Problem Set 0 - Solutions

Please read the sections of the syllabus on problem sets and honor code before starting this homework.

1. [**8 points**] Prove that for every odd integer n, $n^2 - 1$ is divisible by 8.

Solution Let n be an odd integer. Then we can write $n = 2k + 1$, where $k \in \mathbb{Z}$. Then

$$n^2 - 1 = (2k+1)^2 - 1 = 4k^2 + 4k = 4(k^2 + k). \tag{1}$$

Note that if $k^2 + k$ is even, then $n^2 - 1$ is divisible by 8, and we have our result.

We conider two cases: k is even, or k is odd. If k is even, then k^2 is even, and so $k^2 + k$ is even, because the sum of two even numbers is another even number. If k is odd, then k^2 is odd, and so $k^2 + k$ is odd, because the sum of two odd numbers is an even number.

Therefore, in either case, $k^2 + k$ is even, so $4(k^2 + k)$, and hence $n^2 - 1$, is divisible by 8. for every odd integer n.