,j,k+1/2)\right] \quad (23)$$

Velocity Equations

$$V_{ex}^{n+1}(i+1/2,j,k) = \alpha V_{ex}^n(i+1/2,j,k) - C_{vxex}\left[E_{ex}^{n+1}(i+1/2,j,k) + E_{ex}^n(i+1/2,j,k)\right] \quad (24)$$
$$V_{ey}^{n+1}(i,j+1/2,k) = \alpha V_{ey}^n(i,j+1/2,k) - C_{vyey}\left[E_{ey}^{n+1}(i,j+1/2,k) + E_{ey}^n(i,j+1/2,k)\right] \quad (25)$$
$$V_{ez}^{n+1}(i,j,k+1/2) = \alpha V_{ez}^n(i,j,k+1/2) - C_{vzez}\left[E_{ez}^{n+1}(i,j,k+1/2) + E_{ez}^n(i,j,k+1/2)\right] \quad (26)$$