GRAVS2 manual

Software for relative gravity data processing

(Version 20210603)

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Introduction

GRAVS2 is a software package for relative gravity data processing and analysis with two main programs:

- **GRREDU3** for the conversion of readings and the addition of corrections due to calibration, tides, air pressure variations, the height change of gravimeter and polar motion, see details in Ch. 1.
- **GRADJ3** for the estimation of unknown gravity values and the parameters of instrumental drift, calibration etc through the weighted least-squares adjustment (generally described as the network adjustment), see Ch. 2.

GRAVS2 includes also other processing tools, e.g. CG5FORM, TFORM for data conversion (see Sec. 1.1), WZZ for the modeling of local vertical gradient (see Ch. 3) etc.

GRAVS2 uses the CGS (centimetre–gram–second) system of units: 1 mGal = 1000 μGal. The unit of data column (mGal or μGal) is generally listed in the column header. The relation between the SI (The International System of Units) and the CGS units is 1 $\mathrm{m/s^2} = 1 \cdot 10^5$ mGal $= 1 \cdot 10^8$ μGal, so 1 μGal= 10 $\mathrm{nm/s^2}$.

Input and output files of GRAVS2 package are ASCII text files[1], where generally the line with headers, labels (marked by `#` as a first character of line) are followed by lines containing input data, parameters, keys, results etc. It is suggested to use ANSI encoding with `CR+LF` line ending on Win OS and UTF-8 with `LF` on Linux.

The software GRAVS2 has been extensively tested and used for the high precision gravity data processing and network adjustment in Estonia, e.g. see Oja (2012a); Ellmann et al. (2009); Oja et al. (2019); Oja (2019). However, GRAVS2 can be easily adapted for lower precision works like the processing of gravity survey data, e.g. see Oja et al. (2011); Türk et al. (2011); Dmitrijeva et al. (2018).

GRAVS2 development began in 2003 with the aim of creating effective software for advanced gravity data processing with all necessary corrections and gravity network adjustment. Before the software development, several programs were tested:

1) Programs GRREDU and GRADJ (Andersen and Forsberg, 1996), written predominantly in ANSI[2] fortran 77 (along the standards of software package GRAVSOFT) and used through command-line interface (CLI).
2) Programs GRED and GADJD (Microsoft fortran 77, CLI) by FGI[3] (J. Mäkinen 2002, pers. comm.).

[1] ASCII – a character encoding standard for electronic communication (en.wikipedia.org/wiki/ASCII).
[2] ANSI - the American National Standards Institute.
[3] Finnish Geospatial Research Institute, former Finnish Geodetic Institute.

3) Software GravAP (Gravimetric Adjustment Package Software) with the graphical user interface (GUI in Windows) and with closed source code in C (T. Schueler 2000, pers. comm.).
4) Program GRAVNET (in fortran 90, CLI) by Hwang et al. (2002).
5) CG3TOOL tool by Gabalda et al. (2003), written in C (with GUI in Sun Solaris).

After the validation of different software, the programs GRREDU and GRADJ were selected to continue the development of software for advanced gravity data processing. The programs were compilable with GNU[4] compilers (g77, gfortran). Also software used through CLI was preferred to make processing quicker and more automatic by using scripting and task automation. In this way it was possible to extend the software capabilities by using batch processing[5] and other scripting languages like gawk[6], GMT[7] etc. Moreover, GRADJ functional model was preferred which combines the relative gravity readings (not the gravity differences derived from the readings) with unknown parameters (see Sec. 2.3).

From 2004 to 2019 the original programs were developed to the versions GRREDU3 and GRADJ3 (the latest versions GRREDU3.03, GRADJ3.05), bundled into the package GRAVS2. As mentioned before, the GRAVS2 includes additional tools for reformatting (module `tform`, from old GRREDU format to newer version), Scintrex CG5 raw data reformatting/pre-processing (module `cg5form`), a tool for vertical gradient evaluation (module `wzz`) and so on.

[4]The GNU project develops software which is free to copy, edit, and distribute.

[5]For batch processing (or shell scripting in UNIX-like systems) a command-line interface like MS-DOS emulated on Windows 7, 10 etc is used, e.g. by making and executing batch/script files.

[6]gawk - the GNU implementation of the AWK programming language.

[7]GMT - Generic Mapping Tools, an open-source collection of computer software tools for processing and displaying spatial (GIS, geodetic, ...) and temporal datasets (www.generic-mapping-tools.org).

Contents

Chapter 1

GRREDU3

GRREDU3 is used to correct the field data of relative gravimeters for tides, calibration, sensor height change etc. The full list of its capabilities are given below, followed by more detailed descriptions.

The International Absolute Gravity Basestation Network (IAGBN) processing standards (Boedecker, 1988, 1993), created within the Working Group II (World Gravity Standards) of the International Gravity Commission (IGC) and de facto standards in gravimetry, are predominantly followed in GRREDU3 data processing.

GRREDU3 data processing capabilities are as follows:

1) **Conversion of gravimeter's units to CGS units**, which are divided into two parts:
   (a) by using **factory constant or table** to convert the counter units (C.U.) of gravimeters like LCR G-type, GNU, Delta etc. to CGS units mGal, µGal;
   (b) by applying **calibration correction model** with the coefficients for time-dependent (e.g. Scintrex CG-5), polynomial (all kind of gravimeters) and periodic (e.g. LCR G, D) correction functions.
2) **Tidal correction** on the basis of Tamura's tidal potential development with 1,200 waves (Tamura, 1987), with local amplitude factor δ and phase lag κ for the main wave groups interpolated from the global grid (Timmen and Wenzel, 1994), and permanent tide treated according to the zero system concept, i.e. for (time-constant M_0S_0 the amplitude factor is $\delta = 1.0$ in agreement with IAG Resolution (IAG, 1984) and IAGBN standards.
3) **Free-air correction** to remove the effect due to the variable height of gravimeter's sensor during the measurement, by using the conventional constant value of vertical gradient -0.31 µGal/mm or the parameters of linear relation determined locally.
4) **Air pressure correction** to reduce the effect caused by the direct attraction and loading of atmospheric mass redistribution, bu using local air pressure, normal pressure at the sea level from model (The U.S. Standard Atmosphere 1976 or DIN-5450) and a coefficient -0.3 µGal/hPa.
5) **Secular gravity correction** to correct the temporal gravity change, e.g. due to the GIA[1] process in Northern Europe by using land uplift model NKG2016LU_abs

[1]GIA – Glacial Isostatic Adjustment, geophysical process due to the adjustment of Earth crust after massive ice sheet melt.

with coefficient -0.163 µGal/mm (Olsson et al., 2019; Oja et al., 2021).

6) **Polar motion correction** to reduce the effect caused by the motion of the rotation axis of the Earth relative to the crust, based on the pole coordinates x, y of the Celestial Ephemeris Pole (CEP) relative to the IERS[2] Reference Pole (IRP).
NB! The polar motion has a rather insignificant effect on relative gravimetry and is generally not applied.

1.1 Pre-processing

Before the usage of GRREDU3 pre-processing may be needed with tools like:

a) TFORM – to transform the previous GRREDU1 or GRREDU2 files from old format to the latest GRREDU3 format;
b) CG5FORM (previously SFORM) – to convert Scintrex CG-5 observation file (*.txt) to GRREDU3 input format (*.obs);
c) GTPAR – to interpolate parameters for the main tidal wave groups from the global grid by Timmen and Wenzel (1994);
d) WZZ2 – to evaluate the vertical gradient of gravity station along local vertical (plumbline) by using polynomial model or combined RCR method (Ch. 3).

1.1.1 Conversion of CG-5 obs file

The following conversion with CG5FORM is based on the CG-5 observation file (file `*.txt` as the output of Scintrex tool *SCTUTIL*). The CG-5 measurements should be done in "XYm" mode, selected onboard[3] of CG-5, see also CG-5 manual (CG5 Manual, 2012). Then **unique ID number of station** can be recorded during the measurement and used in following data processing. The ID number of station with positive integer value (with a range of max 8 digits) is important for connecting different data sources, therefore station ID should be unique and consistent over these data sources.

Part of CG-5 obs file (full file in App A.1):

/Examples/2010-03-17_GulfofRiga/S36/2010-03-17_S36.txt

```
/        CG-5 SURVEY
/        Survey name:            Gulf-of-Riga(Survey-on-ice)
/        Instrument S/N:         36
/        Client:                 ELB
/        Operator:               T.OJA
/        Date:                   2010/ 3/17
...
Line           3.000N
/------LINE-----STATION-----ALT.------GRAV.---SD.--TILTX--TILTY-TEMP---TIDE---DUR-REJ-----TIME----DEC.TIME+DATE--TERRAIN---DATE
 3.0000000  80006.0000000   20.5565   5120.256 0.020   17.8   30.9 -2.58 -0.037  60   0 07:49:09     40225.32528    0.0000  2010/03/17
 3.0000000  80006.0000000   19.3358   5120.246 0.016  -12.4   -0.1 -2.60 -0.037  60   0 07:50:37     40225.32629    0.0000  2010/03/17
 3.0000000  80006.0000000   18.6033   5120.250 0.024   -6.0    6.4 -2.62 -0.036  60   0 07:51:45     40225.32708    0.0000  2010/03/17
 3.0000  10031711.0000000   14.4530   5110.218 0.024   10.7   13.2 -2.49 -0.023  60   0 08:25:42     40225.35062    0.0000  2010/03/17
 3.0000  10031711.0000000   13.2323   5110.218 0.025   13.5   17.4 -2.52 -0.023  60   1 08:26:48     40225.35138    0.0000  2010/03/17
...
 4.0000000  80006.0000000    9.3260   5120.207 0.020  -15.3    6.3 -2.40 -0.026  40   0 14:02:43     40225.58428    0.0000  2010/03/17
 4.0000000  80006.0000000    9.5702   5120.206 0.013   -3.6   -2.2 -2.43 -0.026  40   0 14:03:47     40225.58502    0.0000  2010/03/17
```

CG5FORM is executed twice to add the numerical values of gravimeter's height from base h_{inst}, the base height h_{base} from reference point (e.g. from benchmark, see Fig. 1.2) and local air pressure p (value -999.9 if not observed). First step makes two files: `*.obs` and file `add.inf` filled with default values for every station. After filling `add.inf` with h, p values (eg from fieldbook) and renaming it (same name with `*.obs` is suggested to use), CG5FORM is run again to get those values to `*.obs` file.

[2] IERS – International Earth Rotation and Reference Systems Service, see https://www.iers.org.
[3] CG5 Setup: Menu → Survey → PARAMS(F1) → Station Designation System: XYm.

Example of `*.inf` file:

```
/Examples/2010-03-17_GulfofRiga/S36/2010-03-17_S36.inf
# S- 36   Gulf-of-Riga(Survey-on-ice)    2010 ELB      T.OJA
   80006  2010-03-17  07:49:39     355  -20   -999.9
10031711  2010-03-17  08:26:12     345    0   -999.9
...
   80006  2010-03-17  14:02:18     357  -20   -999.9
```

For the execution of CG5FORM on CLI use:
`cg5form2f < sform2.inp`
where

```
/Examples/2010-03-17_GulfofRiga/S36/sform2.inp
300 0 -999.9
8
2010-03-17_S36.txt
2010-03-17_S36.inf
```

The content of `sform2.inp` (line by line):

1) default values for h_{inst}, h_{base}, p;
2) max time difference dt in hours allowed between measurements $i, i-1$, ie if $dt > t_i - t_{i-1}$ then new header is entered to `*.obs` file;
3) CG5 `*.txt` file name;
4) name of `*.inf` file.

First run of CG5FORM should be done without name of `*.inf` file (also arbitrary name of non-existing file suits, e.g. ECHO, temp etc) to make file `add.inf` with default values. By running script file (e.g. `cg5form.bat`) without second argument (name of `*.inf`) it is done automatically.

1.2 Input data of GRREDU3

1.2.1 Input parameters and filenames

The list of input `parameters` and <filenames> of GRREDU3 (with comments after "!"):

```
/Examples/2010_Calibration/2010_Haanja-Toila_base_S36/grredu3.inp
0 2000-01-01                      !timezone, epoch
T T T T F F                       !ltide, lpres, lfree, ltime, lcali, lpmot
0 2 0                             !iprint,imodel,irigid
-0.3                              !pcoef
d:\GRAVS2\share\coord.sta         !<stations' coordinate file>
d:\GRAVS2\share\coord.tid         !<stations' tidal file>
d:\GRAVS2\share\ETCPOT.DAT        !<tidal potential development file>
nofile                            !<IERS time series (C04) file, use temp name if not used>
d:\GRAVS2\share\LCRmeter.tab      !<inst. calibration table file>
d:\GRAVS2\share\gmeters.par       !<inst. parameters file>
2010-07-06_S36.obs                !<observation file(s)>, one or several...
2010-07-07_S36.obs
2010-07-08_S36.obs
2010-07-09_S36.obs
```

The explanation of input parameters (line by line):

1) parameters `timezone, epoch`:
   `timezone` = 0 for UTC (Coordinated Universal Time), = +2 for EET (Eastern European Time, i.e. winter time in Estonia), = +3 for EEST (Eastern European Summer Time, i.e. summer time in Estonia);

epoch = 2000-01-01, used to transform all observations from measurement time (date+time) to single epoch by using long-term gravity correction;

2) `ltide, lpres, lfree, ltime, lcali, lpmot = t t t t t f`, to set **t**rue/**f**alse (or **T**rue/**F**alse) values to switch corrections (tides, air pressure, free-air, temporal gravity, calibration, polar motion) **ON or OFF**;
3) `iprint, imodel, irigid = 0 2 0`, parameters needed for tidal corrections, no need to change, for more details look into source code of ETGTAB (see below);
4) `pcoef`$= -0.3$, coefficient value [µGal/hPa] used for air pressure correction;
5) list of 6 input files with information about stations (`coord.sta`), tides (`coord.tid, ETCPOT.dat`), polar motion (currently not given) and instruments (`LCRmeter.cal, gmeters.par`);
6) list of all `*.obs` files (current code allows `maxfile=150` names).

It is strongly suggested to record measurements in UTC (e.g by setting gravimeter's clock to UTC) to avoid later any vagueness in data process.

1.3 Computation of corrections

1.3.1 Conversion of gravimeter's units to CGS system

The convention used for **gravimeter's ID name** (consists of type name with single or few letters and unique integer number) in input files is # <letter>-<number>. The examples are

- LCR gravimeters: LCR G # 193 ⇒ `#␣G-193␣␣`, LCR G # 1150 with feedback system (needs additional column of FB values after column of readings in obs file) ⇒ `#␣G-1150F`;
- Scintrex CG-5 # 10092 ⇒ `# S-92` or `# S-10092` or `# CG5-10092`;
- Burris gravimeter #55 ⇒ `# B-55`;
- Other examples like Soviet GAG-2 gravimeter # 21: `# GAG2-21`, GNU-K2 or Delta gravimeters: `# GNUK2-583`, `# Delta-80`, etc.

It is mandatory to give letter "G" and addition "F" for LCR G type[4] gravimeters to ensure correct data processing with calibration table, periodic error corrections and additional FB values in obs file. Note that characters 3...9 of header line are strictly reserved for the ID name of LCR G meter. However, the name can be also longer than 7 chars (see examples above).

Calibration constant or table by manufacturer

If the readings of gravimeter are in instrument's own units, so called counter units (C.U.), then the measurements have to be converted from C.U. to the system of units like CGS or SI. Currently GRREDU3 converts readings to CGS unit (mGal). For the conversion, the single constant (scale factor) or full calibration table is needed. Those are generally delivered by gravimeter's manufacturer, but can be determined also by user (through calibration procedures).

For LCR gravimeter the calibration table provided by the manufacturer is used for the conversion of readings from C.U. to mGal according to the method given in instrument's

[4] Currently LGR D type gravimeter is not supported, but letter "G" can be also used for that type.

manual. For GRREDU3, only the part of the LCR calibration table that covers the measuring range needs to be copied to the input file `LCRmeter.cal`.

The raw readings of Scintrex CG-5 (and other CG-x type) gravimeters are converted automatically with software onboard using single scale factor *GCAL1* determined by manufacturer. However, user can change the value of *GCAL1* as well.

Calibration correction by user

Most of the modern type relative gravimeters are (metal, quartz) spring instruments with complex mechanical and electronic components on board. If the properties of the spring and other components change due to the normal aging (related to permanent deformation, normal creep) or other processes (shock, vibration, mishandling etc.) then the scale change could be expected.

Also the calibration of gravimeter determined by manufacturer in their laboratory and nearby calibration line (in geographically limited area) may cover only limited gravity range (eg 100-200 mGal) which is not enough for the calibration of instrument's full range (about 7-8 Gal). Another issue is the unknown type of calibration errors (not determined by the manufacturer) like LCR periodical scale errors due to the gearbox imperfection, or temporally changing scale.

For example, the calibration factor *GCAL1* of CG-5 is determined with an accuracy about 85 ppm[5] by Scintrex. After production, *GCAL1* may initially change 1...2 ppm per day (during few months period), due to the stress relaxation effects in the newly fused quartz spring (CG5 Manual, 2012). Thus, after 2-3 months, the scale change of newly purchased CG-5 may be 60...180 ppm. However, repeated scale check of Scintrex CG-5 gravimeters along the calibration lines in Estonia by Oja (2019) showed long-term scale changes, see Fig. 1.1.

Different type of calibration functions are used in GRREDU3:

1) polynomial model (mostly with degree $n = 1$) for all kind of gravimeters;
2) periodic model for LCR G/D type gravimeters due to the systematic effect of their gearbox imperfection on instrument's readings;
3) linear time-dependent model for gravimeters with single scale factor (e.g. Scintrex CG-5) to correct the effect of time-varying scale.

Polynomial model is

$$\Delta F(z)_{pol} = \sum_{i=1}^{n} \Delta c_i z^i, \tag{1.1}$$

where n is the degree of polynomial to evaluate, Δc_i $(i = 1 : n)$ are inserted calibration coefficients from gravimeter's parameter file and z is the reading of gravimeter to correct. It is noted that the degree of polynomial n should be kept rather small ($n \leq 3$) to keep numerical computation errors low[6] and also to avoid overfit. This is true for other polynomial models (e.g. drift) as well.

[5] ppm (parts-per-million, 1 ppm $= 1 \cdot 10^{-6}$) is useful unit to describe the scale change and calibration of relative gravimeter. Scale or calibration error 100 ppm means that by measuring gravity difference with range of 100 mGal the error is $100 \cdot 100 \cdot 10^{-6} = 1 \cdot 10^{-2} = 0.01$ mGal = 10 µGal.

[6] The coefficients of the polynomial higher degree terms could became very large or small and thus cause numerical computation errors.

If user enters value $n = 99$ through the input file then $i = 1$ and calibration function is evaluated as

$$\Delta F(z)_{pol} = (1 - c_1)z, \tag{1.2}$$

where $c_1 = 1 - \Delta c_1$ can be found as the adjusted scale factor from the output of GRADJ3, see Sec. 2.3.2.

Periodical model for LCR meters is

$$\Delta F(z)_{per} = \sum_{i=1}^{r} \Delta A_i \sin(2\pi z/P_i + \Delta\varphi_i), \tag{1.3}$$

where ΔA_i and $\Delta\varphi_i$ ($i = 1 : r$) are amplitude [µGal] and phase [rad] values given by user[7] for the r waves.

With negative integer values ($n = -2, -3, \ldots$)[8] entered by user a time-dependent linear model

$$\Delta F(z)_{tem} = c(t) \cdot z \; (\cdot 10^{-6}) \tag{1.4}$$

is evaluated, where scale change $c(t)$ at observation epoch t ($t_{i-1} \leq t \leq t_i$) is interpolated linearly

$$c(t) = \frac{c(t_i) - c(t_{i-1})}{t_i - t_{i-1}}(t - t_{i-1}) + c(t_{i-1})$$

from the $i = 2 : (-n)$ tabulated values. Note that if $i = 3 : (-n)$, a piecewise linear function is actually used to model the scale change of gravimeter. To avoid extrapolation, t_1 and t_n values should not be lower or higher than the epoch of first and last calibration measurement. If the measurement epoch is outside of table range ($t < t_1$ or $t > t_n$) then the first $c(t_1)$ or last tabulated value $c(t_n)$ as a constant value is used by GRREDU3.

As an example the values $t_i, c(t_i)$ (for case $n = -2, i = 1 : 2$) for Scintrex CG-5 gravimeters used in Estonia are presented (Table 1.1 and Fig. 1.1), estimated from the precise gravity measurements along dedicated calibration lines (Oja, 2019).

Table 1.1: Tabulated values $c(t_i)$ ($n = -2 \Rightarrow i = 1 : 2$) at epoch t_i from the fit of linear regression in Fig. 1.1 to model scale change of Scintrex CG-5 gravimeters.

Gravi-meter	t_i [yr]	$c(t_i)$ [ppm]
S36	2004.40	-185.3
	2018.54	453.6
S92	2005.60	315.4
	2018.54	636.0
S156	2014.37	-196.1
	2018.54	-283.6

All the input coefficients and thus also $\Delta F(z)$ value describes the scale error. Therefore, the calibration correction is computed as

1) $dg_{cal} = -[\Delta F(z)_{pol} + \Delta F(z)_{per}]$ for LCR gravimeters;
2) $dg_{cal} = -\Delta F(z)_{pol}$ for all type of gravimeters (LCR, CG-5, Burris etc);
3) $dg_{cal} = -\Delta F(z)_{tem}$ for gravimeters with the time-dependent scale factor change.

[7]Actually phase values are given in degrees [°], the conversion to radians is made internally.
[8]The case $n = -1$ would give constant scale factor which are covered by above cases $n = 1, 99$.

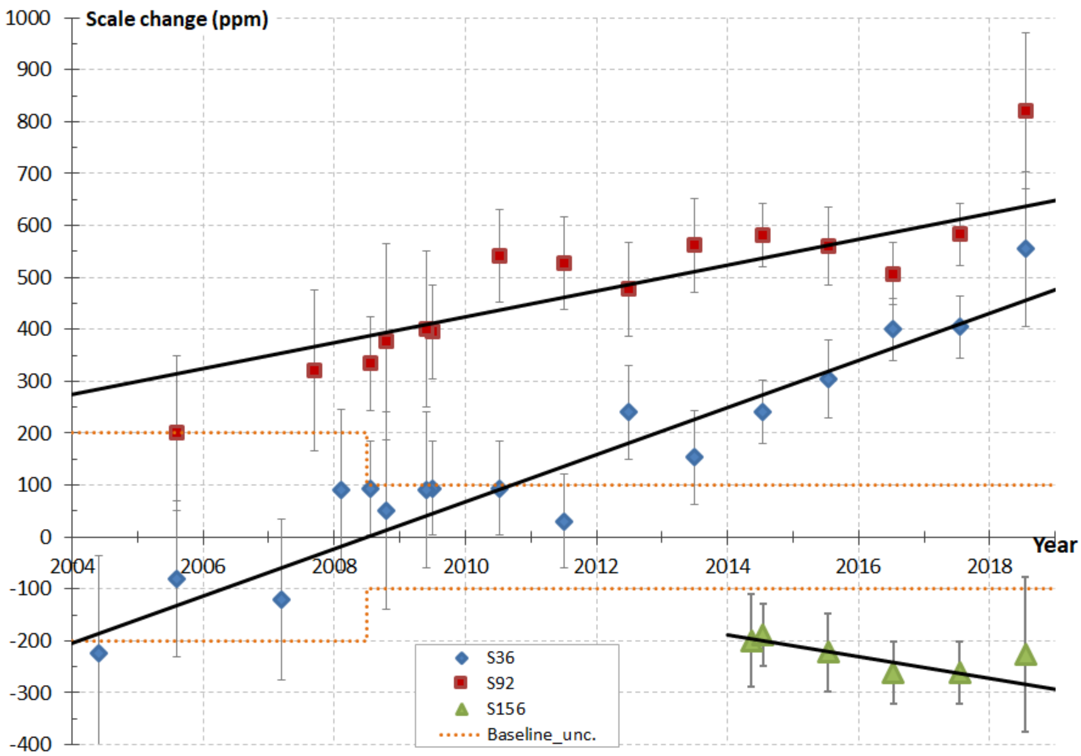

Figure 1.1: The result of scale check of Scintrex CG-5 gravimeters along the calibration lines in Estonia from 2004 to 2018 (Oja, 2019). The weighted ordinary least squares fit of linear regression was used to model time-dependent scale change.

1.3.2 Tidal correction

For the tidal correction dg_{tide} the ETGTAB software by Wenzel (1994) was modified and integrated into GRREDU3. The tidal potential development by Tamura (1987) with 1200 waves is used as a default option (`imodel=2`). Other potential development with less waves (e.g. Cartwright-Tayler-Edden model with 505 waves, `imodel=1`) may be selected, e.g. for testing purpose. However, the default Tamura's development gives the most accurate tidal corrections, with maximum error below 0.1 µGal in time domain (Van Camp and Vauterin, 2005). For the accurate results in the frequency domain, local amplitude factor δ and phase lag κ of the main wave groups are interpolated (by using program GTPAR) from the global 1°x1° grid predicted by Timmen and Wenzel (1994). The permanent tide is treated according to the zero system concept, i.e. for M_0S_0 $\delta = 1.0$ (IAG, 1984). More information about the global grid with δ and κ values in file `WPARM.DAT` are found in App. B.

1.3.3 Air pressure correction

The air pressure effect on gravity due to varying physical properties of atmosphere is approximated by model

$$\delta g_{atm} = C_p(p - p_n), \tag{1.5}$$

where C_p is input coefficient `pcoef` with recommended value -0.3 µGal/hPa (e.g. by IAGBN standards) to relate changing air pressure with gravity change, p [hPa] is air pressure value measured during the measurement at observation location and p_n [hPa] is normal air pressure relative to the sea level.

The normal pressure is aproximated by the model

$$p_n(H_s) \approx P_b \left(1 + \frac{L_b H}{T_b}\right)^{-\frac{g_0 M}{R^* L_b}} = 1013.25 \left(1 + \frac{-6.5 \cdot 10^{-3} \cdot H}{288.15}\right)^{5.2559} \text{[hPa]}, \tag{1.6}$$

where H_s [m] is the height above sea level (approximated by normal/orthometric height H of observation site). The values of parameters (especially M, g_0, and R^*) are in accordance with the U.S. Standard Atmosphere, 1976 (NOAA, 1976): $P_b = 1013.25$ kPa is static pressure, $T_b = 288.15$ K is standard temperature, $L_b = -6.5\text{E}{-03}$ K/m is standard temperature lapse rate, $R^* = 8.31E{+}03$ N·m/(kmol·K) is universal gas constant, $g_0 = 9.80665$ m/s^2 is standard gravity, $M = 28.9644$ kg/kmol is molar mass of Earth's air.

Notice that the normal/orthometric height H of observation location should be known, otherwise it is impossible to calculate the air pressure correction correctly.

The air pressure correction is $dg_{airp} = -\delta g_{atm}$.

1.3.4 Free-air correction

Free-air correction $dg_{fa} = dg_{red}$ helps to reduce the gravity readings from sensor height $h_{sens} = h_{inst} - h_{sys}$ to reference height ($h = 0$), e.g. to the benchmark level (Fig. 1.2).

If a linear term a with unit [$\cdot(-1/10)$ µGal/m], and a quadratic term b with unit $\cdot(-1/10)$ µGal/m^2] of polynomial function ($n = 2$) to model the change of gravity along local vertical are given in coordinate file (Sec. 1.3.7), then the correction is evaluated by

$$dg_{red} = (a\ dh + b\ dh^2)/10, \tag{1.7}$$

where $dh = (h_{inst} - h_{sys})$.

In case of unknown gradient the conventional values of coefficients $a = 3086$ [$\cdot(-1/10)$ µGal/m], $b = 0$ [$\cdot(-1/10)$ µGal/m^2] are used.

If negative value $a < 0$ is set by user (unrealistic case) then the link of WZZ2 output file (`*.vgg`) is expected as b value in coordinate file. Now correction is difference between $g(0)$ at zero level ($h = 0$) and $g(h)$ at level h taken from the second column of WZZ file (`*.vgg`)

$$dg_{red} = g(0) - g(h). \tag{1.8}$$

The columns of WZZ2 output file are

1) h [m]
2) $g(h)$ [µGal]
3) VG(h) as constant gradient [µGal/mm]
4) $dg = g(h) - g(0)$
5) uncertainty of dg

Few rows of WZZ file (consists of 1501 rows, see Sec. 3.1) are shown here as an example of WZZ output:

```
/share/vgg-files/TORA_1995-2017.vgg
   0.000    981759669.747    -327.615  0.127        0.000  0.000
   0.001    981759669.420    -327.602  0.127       -0.328  0.000
...
   0.300    981759572.237    -322.179  0.127      -97.510  0.038
```

```
   0.301   981759571.915    -322.160  0.127     -97.832  0.038
...
   1.499   981759194.541    -311.485  0.127    -475.207  0.190
   1.500   981759194.229    -311.483  0.127    -475.518  0.190
```

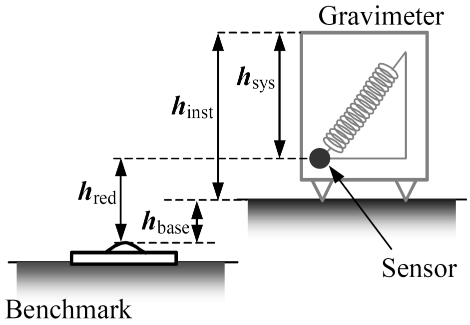

Figure 1.2: Position of the gravimeter's sensor relative to the reference height on top of the benchmark (Oja, 2012b).

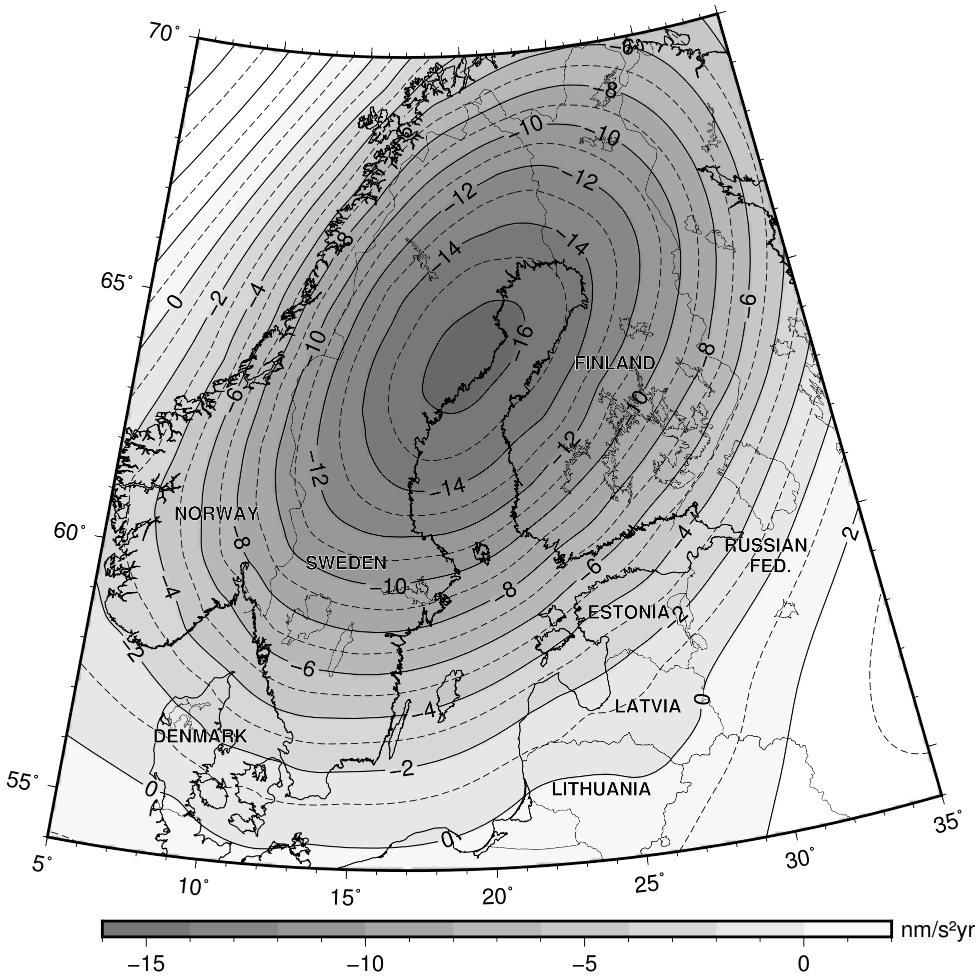

Figure 1.3: The contour lines present secular gravity change (units nm/s^2 per yr) in Northern Europe from the land uplift model NKG2016LU_abs by applying the relation of -1.63 nm/s^2 per mm between gravity rates and vertical rates.

1.3.5 Secular gravity correction

For secular gravity correction a linear function is used

$$dg_{sec} = \dot{g}(T_0 - t) \tag{1.9}$$

where $\dot{g}$ is a constant rate of gravity change [µGal/yr] at observation point (its value is taken from coordinate file), T_0 is epoch given as an input parameter `epoch` and t is the measurement time (observation epoch). For countries in Northern Europe the $\dot{g}$ values can be predicted from NKG2016LU_abs model (Vestøl et al., 2019) multiplied by the coefficient -0.163 µGal/mm (Fig. 1.3), for more details see Olsson et al. (2019).

1.3.6 Polar motion correction

For the polar motion correction dg_{polm} the `POL` subroutine (from FGI software GRED) was integrated into GRREDU3. It implements a spherical model for polar motion effect

$$dg_{polm} = -\omega^2 R\delta \sin 2\varphi \; d\varphi = -\omega^2 R\delta \sin 2\varphi (x_p \cos\lambda - y_p \sin\lambda), \tag{1.10}$$

where δ is amplitude factor (conventionally as $\delta = 1.16$), ω is the rate of Earth's rotation ($\omega = 7.2921151467 \cdot 10^{-5}$ [1/s]), R is the geocentric radius (derived from geodetic coordinates and height of observation point on GRS-80 ellipsoid), φ and λ are geocentric latitude and longitude, $d\varphi$ is the latitude variation due to polar motion and is related to the pair of coordinates x_p, y_p presents the pole position relative to the IRP (the IERS Reference Pole) on a XY plane over North pole. Note that the convention is to define x_p to be positive along 0° longitude and y_p along 90°W longitude.

Observed and predicted polar motion data is available from the IERS EOP (Earth Orientation Parameters) products (IERS EOP products or `http://hpiers.obspm.fr/eop-pc/index.php`). Suitable input file for GRREDU3 is *EOP (IERS) 05 C04* file `eopc04.62-now` with daily time series from ftp://hpiers.obspm.fr:

```
                                      /share/eopc04.62-now
     Date      MJD      x          y        UT1-UTC      LOD        dPsi      dEps       x Err     y Err
                        "          "           s          s           "         "          "         "
...
2006   8   3  53950   0.110255   0.269987   0.1805768  0.0001652  -0.064623  -0.005785  0.000012  0.000017 ..
2006   8   4  53951   0.109349   0.269044   0.1804962  0.0000020  -0.065151  -0.006198  0.000010  0.000013 ..
2006   8   5  53952   0.108806   0.267900   0.1805554 -0.0001196  -0.065287  -0.006365  0.000010  0.000014 ..
...                                                                                                        ..
2011   7   4  55746   0.048346   0.437909  -0.2906765  0.0002643  -0.070677  -0.011023  0.000100  0.000082 ..
2011   7   5  55747   0.050433   0.439020  -0.2910896  0.0004549  -0.071077  -0.010782  0.000103  0.000088 ..
2011   7   6  55748   0.052629   0.440137  -0.2916145  0.0005111  -0.071626  -0.010595  0.000200  0.000125 ..
...
```

1.3.7 GRREDU3 input

GRREDU3 input files are:

1. **Coordinate file** with format given by columns:

   Col 1-2: Point ID no (int*8, %8*i*) & name (text*12, %12*a*)

   Col 3-4: Lat & Lon coordinates [°] (float*.6, %.6*f*)

   Col 5: Normal height [m] (%.3*f*)

   Col 6: Gravity rate [µGal/yr] (%.2*f*)

Col 7-8: Linear* [µGal/m $\cdot(-1/10)$] and quadratic terms [µGal/m$^2 \cdot (-1/10)$] (both integer variables, %i) for polynomial model of local vertical gradient

*If linear term has negative integer value, the path of special VGG file is expected instead of quadratic term, see also Sec. 1.3.4.

Example file:

/share/coord.sta
```
80003  TõravereAG 58.264339  26.463292  71.964 -0.19 -9999 d:\GRAVS2\share\vgg-files\TORA_1995-2017.vgg
80702  HaanjaAG   57.721700  27.050789 245.470 -0.08 -9999 d:\GRAVS2\share\vgg-files\Haanja_2008-2019.vgg
80704  ToilaAG    59.422270  27.536341  36.469 -0.32 -9999 d:\GRAVS2\share\vgg-files\Toila_2008-2017.vgg

807022 Haanja2    57.721700  27.050790 245.40  -0.08  2885 -64
80006  ReiuGR     58.298770  24.610295   6.288 -0.27  3238   0
...
```

2. **Tidal parameters' file** with interpolated synthetic gravity tide parameters for the 13 main tidal wavegroups from the 1°x1° global grid (see Sec. 1.3.2), as an output of program `GTPAR`:

/share/coord.tid
```
#     80003 TõravereAG     58.264339  26.463292     71.964
    1     1  MOSO  1.0000  0.0000
    2   285    MF  1.1715 -0.1877
  286   428    Q1  1.1475  0.0489
...
 1005 1121    K2  1.1671 -0.2093
 1122 1214    M3  1.0682  0.0000
#     80702 HaanjaAG       57.721700  27.050789   245.470
    1     1  MOSO  1.0000  0.0000
    2   285    MF  1.1713 -0.1804
...
```

3. Separate file with **calibration tables for LCR** type gravimeters:

/share/LCRmeter.tab
```
# G-191
5000        5190.380        1.04170
5100        5294.550        1.04170
5200        5398.720        1.04168
...
# G-193
5000        5155.794        1.03300
5100        5259.094        1.03296
...
```

4. **Gravimeter's parameter file** with sensor height (from the top plate) and calibration coefficients for time-dependent or polynomial and/or periodic correction functions (see Sec. 1.3.1):

/share/gmeters.par
```
# S-36
211
1
0.976270E-04        !scale change found 97.6 +/- 7.5 ppm from Haanja-Toila AG baseline
# S-92
211
-2
2005.60   315.4
2018.54   636.0

# G-191
159
1
0.673860E-03        ! 673.9 +/- 30.4 ppm
1
70.9412,   66.7742,  185.7637
```

5. **Observation file** containing field measurements, formatted as:

   Col 1: Point ID number (%8i)

   Col 2-3: Date (yyyy-mm-dd), time (hh:mm:ss)

   Col 4-5: Reading of gravimeter & st.dev [C.U. or mGal] (%.4*f*)

   Col 6: Height of gravimeter [mm] (%i)

   Col 7: Air pressure [hPa] (%.1f)

   Example file:

   /Examples/2010_Calibration/2010_Haanja-Toila_base_S36/2010-07-06_S36.obs

```
# S- 36    TOIL-HAAN         2010 Maa-amet T.OJA
   80003  2010-07-06  09:41:16   6824.9910  0.0130   335   1003.0
   80003  2010-07-06  09:42:03   6824.9910  0.0170   335   1003.0
...
   80702  2010-07-06  11:41:02   6744.2050  0.0130   374    982.2
   80702  2010-07-06  11:41:49   6744.2060  0.0140   374    982.2
...
   80003  2010-07-06  17:34:05   6825.1150  0.0170   335   1000.3
   80003  2010-07-06  17:34:50   6825.1160  0.0150   335   1000.3
```

   Note that values in two last columns are tested by the program:

   - If observed height of gravimeter ≤ -9999 (can be used as unknown value) then no free-air correction is computed.
   - If observed air pressure p differs by more than 100 hPa from the normal pressure p_n by Eq. (1.6) (this includes $p = -999.9$ as unknown value) then no air pressure correction is computed.

1.3.8 GRREDU3 output

Output file of GRREDU3 is the reduced observation file where the headers explain the contents of columns and give units of numerical values:

/Examples/2010_Calibration/2010_Haanja-Toila_base_S36/2010-07-06_S36.redu

```
                                           |-C.U./mGal----uGal|-----------CORRECTIONS-(uGal)-----------|--------mGal--------|
 station       date,      time      obs     reading    stdev     tides    air-    free-    polar     gdot      calibr.  reduced   station
      ID                (UT + 0)     ID                                   pres.   air      motion              error    reading   name
# S- 36   TOIL-HAAN         2010 Maa-amet T.OJA
   80003  2010-07-06, 09:41:16       1  6824.9910     13.0       9.0    -0.5     40.5      0.0      2.0     0.0000  6825.0421 TõravereAG
   80003  2010-07-06, 09:42:03       2  6824.9910     17.0       8.8    -0.5     40.5      0.0      2.0     0.0000  6825.0418 TõravereAG
...
   80702  2010-07-06, 11:41:02      17  6744.2050     13.0     -24.6    -0.6     46.9      0.0      0.8     0.0000  6744.2276 HaanjaAG
   80702  2010-07-06, 11:41:49      18  6744.2060     14.0     -24.8    -0.6     46.9      0.0      0.8     0.0000  6744.2284 HaanjaAG
...
   80003  2010-07-06, 17:34:05      79  6825.1150     17.0     -69.7    -1.3     40.5      0.0      2.0     0.0000  6825.0865 TõravereAG
   80003  2010-07-06, 17:34:50      80  6825.1160     15.0     -69.7    -1.3     40.5      0.0      2.0     0.0000  6825.0875 TõravereAG
```

Chapter 2

GRADJ3

GRADJ3 performs the adjustment of unknown parameters (through the pre-defined functional model) based on the reduced readings of relative gravimeters. The weighted least squares method is used for the adjustment where different weights can be defined for different set of measurements. More loosely speaking, GRADJ3 can be used for gravity network processing with simultaneous drift computation.

GRADJ3 comprises

- **Weighted least squares adjustment** by using fast and memory efficient Cholesky matrix decomposition
- **Functional model with drift correction** by using polynomial drift model $D_n(t)$ with degree n (recommended $n \leq 3$, limited $n \leq 5$ by software)
- **Stochastic model** with different weighting options for input data (through diagonal weight matrix)
- **Statistical testing** such as Student's t-test, χ^2-test etc to test statistical significance of estimated parameters within defined confidence intervals
- **Blunder detection** (outlier detection) by using standardized residuals and Pope's τ-test (Pope, 1976) at confidence level fixed by user
- **Additional statistics:** redundancy estimation, full covariance matrix of adjusted parameters, Akaike and Bayesian information criteria etc

2.1 Input parameters and files

Input text file of GRADJ3 has following lines:

1) Project file name `<project_name>.proj` with several (global) adjustment parameters where `<project_name>` is used to name all output files of current adjustment project, for more details see Sec. 2.1.1
2) The name of fixed stations file (e.g. file `FIXED`) with the list of stations' ID, gravity values and uncertainties to constrain adjustment, Sec. 2.1.3
3) Scale factor (μGal/div) for residual graph in output `*.resi`, `*.resi.mean` files
4) Verbose mode `ltest = T/F` for program testing purposes, which by default is `ltest = F`
5) The list of file names (from 1 to 500 names[1]) with reduced observations (GRREDU3

[1]Limited by parameter `maxfile`=500 in source code.

output with `*.redu` extension, see Sec. 1.3.8)

Example of GRADJ3 input file:

/Examples/2010_Calibration/2010_Parnu_base/AdjALL/Parnu_base2010_gradj3.inp

```
Parnu_base2010.proj
FIXED
2
F
d:\GRAVS2\data\Examples\2010_Calibration\2010_Parnu_base\S36\2010-04-22_S36.redu
d:\GRAVS2\data\Examples\2010_Calibration\2010_Parnu_base\S36\2010-04-23_S36.redu
d:\GRAVS2\data\Examples\2010_Calibration\2010_Parnu_base\G191\2010-04-21_23_G191.redu
```

Notice that:

- If no stations are given in fixed station file (in case of empty file or all fixed stations commented out by using "`!`" at the beginning of line) then adjustment is generally computed successfully. Warning is given about singular entries in normal equations but inversion is computed and results generated. Practically it is the unconstrained (free) adjustment, although there are better and more suitable methods (eg pseudoinverse) than the Cholesky decomposition (Sec. 2.4.5) for such a task. The gravity values from the unconstrained adjustment are useless, but it can be useful for the evaluation of adjusted ties, drift etc parameters and the inner consistency of measurements.

- The list of `*.redu` file names is used to add reduced observations e.g. from different campaigns or projects, separate days etc.

- There are optional files (searched automatically by GRADJ3) from the folder contains project file `*.proj`, or folders containing `*.redu` file(s):

  (i) Control key file `*.par` to control reduced obs file (`*.redu`) processing by introducing observation ID number with keys for drift, tare, skip, weight, see Sec. 2.1.2

  (ii) Parameters in file `<project name>.cal` to control the estimation of scale change functions, Sec. 2.1.4

  (iii) Station list in file `nostat.list` to exclude selected stations from the estimation of gravity rates, Sec. 2.1.5

2.1.1 Parameters for the adjustment

The input parameters for the adjustment computation in a project file:

Line 1: `dtmax, lsc, driftpar`

`dtmax` – time interval (real value, unit hr) between two measurements to set tare automatically, i.e. for longer interval tare is set, e.g. to separate days from multi-day campaign (e.g. `dtmax=6`);

`lsc=f` – no calibration, `=t` – calibration correction function is evaluated based on the supplemental *.cal file (by default `lsc=f`);

`driftpar` – numerical value to constrain the drift computation in case of poor drift control, e.g. `=0` – no drift estimated, `=0.05` – typical constraint, `=99` – no constraint on drift (def. `driftpar=99`).

Line 2: `sigma, sc_std, confl`

`sigma` – apriori standard deviation of unit weight [mGal] (e.g. `sigma=0.005`);
`sc_std` – the factor to scale uncertainty of fixed and adjusted gravity values, see factor k, equations and discussion in Sec. 2.4.3 (def. `sc_std=1`);
`confl` – confidence level ($0 \leq$`confl`≤ 1) for statistical tests and blunder detection (e.g. `confl=0.95`).

Line 3: `rbias, stdevr`

`rbias` – correctional value to reduce numerical value of gravimeter's readings [mGal] (e.g. `rbias=5000`);
`stdevr` – apriori standard deviation of readings [mGal] (def. `stdevr=0.005`).

Line 4: `ldot, epoch`

`ldot=t` – add parameters of linear trends to estimate gravity changes (from repeated measurements), `=f` no gravity change estimated (def. `ldot=f`);
`epoch` – zero epoch for gravity change estimation (e.g. `2000-01-01`).

Example of input parameters in project file `GoF2010_G191.proj`:

`/Examples/2010-03-17_GulfofRiga/G191/GoF2010_G191.proj`

```
1 6 F 99
2 0.05 1.0 0.95
3 5000 0.025
4 F 2008-07-31
```

2.1.2 Control file for reduced observations

For every `*.redu` file with reduced observations, an ***optional parameter file*** `*.par` can be made (as a separate text file with same name but different extension) to introduce specific keys to control data processing in the adjustment. Both `*.redu` and `*.par` files should be stored in same folder, with **same number of headers** followed by control keys with correct observation ID (oID) numbers.

In `*.redu` file every reduced reading in a set of readings separated by headers (line starting with `#` and gravimeter's ID, eg `# S- 36`) get unique sequential number (i.e. observation ID, hereafter labeled as `obs ID` or `oID`) stored after time column – this integer valued ID is used to add control keys as tare, drift, weight, skip etc with correct entry point in the adjustment computation.

The keys to control adjustment process:

- **s** – to skip single or multiple readings from adjustment, e.g. `s15` or `s15-19` are used to skip reading oID$=15$ or readings with oID$=15,\dots,19$;
- **t** – to add new tare for instrument's scale offset from specific reading, e.g. `t20` introduces new tare just before the reading with oID$=20$;
- **d** – to add new tare and drift function with linear (default) or polynomial model, e.g. `d30` or `d30-3` gives offset with 1st or 3rd order polynomial since reading oID$=30$, respectively;
- **u, w** – to change weights for one or several readings:
  a) by replacing the standard deviation of reading(s), e.g. key `u30 0.2` replaces default uncertainty with 0.2 mGal for readings with oID≥ 15, key `u35-44 0.05` presents the uncertainty 0.05 mGal for readings with oID$=35,\dots,44$;

   b) by using factor to scale weights down, e.g. `w10 2` reduces weights twice for readings with oID$\geq$ 10, for a group of readings (`w10-24 0.2`) the weights increase 5 times (ie the factor < 1 actually scales weight up);

- **x** – to duplicate observation with new offset and drift function, e.g. `x19-2` dublicates reading # 19 with tare and 2nd order polynomial drift (this option should only be used in special cases, e.g. separating local observations from long-range ties).

Note that the key **u** effect on weight depends on the value of input parameters `sigma` and `stdevr`, see Sec. 2.1.1 and Eq. (2.14), therefore the scaling of weights (which are relative quantities in adjustment) with **w** is recommended method for re-weighting. Generally re-weighting of observations is not needed due to the similar conditions (like site stability, roads and weather condition, methods used by operator etc) during the measurement campaign. However, the re-weighting of data measured under significantly different conditions (eg survey on ice with connections to land points) is strongly recommended.

The sequence of keys with overlapping oID values can sometimes lead to conflicting results. As an example, the following list of keys (with comments after definitions, separated by few spaces and "!" to give the content)

```
# G-191
d1-2   !second deg drift from oID=1
s1     !skip 1st reading
t10    !tare(offset) since reading oID=10 <- accident kick by operator after obs taken
d11    !new linear drift from oID=11
w23 2  !suspicious readings oID=23...25
s23
w26 1  !readings with def.weights
...
```

presents several conflicts like the overlap of drift with skipped reading, the offset with immediately following drift (new drift introduces offset anyway) etc. Thus user should define the keys with great care. For instance, the list of keys above can be modified to avoid overlap:

```
# G-191
!d1-2
s1
d2-2
!t10    !accident kick by operator after obs taken
d11    !new drift to separate 1st part (with unstable drift) with more stable 2nd part
s23    !clearly blunder -> removed
w24 2  !suspicious readings oID=24...25, but keep in adj with lower weights
w26 1
...,
```

where lines beginning with " ! " are not used – they are commented out.

An example of file with a list of keys to control the processing of observation file:

/Examples/2010_Calibration/2010_Haanja-Toila_base_S36/2010-07-06_S36.par

```
# S-36
d1-2
!t17
s66
```

2.1.3 Fixed gravity values

For constrained adjustment the station list with fixed gravity values (Station ID, g value, g uncertainty, optionally station name) are introduced into the adjustment as a separate text file:

/Examples/2010_Calibration/2010_Haanja-Toila_base_S36/FIXED

```
80003      981759.6697     0.0030     TõravereAG
80702      981678.8601     0.0040     HaanjaAG
80704      981848.7583     0.0040     ToilaAG
```

2.1.4 Parameters for calibration

Control keys to operate the scale change estimation of relative gravimeter are recorded in file <project name>.cal, eg. file Parnu_calbase_2008.cal contains:

```
# S-36
1
# S-92
99
# S-156
0
# G-191
2
3
7.8824
35.4706
70.9412
...
# G-55
0
0
```

According to the example above, the constant scale change parameters Δc_1 ($n = 1$) and c_1 ($n = 99$) for gravimeters S-36 and S-92 are estimated. Not ehere that $\Delta c_1 = 1 - c_1$. No scale change is computed for S-156, and no scale change as well as periodic scale variation is estimated for G-55. The polynomial function $\Delta c_1 z + \Delta c_2 z^2$ and also the periodical model with 3 waves (with periods given by user in C.U.) are fitted to estimate non-linear scale change of G-191, see also Sec. 2.3.2.

2.1.5 Gravity rate estimation

This is rather experimental part of GRADJ3. For the estimation of constant gravity rates, high precision repeated measurements of common stations over longer time period have to be collected. If there are stations without any or with limited repetitions, a station list file nostat.list can be made to exclude selected stations from the estimation of linear gravity rates.

The list file nostat.list contains station ID and optionally its name:

```
80047 Aluste
80049 Rakke
80053 Roela
890100
...
```

2.2 Output files of GRADJ3

The information given in the output files of GRADJ3 (with specific extension) are following:

1) `*.grav` – adjusted gravity result file (see App. C.1) that contains...
   (i) adjusted gravity values and residuals of fixed and observed stations;
   (ii) adjusted gravity values and uncertainties of non-fixed and observed stations;
   (iii) optionally adjusted time rates $\dot{g}$ (see Sec. 2.1.5);
   (iv) optionally adjusted coefficients of scale change functions (Sec. 2.1.4);
   (v) apriori SD (`SIGMA1`) and aposteriori SD (`SIGMA2`) of unit weight (Sec. 2.4.2);
   (vi) chi-square test of the `SIGMA1` (Sec. 2.4.4);
   (vii) Information Criteria (AIC, BIC) estimations (Sec. 2.5).
2) Files with residuals `*.resi` and mean residuals per site occupation in `*.resi.mean` (App. C.2), containing also...
   (i) drift, bias and tare parameters;
   (ii) standard residuals and redundancy numbers;
   (iii) Student's, Pope's test statistics to test the significance[2] of parameters and residuals (Sec. 2.4.4);
   (iv) RMS and WRMS of residuals.
3) Observed connections or ties file `*.ties` with sorted ties in `*.ties.sort` (App. C.3), containing...
   (i) observed gravity differences `dg` computed from reduced readings;
   (ii) corrected for drift `dD` and residuals `dv`, which is equivalent to the adjusted difference (see below).
4) Adjusted ties file `*.ties.adj` with adjusted gravity differences between all observed stations and their combined uncertainties, estimated from the full covariance matrix of adjusted results.
5) Supplemental files `*.cov` containing full covariance matrix of adjusted results, and `resid4hist.dat` containing a column with residuals of single readings (see also file `*.resi`) for quick production of histogram.

[2]Insignificant parameters (no need to estimate) and significant residuals (suggest outliers, bias etc.) are flagged with “ ! ” sign.

2.3 GRADJ3 theory

The relation between the observations and unknown parameters, and the propagation of errors are established through the mathematical model which can be divided into two parts: (1) functional model, (2) stochastic model. Based on the linear or linearized functional model, the observation equations are formed into a linear system of equations. In case of non-linear functional model the equations are linearized first. The weighted least squares (WLS) method can then be used to find the best linear unbiased estimation of the linear system.

2.3.1 Original functional model

GRADJ original functional model (or adjustment model) combines the relative gravity readings *as independent observations* with unknown parameters (Andersen and Forsberg, 1996)

$$y(t) = \frac{1}{s}\left[g + a(t_0) + b(t - t_0)\right], \tag{2.1}$$

where $y(t)$ is reduced reading at observation epoch t, g is station gravity, s is gravimeter's scale factor (to model constant scale change), a and b are parameters for tare (for instrument's scale offset/shift) and a constant drift relative to the epoch t_0, respectively.

The model (2.1) has two distinct features:

- the inclusion of a scale factor s implies a non-linearity;
- it relies directly on the reduced readings of gravimeter, not on the observed differences of readings.

The former needs a linearization and iterative computations steps, see 2.3.3. The latter results simple, sparse structure of a normal matrix with lower memory needs and fast adjustment computation. Moreover, the direct usage of reduced readings gives correct weight matrix (by assuming uncorrelated observations) which is advantage over the adjustment of differences (with correlation of -0.5 between these) formed from successive readings (Torge, 1989; Andersen and Forsberg, 1996).

The scale offset a has been assumed to be constant on a daily or near-daily basis (generally t_0 is time of the first reading of a day). New offset parameter should be introduced if a tare (or jump) in the readings of relative gravimeter is assumed. Such tares can correspond to sudden discontinuities in observation series (caused e.g. by physical shocks due to accidental kicks to gravimeter's body, rapid temperature change, extremely bad road conditions between observed points etc). The new offset could also be useful to separate (decouple) more precise measurements from less precise data, as well as to absorb the non-linear behavior of the instrument drift. However, the introduction of too many bias parameters in the adjustment could reduce the redundancy of observations, with lower value of DoF – degrees of freedom.

2.3.2 New functional model

GRADJ3 functional model is the extended version of GRADJ model Eq. (2.1) with several new parameters and functions (sub-models) added to estimate non-linear drift and instrumental scale change, the rate of gravity change etc:

$$s\,y(t) = g(T_0) + \dot{g} + a(\tau) + D(t) + \Delta F(z). \tag{2.2}$$

Some terms are similar with the terms of Eq. (2.1): $y(t)$ is corrected reading of gravimeter (from the output of GRREDU3 in CGS units) at observation epoch t, $g(T_0)$ is gravity value [mGal] (at epoch T_0 fixed by user, in case if gravity rate $\dot{g}$ [µGal/yr] is also estimated or used in pre-processing to convert readings to epoch T_0), $a(\tau)$ is scale offset function to model the tare between the readings of gravimeter and s is the scale correction parameter. The new functions with additional unknown parameters are polynomial drift function $D(t)$ and calibration correction function $\Delta F(z)$.

The offset function

$$a(\tau) = a(t - t_0) = N_0\,\mathrm{H}(t - t_0) \tag{2.3}$$

can be defined using the Heaviside step function

$$\mathrm{H}[\tau] = \begin{cases} 0, & \tau < 0 \\ 1, & \tau \geq 0 \end{cases},$$

where N_0 is a tare within a period of time $\tau = t - t_0$ (or since time t_0). Generally t_0 is taken as an epoch of first reading during the measurement day. The purpose of tare parameter was discussed in sec. 2.3.1.

Drift function $D(t)$ is defined by degree p polynomial

$$D(t) = \sum_{i=1}^{p} D_i t^i, \tag{2.4}$$

where D_i ($i = 1 : p$, where $p_{\max} \leq 5$) is drift parameter with unit µGal/(day)^i.

Calibration correction function $\Delta F(z)$ after Torge (1989) is

$$\Delta F(z) = \sum_{i=1}^{q} \Delta c_i z^i + \sum_{i=1}^{r} \Delta A_i \sin(2\pi z/P_i + \Delta\varphi_i), \tag{2.5}$$

where Δc_i ($i = 1 : q$) are the coefficients of polynomial function, ΔA_i and $\Delta\varphi_i$ ($i = 1 : r$) are the amplitude [µGal] and zero phase [rad] of periodic function, respectively.

For the real estimation of scale change with GRADJ3 only one or another option (the evaluation of the function $\Delta F(z)$ or scale factor s) can be used, see also Sec. 1.3.1 and Sec. 2.1.4.

By using notation

$$\begin{aligned} \Delta\alpha &= \Delta A \sin(\Delta\varphi), \\ \Delta\beta &= \Delta A \cos(\Delta\varphi), \end{aligned}$$

and known relations in trigonometry, the linear function with unknown parameters $\Delta\alpha$, $\Delta\beta$ for periodic part is

$$\Delta A \sin(2\pi z/P + \Delta\varphi) = \Delta\alpha \cos(2\pi z/P) + \Delta\beta \sin(2\pi z/P). \tag{2.6}$$

The relation (2.6) can now be used to set up linear system of equations.

After the adjustment, the values of ΔA and $\Delta\varphi$ in (2.5) are found

$$\begin{aligned}\Delta A &= \sqrt{\Delta\alpha^2 + \Delta\beta^2},\\ \Delta\varphi &= \arctan\left(\frac{\Delta\alpha}{\Delta\beta}\right),\end{aligned} \tag{2.7}$$

and their standard deviations according to error propagation (see App. D) are

$$\begin{aligned}s_{\Delta A} &= \pm\frac{\sqrt{\left(\Delta\alpha\, s_{\Delta\alpha}\right)^2 + \left(\Delta\beta\, s_{\Delta\beta}\right)^2 + 2\Delta\alpha\,\Delta\beta\,\mathrm{Cov}(\Delta\alpha,\Delta\beta)}}{\Delta A},\\ s_{\Delta\varphi} &= \pm\frac{\sqrt{\left(\Delta\beta\, s_{\Delta\alpha}\right)^2 + \left(\Delta\alpha\, s_{\Delta\beta}\right)^2 - 2\Delta\alpha\,\Delta\beta\,\mathrm{Cov}(\Delta\alpha,\Delta\beta)}}{\Delta A^2},\end{aligned} \tag{2.8}$$

where $s^2_{\Delta\alpha}, s^2_{\Delta\beta}$ are the variances $\mathbf{D}(\Delta\alpha, \mathbf{D}(\Delta\beta)$ of $\Delta\alpha, \Delta\beta$, and $\mathrm{Cov}(\Delta\alpha,\Delta\beta) = \mathbf{D}(\Delta\alpha\Delta\beta)$ is their covariance from the adjustment.

The long-term (secular) gravity change can be estimated from the repeated measurements using linear relation

$$\dot{g}(t) = \dot{g}_c[t - T_0], \tag{2.9}$$

where $\dot{g}_c$ is the rate of gravity change [μGal/yr].

2.3.3 Linearization of functional model

Due to the scale correction s in (2.2) the functional model is non-linear

$$y = \frac{1}{s}\left[g + \dot{g} + a + D + \Delta F\right] = f(\boldsymbol{x}), \tag{2.10}$$

where $\boldsymbol{x} = (g, \dot{g}, a, D, \Delta F, s)$ represents the vector of parameters.

For the WLS adjustment of linear system of equations, the model is linearized by using the Taylor series expansion at $\boldsymbol{x}_0 = (g_0, \dot{g}_0, a_0, D_0, \Delta F_0, s_0)$

$$\begin{aligned}f(\boldsymbol{x}) =& f(\boldsymbol{x}_0) + (\boldsymbol{x} - \boldsymbol{x}_0)^{\mathrm{T}}\frac{d}{d\boldsymbol{x}}f(\boldsymbol{x}_0) + \\ &+ \frac{1}{2!}(\boldsymbol{x} - \boldsymbol{x}_0)^{\mathrm{T}}\left\{\frac{d^2}{d\boldsymbol{x}^2}f(\boldsymbol{x}_0)\right\}(\boldsymbol{x} - \boldsymbol{x}_0) + \cdots,\end{aligned}$$

where the vector $\boldsymbol{x}_0$ contains initial values.

By omitting terms of 2nd and higher order

$$f(\boldsymbol{x}) \approx f(\boldsymbol{x}_0) + \delta\boldsymbol{x}\frac{d}{d\boldsymbol{x}}f(\boldsymbol{x}_0), \tag{2.11}$$

where $\delta\boldsymbol{x} = \boldsymbol{x} - \boldsymbol{x}_0$. By using notation $f = f(\boldsymbol{x})$ ja $f_0 = f(\boldsymbol{x}_0)$ and partial derivatives, we can write

$$f = f_0 + \frac{\partial f_0}{\partial g}(g - g_0) + \frac{\partial f_0}{\partial \dot{g}}(\dot{g} - \dot{g}_0) + \frac{\partial f_0}{\partial a}(a - a_0) + \cdots. \tag{2.12}$$

By evaluating partial derivatives (also noting that $\delta g = g - g_0$, ...)

$$f = f_0 + \frac{1}{s_0}\delta g + \frac{1}{s_0}\,\delta \dot{g} + \frac{1}{s_0}\,\delta a + \frac{1}{s_0}\,\delta D + \frac{1}{s_0}\,\delta(\Delta F) - \frac{f_0}{s_0}\,\delta s. \tag{2.13}$$

By replacing the right side in Eq. (2.10) with linearized model, by multiplying with the weight of the observation w and noting that $y_0 = f_0$ (y_0 is the function of initial values of parameters)

$$w(y - y_0) = \frac{w}{s_0}\left[\delta g + \delta \dot{g} + \delta a + \delta D + \delta(\Delta F) - y_0\,\delta s\right] - wv, \tag{2.14}$$

where $w = \sigma_0^2/\sigma^2$, σ_0^2 is a priori variance of unit weight (input parameter `sigma`, see Sec. 2.1.1), σ^2 is variance of reading y (another parameter `stdevr`, Sec. 2.1.1). By default $w = 1$ (`sigma` = `stdevr`), i.e. single observation has unit weight.

Known gravity value g_{fix} (measured at epoch t) of fixed stations is introduced into the functional model

$$g_{fix}(t) + v = g(T_0) + \dot{g}(t - T_0), \tag{2.15}$$

where $g(T_0)$ is adjusted gravity value at epoch T_0 and $\dot{g}$ is adjusted gravity rate.

By linearizing (2.15) and adding the weights for absolute gravity values (similar to (2.14))

$$w(g_{fix} - g_{fix(0)}) = w(\delta g(T_0) + \delta \dot{g}) - w\,v. \tag{2.16}$$

2.3.4 Initial values and iteration

The solution of the system by Eq. (2.14) and Eq. (2.16) are corrections $\delta g, \delta \dot{g}, \delta a$ etc. which are used to estimate new set of parameters: $g_1, \dot{g}_1, \ldots$ into the adjustment

$$\begin{aligned} g_1 = g_0 + \delta g, \dot{g}_1 = \dot{g}_0 + \delta \dot{g},\ \ldots \Rightarrow &\qquad y_1 = y_0 + \delta y, \\ \ldots \Rightarrow &\quad g_{fix(1)} = g_{fix(0)} + \delta g_{fix}. \end{aligned} \tag{2.17}$$

Now the adjusted values of y_1 and $g_{fix(1)}$ are used again to estimate another set of parameters from the (2.14) and (2.16). The system of equations is solved iteratively as long as the change of parameter values and residuals v are getting small enough (if variable `solmax < 1.d-6`). If the iteration process is not converging well (i.e. `solmax` is not getting small enough), the process is stopped after 50 steps (`maxloop = 50`). If correctional calibration parameters are estimated then `maxloop = 100`.

Iteration is started from initial values based on parameters fixed in source code and input files:

1) $g_0 = 981800.0$ mGal (parameter `gbias`);
2) $\dot{g}_0 = 0.0$ µGal/yr (parameter `gdot` through array `sol[i]`);
3) $a_0 = 5500$ mGal (through input parameter `rbias`, see Sec. 2.1.1);
4) $D_j = 0$ (through array `sol[i]`) with $j = 1$ (linear drift is estimated by default);
5) $\Delta F = 0$ (through array `sol[i]`);
6) $s = 1.0$ (parameter `s0`).

2.4 Solving linear system of equations

The linearization of Eq. (2.14), (2.16) gives the system of linear equations in matrix form (Wolf and Ghilani, 1997; Strang and Borre, 1997)

$$A\boldsymbol{x} = \boldsymbol{b} - \boldsymbol{r}, \tag{2.18}$$

where $\boldsymbol{b}$, $\boldsymbol{r} \in \mathbf{R}^m$ are the vectors of observations and residuals, respectively, $A \in \mathbf{R}^{m\times n}$ is design (or coefficient) matrix, $\boldsymbol{x} \in \mathbf{R}^n$ is a vector of unknown parameters (coefficients), m is the number of observations and n is the number of parameters.

From the diagonal covariance matrix of observations $\Sigma_b \in \mathbf{R}^{m\times m}$ the weight matrix is estimated

$$W_b = \sigma_0^2 \Sigma_b^{-1} = \sigma_0^2 Q_b, \tag{2.19}$$

where σ_0^2 is an a priori variance of unit weight and $Q_b = \Sigma_b^{-1}$ denotes the matrix of cofactors or weight coefficients. The variance of unit weight σ_0^2 is used to scale the stochastic model through the a priori weight matrix, thus it is also called variance factor.

The system of equations can be referred to as a Gauss-Markov model (Koch, 1999)

$$E(\boldsymbol{b}) = A\boldsymbol{x}, \quad D(\boldsymbol{b}) = \sigma_0^2 W_b^{-1} = \Sigma_b, \tag{2.20}$$

where the design matrix A is assumed to be of full column rank, i.e., $\mathbf{rank}(A) = n$ (otherwise $A^{\mathrm{T}}A$ is not invertible), provided that $m \geq n$, and Σ_b, W_b are symmetric and positive-definite.

2.4.1 *WLSQ BLUE* solution

The weight matrix inversely proportional to the covariance matrix (see Eq. 2.19) in the weighted least squares (WLSQ) adjustment leads to *the best linear unbiased estimate* (BLUE) $\hat{\boldsymbol{x}}$ of the unknown parameter vector $\boldsymbol{x}$ (Strang and Borre, 1997).

By multiplying (from the left) the linear system of equations (2.18) by $A^{\mathrm{T}}W_b$ the normal equations become

$$A^{\mathrm{T}}W_b A\hat{\boldsymbol{x}} = A^{\mathrm{T}}W_b\boldsymbol{b}, \tag{2.21}$$

where $\hat{\boldsymbol{x}}$ is an estimate of unknown parameter vector and $N = A^{\mathrm{T}}W_bA$ is called the normal matrix and its inversion $N^{-1} = (A^{\mathrm{T}}W_bA)^{-1}$ is the information matrix.

Now the *WLSQ BLUE* solution of the system is

$$\hat{\boldsymbol{x}} = (A^{\mathrm{T}}W_bA)^{-1}A^{\mathrm{T}}W_b\boldsymbol{b} = N^{-1}A^{\mathrm{T}}W_b\boldsymbol{b}. \tag{2.22}$$

Now also other BLUE estimates can be found for vectors $\boldsymbol{b}, \boldsymbol{r}$

$$\hat{\boldsymbol{b}} = A\hat{\boldsymbol{x}} = A(A^{\mathrm{T}}W_bA)^{-1}A^{\mathrm{T}}W_b\boldsymbol{b}, \tag{2.23}$$

and

$$\hat{\boldsymbol{r}} = \boldsymbol{b} - A\hat{\boldsymbol{x}} = \boldsymbol{b} - \hat{\boldsymbol{b}}. \tag{2.24}$$

2.4.2 Stochastic solution

An estimate of the weighted sum of squared deviations are found from

$$\hat{\boldsymbol{r}}^{\mathrm{T}} W_b \hat{\boldsymbol{r}} = \boldsymbol{b}^{\mathrm{T}} W_b \boldsymbol{b} - \boldsymbol{b}^{\mathrm{T}} W_b A \hat{\boldsymbol{x}}, \tag{2.25}$$

which yields unbiased estimate of the variance of unit weight (the a posteriori variance factor for stochastic model)

$$\hat{\sigma}_0^2 = \hat{\boldsymbol{r}}^{\mathrm{T}} W_b \hat{\boldsymbol{r}}/(m-n) = \frac{\chi^2}{v} = \chi_v^2, \tag{2.26}$$

where $v = m - n$ is degrees of freedom (*DoF*, also the number of redundant observations), and χ^2 and χ_v^2 are chi-square and reduced chi-square statistics respectively, describing *the goodness of fit.*

The variance of unit weight (both a priori and a posteriori) plays a valuable role since it allows to rescale the weight and covariance matrices when the estimates of these matrices prove (from the actual data) to be unrealistic (Strang and Borre, 1997).

By using realistic weight matrix and $\hat{\sigma}_0^2$ value, the BLUE estimate for the covariance matrix of adjusted parameters is

$$\Sigma_{\hat{x}} = \hat{\sigma}_0^2 (A^{\mathrm{T}} W_b A)^{-1} = (A^{\mathrm{T}} \Sigma_b^{-1} A)^{-1}. \tag{2.27}$$

For statistical analysis and testing (to detect statistically significant parameters' values), and blunder detection (to detect offsets, gross errors etc) it is useful to estimate the covariance matrix of observations $\hat{\boldsymbol{b}} = A\hat{\boldsymbol{x}}$

$$\Sigma_{\hat{b}} = \Sigma_{A\hat{x}} = \hat{\sigma}_0^2 A (A^{\mathrm{T}} W_b A)^{-1} A^{\mathrm{T}} = A \Sigma_{\hat{x}} A^{\mathrm{T}}, \tag{2.28}$$

as well as the covariance matrix of residuals

$$\Sigma_{\hat{r}} = \hat{\sigma}_0^2 (W_b^{-1} - A(A^{\mathrm{T}} W_b A)^{-1} A^{\mathrm{T}}) = \hat{\sigma}_0^2 W_b^{-1} - \Sigma_{A\hat{x}} = \Sigma_b - \Sigma_{\hat{b}}. \tag{2.29}$$

2.4.3 Scaling the variances

With GRADJ3 the factor k (defined by input parameter `sc_std`, see ch. 2.1.1) is used to scale the variance of input data so that the input weight matrix from Eq. (2.19) is represented as a block diagonal matrix

$$W_b = \begin{bmatrix} k^2 W_{\mathrm{fix}} & 0 \\ 0 & W_{\mathrm{obs}} \end{bmatrix}, \tag{2.30}$$

where W_{fix} is a diagonal matrix with weights $w_{\mathrm{fix}} = (\sigma_0/\sigma_{\mathrm{fix}})^2$ for fixed points and W_{obs} is a diagonal matrix with weights $w_{\mathrm{obs}} = (\sigma_0/\sigma_{\mathrm{obs}})^2$ of readings.

After the adjustment the covariance matrix by (2.27) is now scaled

$$\Sigma_{\hat{x}} = \begin{bmatrix} \Sigma_{\mathrm{fix}} & 0 \\ 0 & k^2 \Sigma_{\mathrm{obs}} \end{bmatrix}. \tag{2.31}$$

By default $k = 1$, and for scaling the interval $1 < k \leq 30$ is suggested to use (according to the hundreds of test computations). In other words, at first the weights of fixed values are scaled up to increase their influence and reduce their residuals in adjustment.

Such scaling is necessary if the influence of relative gravity data (with many repeated readings with small RMS) tend to be too high in adjustment. Yet the increased weights of fixed values reduce also the standard deviations of adjusted gravity values and second scaling in (2.31) is used to avoid the estimation of unrealistic small uncertainties. However, several test runs should be made to find reasonable balance (not too high or low values) between the residuals of fixed values and the standard deviations of adjusted gravity estimations. The finding of such balance needs good knowledge about data (about their precision as well as accuracy), experience with software and many test computations.

2.4.4 Statistical tests

GRADJ3 uses different statistical tests to check the significance of adjusted parameters and other statistical estimates. For more details, see Wolf and Ghilani (1997); Strang and Borre (1997); Koch (1999); Vanicek and Krakiwsky (1986).

For statistical testing the confidence level $1 - \alpha$ has to set by user, e.g. the value of 0.95 is common choice for confidence interval $\pm 1.96\sigma$. The statistical tests used in GRADJ3 are

1) Chi-squared χ^2-test for the variance of unit weight σ_0^2;
2) Student's t-test for the significance of parameter;
3) Pope's τ-test for the significance of residual to detect outlier (gross error, blunder, bias etc) from readings.

χ^2-test
The probability P for confidence interval of confidence level $1-\alpha$ of two-tailed (two-sided) χ^2-test for the variance of unit weight σ_0^2 is

$$P\left(\chi^2_{v,\alpha/2} < v\frac{\hat{\sigma}_0^2}{\sigma_0^2} < \chi^2_{v,1-\alpha/2}\right) = 1 - \alpha, \tag{2.32}$$

where the critical values of the χ^2 distribution $\chi^2_{v,p} = \chi^2_v(p)$ with parameter v (DoF) is computed by subroutine `CHI2INV`, which finds the χ^2 quantile $Q(p)$ at probability level p, where function Q is inverse of F - the cumulative distribution function[3].

In GRADJ3 the test statistic is found

$$\chi^2 = \frac{\hat{\sigma}_0^2}{\sigma_0^2} = \frac{\chi_v^2}{\sigma_0^2}, \tag{2.33}$$

where the notation of (2.26) is used.

Now the null hypothesis $H_0 : \sigma_0^2 = \hat{\sigma}_0^2$ is true if $\frac{\chi^2_{v,\alpha/2}}{v} < \chi_t < \frac{\chi^2_{v,1-\alpha/2}}{v}$. If H_0 is true then some conclusions can be made, e.g. a priori variance of unit weight to scale weight matrix is correctly selected. However, the rejection of H_0 could mean that the mathematical model is incorrectly defined (e.g. under- or over-fitting), the residuals are not normally distributed, there are too many blunders in data, and so forth.

Student's t-test
Student's two-tailed t-test is used to test the significance of adjusted parameters.

[3]In terms of the distribution function F, the quantile function Q returns the value x such that $F_X(x) := \Pr(X \leq x) = p$, see https://en.wikipedia.org/wiki/Quantile_function.

The null hypothesis $H_0 : \hat{x} = 0$ (parameter is not statistically significant). The alternative hypothesis $H_a : \hat{x} \neq 0$ (it is statistically significant).

Test statistic for parameter x_i is

$$t_i = \frac{|\hat{x}_i|}{\hat{\sigma}_0 q_{\hat{x}_i}} = \frac{|\hat{x}_i|}{\sigma_{\hat{x}_i}}, \tag{2.34}$$

where $\hat{x}_i$ is an estimated value of parameter in adjustment, with its estimated standard deviation $\sigma_{\hat{x}_i}$ (cofactor $q_{\hat{x}_i} = \sqrt{c_{ii}}$ is found from the main diagonal of covariance matrix $\Sigma_{\hat{x}}$, see Eq. (2.27)). For the function of parameters $f(\hat{x}_i, \hat{x}_j)$ (e.g. tares, gravity differences etc) the error propagation with covariances are rigorously computed to estimate $\sigma_{f(\hat{x})}$, by using off-diagonal elements c_{ij} $(i \neq j)$ of $\Sigma_{\hat{x}}$.

Now the probability P for confidence level $(1 - \alpha)$ of two-tailed test is

$$P\left(-t_{v,1-\alpha} < t < t_{v,1-\alpha}\right) = 1 - \alpha. \tag{2.35}$$

Accordingly, H_0 is invalid (parameter is statistically significant) if $|t| > t_{v,1-\alpha/2}$. The critical t-value $t \sim t_{v,1-\alpha/2}$ is found by subroutine `STUDIN` (which computes the two-tailed inverse of the Student's t-distribution).

Pope's τ-test

The significance test of residuals ($H_0 : \hat{r} = 0$, $H_a : \hat{r} \neq 0$) with τ-test by Pope (1976) helps to detect outliers and thus remove possible gross errors (blunders) from the readings.

The probability P for confidence level $1 - \alpha$ of one-tailed test is

$$P\left(-\tau_{v,n,1-\alpha} < \frac{|\hat{r}_i|}{\sigma_{\hat{r}_i}} < \tau_{v,n,1-\alpha}\right) = 1 - \alpha. \tag{2.36}$$

Thus the test statistic for residual r_i is

$$\tau_i = \tilde{r}_i = \frac{|\hat{r}_i|}{\hat{\sigma}_0 q_{\hat{r}_i}} = \frac{|\hat{r}_i|}{\sigma_{\hat{r}_i}}, \tag{2.37}$$

where $\hat{r}_i$ is an estimated residual of reading i, $\sigma_{\hat{r}_i}$ is a standard deviation of the residual, found from the main diagonal $q_{\hat{r}_i} = \sqrt{c_{\hat{r}}(ii)}$ of covariance matrix $\Sigma_{\hat{r}}$ using Eq. (2.29). Eq. (2.37) reveals that test statistic is equivalent to standardized residual $\tilde{r}_i$.

Now H_0 is valid (residual is statistically insignificant, i.e. equal to zero) if $|\tau| < \tau_{v,n,1-\alpha}$, otherwise residual could be outlier. Critical value $\tau \sim \tau_{v,n,1-\alpha}$ is found by subroutine `TAURE`.

For outlier detection the local redundancy number $(0 < r_i < 1)$

$$r_i = 1 - \frac{\hat{\sigma}_i^2}{\sigma_i^2} = 1 - w_i^2 q_{\hat{b}_i}^2 = w_i^2 q_{\hat{r}_i}^2,$$

is useful to estimate, because $r_i < 0.5$ indicates poorly controlled observation y_i. In that case, there is not enough information (e.g. repeated measurements) to correctly detect outliers. Only well controlled observations with $r_i \geq 0.5$ can be used for reliable outlier detection.

2.4.5 Matrix computation in GRADJ3

The linear system represented as an augmented matrix $N_a = \left[A^{\mathrm{T}}WA : \mathbf{b}\right]$ (only upper triangular part of it) is stored into vector $c(i)(i = 1 : n)$ (build-up by subroutine `ADDOBSD2`).

For solving the system with symmetric positive-definite matrix $N = A^{\mathrm{T}}WA$, the Cholesky decomposition is used for quick and efficient numerical solution. This decomposition is the product of a lower triangular matrix G^{T} and its transpose G

$$N = A^{\mathrm{T}}WA = G^{\mathrm{T}}G. \tag{2.38}$$

It yields now

$$\hat{\boldsymbol{x}} = (G^{\mathrm{T}}G)^{-1}A^{\mathrm{T}}W\boldsymbol{b}. \tag{2.39}$$

However, no inverse matrix is actually computed with the Cholesky decomposition because the linear system with lower triangular matrix is first solved

$$G^{\mathrm{T}}\boldsymbol{y} = \boldsymbol{b},$$

followed by

$$G\boldsymbol{x} = \boldsymbol{y}$$

for $\boldsymbol{y}$ by forward substitution, and for $\boldsymbol{x}$ by back substitution. Both substitutions are very easy to solve by an iterative process. Accordingly, the matrix inversion is needed only for covariance information and not for the solution of linear system.

Matrix G is computed by subroutine `CHOLD`. The main diagonal of inversion

$$N^{-1} = (G^{\mathrm{T}}G)^{-1} = G^{-1}(G^{\mathrm{T}})^{-1} = G^{-1}(G^{-1})^{\mathrm{T}}$$

is found quickly by subroutine `CHOLINV` to estimate variances of parameters (see (2.27)). However, the subroutine `CHOLINV` does not yield the fully inverted matrix (e.g. for covariances and another covariance matrices like Eqs. 2.28,2.29).

For the estimation of G^{-1} and the triangular part of $N^{-1} = G^{-1}(G^{-1})^{\mathrm{T}}$ another subroutine `CHINV2` is used. Since the first part of normal equations (the first `ndia` diagonal elements of matrix N corresponding to the unknown gravity values, where `ndia` is a number of stations) is diagonal, the computations described above are fast with low memory usage (only nonzero elements of N and other matrices are stored).

According to Eqs. (2.28) and (2.29), the covariance matrices $\Sigma_{\hat{b}} = \sigma_0^2 Q_{\hat{b}}$ and $\Sigma_{\hat{r}} = \sigma_0^2 Q_{\hat{r}}$ can be computed

$$Q_{\hat{b}} = AN^{-1}A^{\mathrm{T}} = AG^{-1}(AG^{-1})^{\mathrm{T}}$$

and

$$Q_{\hat{r}} = W^{-1} - Q_{\hat{b}}.$$

Now degrees of freedom v (overall redundancy) can be found from the trace of matrix multiplication (Koch, 1999, p. 305)

$$\mathbf{tr}(Q_{\hat{r}}W) = \mathbf{tr}(I - Q_{\hat{b}}W) = m - n = v.$$

This trace is found by GRADJ3 to check the correctness of matrix computation.

2.5 The Akaike and Bayesian Information Criteria

The selection of the best functional model (which has the best fit with observations) from the set of models by using the minimum set of needed parameters in the adjustment computation, the Akaike (AIC) or Bayesian Information Criterion (BIC) are useful to estimate. The original formulation (Akaike, 1974; Schwarz, 1978) for the maximum likelihood estimation can be transformed also for the least squares model fitting (Burnham and Anderson, 2002, p. 63)[4]

$$\mathrm{AIC} = m \ln(\mathrm{RSS}) + 2k, \tag{2.40}$$

where $k = n + 1$, n is the number of parameters adjusted ($n + 1$ to add RSS part), m is the number of all observations and the the residual sum of squares (RSS) is

$$\mathrm{RSS} = \sum_{i=1}^{n} (y_i - f(\hat{\boldsymbol{x}}))^2 = \sum_{i=1}^{n} r_i^2.$$

In other words, as its starting point, AIC is a measure of the discrepancy between the data and an estimation model with penalty for adding parameters (in order to avoid overfitting).

AIC may perform poorly if there are too many parameters in relation to the size of the sample n (e.g. if $m/k < 40$). Then the correction part is added (Burnham and Anderson, 2002, p. 66)

$$\mathrm{AIC}_c = \mathrm{AIC} + \frac{2k(k+1)}{n-k-1}. \tag{2.41}$$

From the AIC_c (used by GRADJ3) another criteria BIC (Burnham and Anderson, 2004) is estimated

$$\mathrm{BIC} = \mathrm{AIC}_c + k\left[\ln(m) - 2\right]. \tag{2.42}$$

The preferred model is the one with the minimum AIC/BIC value. Note that these are relative measures within a set of models, not absolute criteria (Hector manual ver. 1.7.2). Thus effort by user must be made to ensure well founded models for selection.

To find the preferred model, AIC/BIC differences are computed over all candidates in the current set of models

$$\Delta_i = \mathrm{AIC}_i - \min(\mathrm{AIC}),$$
$$\Delta_i = \mathrm{BIC}_i - \min(\mathrm{BIC}).$$

Thus the model estimated to be the best has $\Delta_i \equiv \Delta_{\min} \equiv 0$ (Burnham and Anderson, 2002, p. 71).

[4]See also wikipedia.org/wiki/Akaike_information_criterion.

Chapter 3

Program WZZ

For different purposes (comparison of AG results, RG height correction, connections between AG and RG sites) it is important to know the change of gravity as a function of height above the benchmark of gravity station. However, there are usually local mass anomalies, e.g massive piers (constructed to make site stable for precise gravimetric measurements) that could make gravity a strongly non-linear function of height, i.e. the vertical gradient of gravity is not constant.

Program WZZ helps to model gravity change along the positive down local vertical Z (plumb line), i.e. to evaluate vertical gravity gradient (VGG): $dg/dz = g_z = d^2W/dz^2 = W_{zz}$ (thus the name WZZ), where W is the potential of Earth's gravity field. Exact gravity values at different height levels can then be estimated from the model of VGG.

The gravity differences Δg_{ij} between height levels $dz = z_j - z_i$ and their uncertainties (measured with relative gravimeters) are adjusted with WLS method to estimate the parameters of functional model. For the weights the relation $w \propto 1/[u^2(\Delta g)]$ is used, where $u(\Delta g)$ is the standard uncertainty of observed gravity difference estimated from the processing of relative gravity data.

The WLS adjustment of WZZ uses the fast and memory efficient Cholesky matrix decomposition (see Sec. 2.4.5).

3.1 Functional model

There are two functional models available in WZZ:

a) Polynomial with degree n
b) Remove-compute-restore (RCR)

Polynomial function

Conventionally the polynomial function is used for the VGG modelling, recommended also by the IAGBN standards. For the modelling of gravity change $g(z)$ along the local vertical with changing height z, the polynomial with degree n is used to approximate the relation

$$g(z) = \sum_{l=1}^{n} c_l z^l. \tag{3.1}$$

Now for the modelled gravity difference dg_{ij} between two height levels z_i, z_j is

$$g(z_j) - g(z_i) = dg_{ij} = \sum_{l=1}^{n} c_l \left(z_j^l - z_i^l\right), \tag{3.2}$$

where the coefficients c_l are estimated from the WLS adjustment of relative gravity data.

In general, the first ($dg = c_1 dh$) or second degree polynomial is used

$$dg_{ij} = c_1(z_j - z_i) + c_2\left(z_j^2 - z_i^2\right), \tag{3.3}$$

where parameter c_1 describes the constant part of VGG (conventionally -308.6 µGal/m) and c_2 is for a linear change of VGG. The covariance matrix of adjusted parameters $\Sigma_{\hat{x}}$ helps to estimate the uncertainty of gravity difference

$$\begin{aligned} u(dg_{ij}) = \big\{&u^2(c_1)(z_2 - z_1)^2 + u^2(c_2)(z_2^2 - z_1^2)^2 \\ &+2u(c_1)u(c_2)\mathrm{Corr}[c_1, c_2](z_2 - z_1)(z_2^2 - z_1^2)\big\}^{1/2}. \end{aligned} \tag{3.4}$$

Here $u(c_1)$ and $u(c_2)$ are the standard uncertainties of the estimated coefficients, $\mathrm{Corr}[c_1, c_2]$ is the Pearson's correlation coefficient between them.

It is intuitive to understand that the gravity differences Δg at least between two heigts for constant VGG and between three heights for linear VGG should be determined. However, repeated measurements of Δg between more than 3 heights should be made (as accurate as possible by measuring repeatedly with calibrated gravimeter) for the reliable estimation of polynomial VGG model.

RCR approach

Neither second nor higher degree polynomial could be enough to model the effect of local mass anomalies like massive pier at gravity station, especially near the pier surface. To deal with this problem, an approach based on the known remove-compute-restore (RCR) method in geodesy has been proposed by Dr Jaakko Mäkinen from FGI (Mäkinen, 2012).

The RCR method combines the polynomial function with the theoretically modelled vertical attraction of local masses. It contains three steps: (i) the theoretical influence of local masses from the data is removed, (ii) the polynomial function of height is fitted to the residuals, (iii) the theoretical part is restored (combined with polynomial). According to (3.2)

$$dg_{ij} - dM_{ij} = \sum_{l=1}^{n} c_i \left(z_j^l - z_i^l\right), \tag{3.5}$$

where $dM_{ij} = [M(z_j) - M(z_i)]$ describes the theoretical effect (g_M along vertical) of local mass M (or sum of masses $M = \sum M_i$), and right side is used in fitting of reduced data from left side.

Currently gravity effect by WZZ can be theoretically modelled for two types of a right prism[1] (see also Fig. 3.1):

[1] A right prism is a geometric solid that has a polygon as its base and vertical sides perpendicular to the base (www.siyavula.com).

1) a rectangular prism with top surface perpendicular to and vertical axis parallel to local vertical z (does not need to coincides with z), computed by subroutine `gbox.f`;
2) a vertical cylinder with top surface perpendicular to and vertical axis coincides with local vertical z, computed by subroutine `silinder.f`.

The user can add new models for right prisms by modifying the source code, if such addition is necessary.

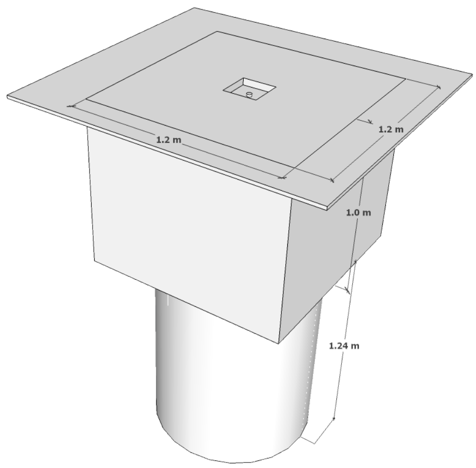

Figure 3.1: Pier's 3D projection of the Estonian I order gravity network point in Haanja. The concrete underground pier with total weight about 4.7 Mg (with $\rho = 1966$ kg/m^3) consists of three parts: (i) upper rectangular prism with top surface coinciding with the floor; (ii) lower cylinder (with diameter 1 m) with bottom rests on a moraine layer; (iii) small empty hollow around benchmark. Such massive pier provides stability for precise absolute and relative gravity measurements, but also disturbs local gravity field.

Input file with dimensions and densities of Haanja pier:

/Examples/Haanja_2008-2019_VGG/body_Haanja.inp

```
# R1               !(R)ectangular parallelepiped body (cuboid)
200                !density (0 or const)
-0.6 -0.6 -0.021   !x1,y1,z1 (two sides relative to center(BM) and top height)
0.6 0.6 0.979      !x2,y2,z2 (opposite sides and bottom height)

# C1               !(C)ylinder
200                !density (0 or const)
0.98 2.22 1.0      !z1,z2,D - depth of top,bottom and diameter(m)

# R2               !Empty space around BM
-1966
-0.11 -0.11 -0.021
0.11 0.11 0.011
```

Accordingly, density contrast $d\rho = 200$ kg/m^3 of pier-soil, and the density $d\rho = -1966$ kg/m^3 of hollow around BM on top of pier are assumed.

Negative density value helps to remove the effect of hollow around BM. Exact density of pier was determined in special laboratory (by using test mass) during the construction of pier in 2007-2008 (Oja, 2019).

Density parameter

It is possible to estimate additional density parameter ρ with WZZ by using model

$$dg_{ij} = \sum_{l=1}^{n} c_i \left(z_j^l - z_i^l\right) - \rho\, dM'_{ij}, \tag{3.6}$$

where M' is the theoretical effect of local mass with unit density ($\rho' = 1$ kg/m^3). Only one parameter ρ for all local mass bodies or density contrasts can be estimated. For the parameter ρ estimation in adjustment the body density in input file (e.g. `body_Haanja.inp`) should be set to zero.

/Examples/Haanja_2008-2019_VGG/adj_dens/body0_Haanja.inp

```
# R1
0
-0.6 -0.6 -0.021
0.6 0.6 0.979
# C1
0
0.98 2.22 1.0
# R2
-1966
-0.11 -0.11 -0.021
0.11 0.11 0.011
```

For the simulation of theoretical attraction of local mass bodies M and M' the exact dimensions and locations of these bodies as well as location of gravimeter's sensor (used for gradient measurement) relative to the benchmark should be known. However, if the exact size of e.g. underground body is not well known, approximate dimensions (e.g. estimated from the size of pier's top surface) could still be helpful in modelling.

In near ideal case also the densities of local masses (e.g. the density contrast between underground measurement pier and its surrounding soil) are known. However, such knowledge is often missing, therefore the estimation of ρ parameter could be useful to study the density or density contrast.

It should be noted that the density parameter is very sensitive to the errors in gravity measurements as well as in the dimension values of mass bodies, thus more measurements with higher precision and careful analysis by user must be done to extract reliable density information. For more accurate and independent determination of density contrast the horizontal gravity profiles over mass bodies should be done to fit these with theoretical model, similarly to the method proposed by Nettleton (1939).

Fixed solution

To estimate absolute gravity values g_j at every height level z_j ($z_j = 0 \ldots 1.5$ m with 1 mm step, i.e. $j = 1 \ldots 1501$) above reference point (benchmark), fixed gravity value g_{fix} or several values (e.g. absolute gravity results from the AG campaigns) at observation height are used to fix the solution

$$g_{\text{fix}} = g_j + \sum_{l=1}^{n} c_l \left(z_{\text{fix}}^l - z_j^l\right) + \rho\, dM'_{\text{fix},j}, \tag{3.7}$$

where $dM'_{\text{fix},j} = [M'(z_j) - M'(z_{\text{fix}})]$. In practice the gravity value g_0 at benchmark level $z_0 = 0$ m ($j = 0$) has to be evaluated.

Input and output files

The body file with dimensions and densities (or density contrasts) for mass bodies was introduced above. However, the main input file (separated by headers: lines started with #) includes the fixed height z_0, fixed gravity g_{fix}!! and gravity differences Δg_{ij} between height levels z_i, z_j (the lines starting with "!" flag the outliers which are not used in adjustment):

/Examples/Haanja_2008-2019_VGG/Haanja_2008-2019.dat

```
# Ref.height (where the g value is reduced, eg level of BM)
0.0
# Fixed value(s) (AG values with STD and obs.height)
981678514.0  3.9  1.200
# Gravity ties along the vertical with STD and heights h1, h2
# Optionally columns with additional info can be given
-328.60  1.25  0.1540  1.2890    2008-07-27  MBK  CG5-10052
-304.20  0.79  0.1420  1.1920    2008-07-25  TO   CG5-36
-304.60  0.73  0.1450  1.1950    2008-07-25  TO   CG5-10092
-218.90  1.03  0.1410  0.8890    2008-08-27  TO   CG5-36
!-334.70  0.61  0.1410  1.2910    2008-08-27  TO   CG5-36
-217.30  1.22  0.1430  0.8910    2008-08-27  TO   CG5-10092
-332.00  0.66  0.1420  1.2930    2008-08-27  TO   CG5-10092
!-174.80  1.25  0.1635  0.7730    2017-07-14  TO   CG5-36
-320.30  0.68  0.1635  1.2780    2017-07-14  TO   CG5-36
-176.60  1.16  0.1670  0.7765    2017-07-14  TO   CG5-10092
-319.00  0.63  0.1670  1.2820    2017-07-14  TO   CG5-10092
-187.80  1.40  0.1587  0.8068    2019-05-21  TO   CG5-36
-312.00  0.84  0.1587  1.2411    2019-05-21  TO   CG5-36
```

The control file of WZZ2.1 includes the input file names (lines 1-2), the degree n of polynomial (line 3), the logical T/F value for the adjustment with (T – WLS) or without weights (F – LS) and a priori standard deviation of unit weight (line 4):

/Examples/Haanja_2008-2019_VGG/wzz.inp

```
1  body_Haanja.inp
2  Haanja_2008-2019.inp
3  1
4  T 1
```

There are two output files to contain:

a) all input and output parameters and other information about the calculations, see e.g. file `Haanja_2008-2019_RCR_d1_Wzz21.out` in App. E.1;
b) modelled gravity $g(z)$ [unit µGal] and gradient $g_z(z)$ values [µGal/m] along vertical from $z = 1, 2 \ldots 1500$ mm (with name `*.vgg`).

The model file `*.vgg` has 6 columns: $z, g(z), g_z = dg/dz, u(g_z), dg_{0j} = g_j - g_0, u(dg_{0j})$. The part of file `Haanja_2008-2019.vgg` with 1501 rows is shown here as an example:

/Examples/Haanja_2008-2019_VGG/Haanja_2008-2019.vgg

```
   0.000   981678858.767    -272.980  0.476      0.000  0.000
   0.001   981678858.494    -272.947  0.476     -0.273  0.000
   0.002   981678858.221    -272.919  0.476     -0.546  0.001
...
   0.998   981678572.056    -287.577  0.476   -286.710  0.475
   0.999   981678571.769    -287.575  0.476   -286.998  0.476
   1.000   981678571.481    -287.573  0.476   -287.286  0.476
   1.001   981678571.194    -287.571  0.476   -287.573  0.477
```

```
...
   1.198   981678514.575    -287.263  0.476    -344.192  0.571
   1.199   981678514.287    -287.262  0.476    -344.479  0.571
   1.200   981678514.000    -287.260  0.476    -344.767  0.572
   1.201   981678513.713    -287.259  0.476    -345.054  0.572
...
   1.499   981678428.157    -286.971  0.476    -430.610  0.714
   1.500   981678427.870    -286.970  0.476    -430.897  0.715
```

Bibliography

Akaike, H. (1974). A new look at the statistical model identification. *IEEE transactions on automatic control*, 19(6):716–723.

Andersen, O. B. and Forsberg, R. (1996). Danish precision gravity reference network. Publication vol. 4, National Survey and Cadastre (KMS), Denmark.

Boedecker, G. (1988). International Absolute Gravity Basestation Network (IAGBN). Absolute gravity observations data processing standards station documentation. *Bulletin d'information-Bureau gravimétrique international*, 63:51–57.

Boedecker, G. (1993). The International Absolute Gravity Basestation Network (IAGBN): Status 1992. In *Montag H, Reigber C (eds) Geodesy and Physics of the Earth. IAG Symp. 112*, pages 61–62. Springer.

Burnham, K. P. and Anderson, D. R. (2002). *Model Selection and Multimodel Inference*. Springer, New York, NY.

Burnham, K. P. and Anderson, D. R. (2004). Multimodel inference: understanding aic and bic in model selection. *Sociological methods & research*, 33(2):261–304.

CG5 Manual (2012). *CG-5 Operation Manual*. Scintrex Ltd. Part #867700, Rev. 8 (pdf-file from http://www.scintrexltd.com at 2012-08-29).

Dmitrijeva, M., Plado, J., and Oja, T. (2018). The Luusika potential field anomaly, eastern Estonia: modelling results. *Estonian Journal of Earth Sciences*, 67(4):228–237.

Ellmann, A., All, T., and Oja, T. (2009). Toward unification of terrestial gravity data sets in Estonia. *Estonian Journal of Earth Sciences*, 58(4):229–245.

Gabalda, G., Bonvalot, S., and Hipkin, R. (2003). CG3TOOL: an interactive computer program to process Scintrex CG-3/3M gravity data for high-resolution applications. *Computers & Geosciences*, 29(2):155–171.

Hwang, C., Wang, C.-G., and Lee, L.-H. (2002). Adjustment of relative gravity measurements using weighted and datum-free constraints. *Computers & Geosciences*, 28(9):1005–1015.

IAG (1984). International Association of Geodesy (IAG) Resolution No 16 adopted at the XVIII General Assembly of the IUGG in Hamburg, August 1983. In *The Geodesist's Handbook*, page 321. Bulletin Geodesique 58(3).

Koch, K.-R. (1999). *Parameter Estimation and Hypothesis Testing in Linear Models*. Springer, 2nd edition.

Mäkinen, J. (2012). Gravity gradients above piers in absolute gravimetry: use remove-restore for the pier attraction. In Peshekhonov, V., editor, *Proceedings of the IAG Symposium on Terrestrial Gravimetry (TG-SMM2010)*, pages 90–97.

Nettleton, L. (1939). Determination of density for reduction of gravimeter observations. *Geophysics*, 4(3):176–183.

NOAA (1976). *U.S. Standard Atmosphere, 1976.* National Oceanic and Atmospheric Administration, U.S. Air Force.

Oja, T. (2012a). Gravity system and network in Estonia. In *Geodesy for Planet Earth*, volume 136 of *IAG Symposia*, pages 315–322. Springer.

Oja, T. (2012b). Gravity system and network in Estonia. In *Geodesy for Planet Earth*, volume 136 of *IAG Symposia*, pages 315–322. Springer. Proceedings of the 2009 IAG Symposium, Buenos Aires, Argentina, 31 August 31 - 4 September 2009.

Oja, T. (2019). Estonian first order gravity network and solution EG2000 [Eesti I klassi gravimeetriline võrk ja eg2000 lahendus]. Report of geodetic works, Department of geodesy, Estonian Land Board. 52 p. (in Estonian).

Oja, T., Ellmann, A., and Märdla, S. (2019). Gravity anomaly field over Estonia. *Estonian Journal of Earth Sciences*, 68(2):55–75.

Oja, T., Mäkinen, J., Bilker-Koivula, M., and Timmen, L. (2021). Absolute gravity observations in Estonia from 1995 to 2017. *Journal of Geodesy*, 95(2). (submitted).

Oja, T., Türk, K., Ellmann, A., Gruno, A., Bloom, A., and Sulaoja, M. (2011). Relative gravity surveys on ice-covered water bodies. In *Proceedings of the International Conference on Environmental Engineering. ICEE*, volume 8, pages 1394–1401. Vilnius Gediminas Technical University.

Olsson, P.-A., Breili, K., Ophaug, V., Steffen, H., Bilker-Koivula, M., Nielsen, E., Oja, T., and Timmen, L. (2019). Postglacial gravity change in Fennoscandia—three decades of repeated absolute gravity observations. *Geophysical Journal International*, 217(2):1141–1156.

Pope, A. J. (1976). *The statistics of residuals and the detection of outliers.* US National Oceanic and Atmospheric Administration, National Geodetic Survey, Geodetic Research and Development Laboratory.

Schwarz, G. (1978). Estimating the dimension of a model. *The annals of statistics*, 6(2):461–464.

Strang, G. and Borre, K. (1997). *Linear algebra, geodesy, and GPS.* Wellesley-Cambridge Press.

Tamura, Y. (1987). A harmonic development of the tide-generating potential. *Bull. d'Inf. Marées terrestres*, 99:6813–6855.

Timmen, L. and Wenzel, H. (1994). Worldwide synthetic gravity tide parameters available on internet. *Bulletin d'information-Bureau gravimétrique international*, 75:32–40.

Torge, W. (1989). *Gravimetry.* Walter de Gruyter, Berlin, New York. 465 p.

Türk, K., Sulaoja, M., Oja, T., Ellmann, A., and Jürgenson, H. (2011). Precise gravity surveys in south Estonia from 2009 to 2010. In *Proceedings of the International Conference on Environmental Engineering. ICEE*, volume 8, pages 1499–1505. Vilnius Gediminas Technical University.

Van Camp, M. and Vauterin, P. (2005). Tsoft: graphical and interactive software for the analysis of time series and Earth tides. *Computers & Geosciences*, 31(5):631–640.

Vanicek, P. and Krakiwsky, E. J. (1986). *Geodesy: the concepts.* Elsevier, 2nd edition.

Vestøl, O., Ågren, J., Steffen, H., Kierulf, H., and Tarasov, L. (2019). NKG2016LU: a new land uplift model for Fennoscandia and the Baltic region. *Journal of Geodesy*, 93(9):1759–1779.

Wenzel, H.-G. (1994). Earth tide data processing package ETERNA 3.20. *Marées terrestres (Bruxelles)*, 120:9019–9021.

Wolf, P. R. and Ghilani, C. D. (1997). *Adjustment Computations. Statistics and least squares in surveying and GIS.* John Wiley & Sons Inc. New York.

Appendices

Appendix A

Observation files and pre-processing

A.1 CG-5 observation file

/Examples/2010-03-17_GulfofRiga/S36/2010-03-17_S36.txt

```

/        CG-5 SURVEY
/        Survey name:            Gulf-of-Riga(Survey-on-ice)
/        Instrument S/N:         36
/        Client:                 ELB
/        Operator:               T.OJA
/        Date:                   2010/ 3/17
/        Time:                   06:16:12
/        LONG:                 26.0000000 E
/        LAT:                  58.4000000 N
/        ZONE:                 0
/        GMT DIFF.:            0.0

/        CG-5 SETUP PARAMETERS
/        Gref:                        0.000
/        Gcal1:                 8987.504
/        TiltxS:                 609.765
/        TiltyS:                 570.791
/        TiltxO:                 -45.332
/        TiltyO:                 65.519
/        Tempco:                 -0.128
/        Drift:                 0.575
/        DriftTime Start:         06:16:16
/        DriftDate Start:         2016/03/17

/        CG-5 OPTIONS
/        Tide Correction:     NO
/        Cont. Tilt:         YES
/        Auto Rejection:     YES
/        Terrain Corr.:       NO
/        Seismic Filter:     YES
/        Raw Data:           YES
Line          3.000N
/------LINE-----STATION-----ALT.------GRAV.---SD.--TILTX--TILTY-TEMP---TIDE---DUR-REJ-----TIME----DEC.TIME+DATE--TERRAIN---DATE
 3.0000000  80006.0000000   20.5565   5120.256 0.020   17.8    30.9 -2.58 -0.037  60   0 07:49:09     40225.32528    0.0000  2010/03/17
 3.0000000  80006.0000000   19.3358   5120.246 0.016  -12.4    -0.1 -2.60 -0.037  60   0 07:50:37     40225.32629    0.0000  2010/03/17
 3.0000000  80006.0000000   18.6033   5120.250 0.024   -6.0     6.4 -2.62 -0.036  60   0 07:51:45     40225.32708    0.0000  2010/03/17
 3.0000  10031711.0000000   14.4530   5110.218 0.024   10.7    13.2 -2.49 -0.023  60   0 08:25:42     40225.35062    0.0000  2010/03/17
 3.0000  10031711.0000000   13.2323   5110.218 0.025   13.5    17.4 -2.52 -0.023  60   1 08:26:48     40225.35138    0.0000  2010/03/17
 3.0000  10031712.0000000    9.8143   5107.594 0.067   -6.3    -1.9 -2.75 -0.012  90   2 08:57:12     40225.37246    0.0000  2010/03/17
 3.0000  10031712.0000000    9.5702   5107.591 0.118   -9.9    -1.6 -2.80 -0.012  90  16 08:58:51     40225.37360    0.0000  2010/03/17
 3.0000  10031713.0000000    9.3260   5100.535 0.058   -7.6     4.6 -2.44 -0.002  90   3 09:30:49     40225.39577    0.0000  2010/03/17
 3.0000  10031713.0000000    9.5702   5100.541 0.101   -9.4     8.5 -2.48 -0.002  90   4 09:32:28     40225.39691    0.0000  2010/03/17
 3.0000  10031714.0000000    8.5936   5109.015 0.176  -19.9    -7.9 -2.44 0.005  90   8 10:05:37     40225.41989    0.0000  2010/03/17
 3.0000  10031714.0000000    8.8377   5108.982 0.089  -12.0    -5.3 -2.52 0.005  90  16 10:07:56     40225.42150    0.0000  2010/03/17
 3.0000  10031715.0000000    6.6405   5110.633 0.138   -9.4     7.4 -2.50 0.008  90   0 10:29:09     40225.43621    0.0000  2010/03/17
 3.0000  10031715.0000000    6.1522   5110.625 0.225  -12.0     9.3 -2.52 0.009  90  10 10:30:48     40225.43735    0.0000  2010/03/17
 3.0000  10031604.0000000    4.6874   5109.420 0.099    0.2     6.5 -2.62 0.010  90  13 10:44:55     40225.44714    0.0000  2010/03/17
 3.0000  10031604.0000000    3.9549   5109.409 0.316   -4.7     9.6 -2.63 0.010  90  29 10:46:32     40225.44826    0.0000  2010/03/17
 3.0000  10031717.0000000    4.1991   5111.254 0.161    2.9    -1.3 -2.47 0.011  90   0 11:10:27     40225.46485    0.0000  2010/03/17
 3.0000  10031717.0000000    3.9549   5111.213 0.219    5.7    -2.4 -2.52 0.011  90   0 11:12:06     40225.46599    0.0000  2010/03/17
 3.0000  10031713.0000000    3.2225   5100.471 0.162    4.6     3.1 -2.40 0.009  90   0 11:40:13     40225.48548    0.0000  2010/03/17
 3.0000  10031713.0000000    2.4901   5100.404 0.202    3.6     3.0 -2.44 0.009  90   2 11:41:50     40225.48660    0.0000  2010/03/17
 4.0000  10031711.0000000    4.1991   5110.156 0.043    1.7    -1.8 -2.43 0.005  60   0 12:15:01     40225.50961    0.0000  2010/03/17
 4.0000  10031711.0000000    4.1991   5110.157 0.033    0.4    -1.4 -2.48 0.004  60   0 12:16:09     40225.51040    0.0000  2010/03/17
 4.0000000  80006.0000000    5.9081   5120.188 0.018    5.6    21.9 -2.40 0.000  60   0 12:41:25     40225.52792    0.0000  2010/03/17
 4.0000000  80006.0000000    5.9081   5120.198 0.025   13.9    35.1 -2.42 -0.001  60   0 12:42:33     40225.52870    0.0000  2010/03/17
 4.0000000  80006.0000000    5.6639   5120.188 0.020    3.6     5.8 -2.47 -0.001  60   0 12:44:06     40225.52978    0.0000  2010/03/17
 4.0000  10031601.0000000    7.1288   5105.818 0.017    2.0    14.5 -2.53 -0.006  60   0 13:03:29     40225.54322    0.0000  2010/03/17
 4.0000  10031601.0000000    6.6405   5105.822 0.019    3.1    18.0 -2.54 -0.007  60   0 13:04:35     40225.54398    0.0000  2010/03/17
 4.0000  10031701.0000000    8.1053   5089.966 0.019    2.5     5.6 -2.62 -0.014  40   0 13:29:55     40225.56154    0.0000  2010/03/17
 4.0000  10031701.0000000    8.5936   5089.965 0.014    3.8     1.9 -2.64 -0.015  40   0 13:31:00     40225.56229    0.0000  2010/03/17
 4.0000000  80006.0000000   10.0584   5120.203 0.014  -11.7     4.7 -2.38 -0.025  40   0 14:01:58     40225.58376    0.0000  2010/03/17
 4.0000000  80006.0000000    9.3260   5120.207 0.020  -15.3     6.3 -2.40 -0.026  40   0 14:02:43     40225.58428    0.0000  2010/03/17
 4.0000000  80006.0000000    9.5702   5120.206 0.013   -3.6    -2.2 -2.43 -0.026  40   0 14:03:47     40225.58502    0.0000  2010/03/17
```

A.2 CG5FORM input

Example of `*.inf` file:

/Examples/2010-03-17_GulfofRiga/S36/2010-03-17_S36.inf

```
# S- 36   Gulf-of-Riga(Survey-on-ice)     2010 ELB      T.OJA
   80006  2010-03-17  07:49:39     355  -20   -999.9
10031711  2010-03-17  08:26:12     345    0   -999.9
10031712  2010-03-17  08:57:57     350    0   -999.9
10031713  2010-03-17  09:31:34     335    0   -999.9
10031714  2010-03-17  10:06:22     350    0   -999.9
10031715  2010-03-17  10:29:54     350    0   -999.9
10031604  2010-03-17  10:45:40     340    0   -999.9
10031717  2010-03-17  11:11:12     305    0   -999.9
10031713  2010-03-17  11:40:58     350    0   -999.9
10031711  2010-03-17  12:15:31     340    0   -999.9
   80006  2010-03-17  12:41:55     355  -20   -999.9
10031601  2010-03-17  13:03:59     345    0   -999.9
10031701  2010-03-17  13:30:15     345    0   -999.9
   80006  2010-03-17  14:02:18     357  -20   -999.9
```

A.3 Observation file for GRREDU3

The output of `CG5FORM` is the observation file with correct format for GRREDU3. Without pre-processing (eg data from fieldbook for LCR G), the obs file is made by using text editor, a spreadsheet tool (like Excel) etc.

/Examples/2010-03-17_GulfofRiga/S36/2010-03-17_S36.obs

```
# S- 36   Gulf-of-Riga(Survey-on-ice)     2010 ELB      T.OJA
   80006  2010-03-17  07:49:39   5120.2560  0.0200   335   -999.9
   80006  2010-03-17  07:51:07   5120.2460  0.0160   335   -999.9
   80006  2010-03-17  07:52:15   5120.2500  0.0240   335   -999.9
10031711  2010-03-17  08:26:12   5110.2180  0.0240   345   -999.9
10031711  2010-03-17  08:27:18   5110.2180  0.0250   345   -999.9
10031712  2010-03-17  08:57:57   5107.5940  0.0670   350   -999.9
10031712  2010-03-17  08:59:36   5107.5910  0.1180   350   -999.9
10031713  2010-03-17  09:31:34   5100.5350  0.0580   335   -999.9
10031713  2010-03-17  09:33:13   5100.5410  0.1010   335   -999.9
10031714  2010-03-17  10:06:22   5109.0150  0.1760   350   -999.9
10031714  2010-03-17  10:08:41   5108.9820  0.0890   350   -999.9
10031715  2010-03-17  10:29:54   5110.6330  0.1380   350   -999.9
10031715  2010-03-17  10:31:33   5110.6250  0.2250   350   -999.9
10031604  2010-03-17  10:45:40   5109.4200  0.0990   340   -999.9
10031604  2010-03-17  10:47:17   5109.4090  0.3160   340   -999.9
10031717  2010-03-17  11:11:12   5111.2540  0.1610   305   -999.9
10031717  2010-03-17  11:12:51   5111.2130  0.2190   305   -999.9
10031713  2010-03-17  11:40:58   5100.4710  0.1620   350   -999.9
10031713  2010-03-17  11:42:35   5100.4040  0.2020   350   -999.9
10031711  2010-03-17  12:15:31   5110.1560  0.0430   340   -999.9
10031711  2010-03-17  12:16:39   5110.1570  0.0330   340   -999.9
   80006  2010-03-17  12:41:55   5120.1880  0.0180   335   -999.9
   80006  2010-03-17  12:43:03   5120.1980  0.0250   335   -999.9
   80006  2010-03-17  12:44:36   5120.1880  0.0200   335   -999.9
10031601  2010-03-17  13:03:59   5105.8180  0.0170   345   -999.9
10031601  2010-03-17  13:05:05   5105.8220  0.0190   345   -999.9
10031701  2010-03-17  13:30:15   5089.9660  0.0190   345   -999.9
10031701  2010-03-17  13:31:20   5089.9650  0.0140   345   -999.9
   80006  2010-03-17  14:02:18   5120.2030  0.0140   337   -999.9
   80006  2010-03-17  14:03:03   5120.2070  0.0200   337   -999.9
   80006  2010-03-17  14:04:07   5120.2060  0.0130   337   -999.9
```

A.4 GRREDU3 output

```
/Examples/2010-03-17_GulfofRiga/S36/2010-03-17_S36.redu
                                                   |-C.U./mGal----uGal|-----------CORRECTIONS-(uGal)-----------|--------mGal--------|
  station       date,      time      obs    reading   stdev    tides   air-    free-   polar    gdot    calibr.  reduced   station
       ID              (UT + 0)      ID                                pres.   air     motion           error    reading   name
# S- 36   Gulf-of-Riga(Survey-on-ice)     2010 ELB       T.OJA
    80006  2010-03-17, 07:49:39       1  5120.2560    20.0    -33.6     0.0     40.2     0.0     0.0    -0.4999  5119.7627 ReiuGR
    80006  2010-03-17, 07:51:07       2  5120.2460    16.0    -33.0     0.0     40.2     0.0     0.0    -0.4999  5119.7532 ReiuGR
    80006  2010-03-17, 07:52:15       3  5120.2500    24.0    -32.6     0.0     40.2     0.0     0.0    -0.4999  5119.7577 ReiuGR
 10031711  2010-03-17, 08:26:12       4  5110.2180    24.0    -20.3     0.0     41.4     0.0     0.0    -0.4989  5109.7402 Sunset
 10031711  2010-03-17, 08:27:18       5  5110.2180    25.0    -19.8     0.0     41.4     0.0     0.0    -0.4989  5109.7406 Sunset
 10031712  2010-03-17, 08:57:57       6  5107.5940    67.0     -8.8     0.0     42.9     0.0     0.0    -0.4986  5107.1295 Uus2
 10031712  2010-03-17, 08:59:36       7  5107.5910   118.0     -8.2     0.0     42.9     0.0     0.0    -0.4986  5107.1271 Uus2
 10031713  2010-03-17, 09:31:34       8  5100.5350    58.0      1.9     0.0     38.3     0.0     0.0    -0.4979  5100.0772 Vana1121
 10031713  2010-03-17, 09:33:13       9  5100.5410   101.0      2.4     0.0     38.3     0.0     0.0    -0.4980  5100.0837 Vana1121
 10031714  2010-03-17, 10:06:22      10  5109.0150   176.0     10.6     0.0     42.9     0.0     0.0    -0.4988  5108.5697 Vana1116
 10031714  2010-03-17, 10:08:41      11  5108.9820    89.0     11.0     0.0     42.9     0.0     0.0    -0.4988  5108.5371 Vana1116
 10031715  2010-03-17, 10:29:54      12  5110.6330   138.0     14.7     0.0     42.9     0.0     0.0    -0.4989  5110.1916 Kalamees
 10031715  2010-03-17, 10:31:33      13  5110.6250   225.0     14.9     0.0     42.9     0.0     0.0    -0.4989  5110.1839 Kalamees
 10031604  2010-03-17, 10:45:40      14  5109.4200    99.0     16.6     0.0     39.8     0.0     0.0    -0.4988  5108.9776 Vana1111
 10031604  2010-03-17, 10:47:17      15  5109.4090   316.0     16.7     0.0     39.8     0.0     0.0    -0.4988  5108.9667 Vana1111
 10031717  2010-03-17, 11:11:12      16  5111.2540   161.0     17.8     0.0     29.0     0.0     0.0    -0.4990  5110.8018 Vana1109
 10031717  2010-03-17, 11:12:51      17  5111.2130   219.0     17.8     0.0     29.0     0.0     0.0    -0.4990  5110.7608 Vana1109
 10031713  2010-03-17, 11:40:58      18  5100.4710   162.0     16.7     0.0     42.9     0.0     0.0    -0.4979  5100.0326 Vana1121
 10031713  2010-03-17, 11:42:35      19  5100.4040   202.0     16.6     0.0     42.9     0.0     0.0    -0.4979  5099.9655 Vana1121
 10031711  2010-03-17, 12:15:31      20  5110.1560    43.0     12.1     0.0     39.8     0.0     0.0    -0.4989  5109.7091 Sunset
 10031711  2010-03-17, 12:16:39      21  5110.1570    33.0     11.9     0.0     39.8     0.0     0.0    -0.4989  5109.7099 Sunset
    80006  2010-03-17, 12:41:55      22  5120.1880    18.0      6.3     0.0     40.2     0.0     0.0    -0.4999  5119.7346 ReiuGR
    80006  2010-03-17, 12:43:03      23  5120.1980    25.0      6.0     0.0     40.2     0.0     0.0    -0.4999  5119.7443 ReiuGR
    80006  2010-03-17, 12:44:36      24  5120.1880    20.0      5.6     0.0     40.2     0.0     0.0    -0.4999  5119.7339 ReiuGR
 10031601  2010-03-17, 13:03:59      25  5105.8180    17.0      1.6     0.0     41.4     0.0     0.0    -0.4985  5105.3625 Võiste
 10031601  2010-03-17, 13:05:05      26  5105.8220    19.0      1.3     0.0     41.4     0.0     0.0    -0.4985  5105.3661 Võiste
 10031701  2010-03-17, 13:30:15      27  5089.9660    19.0     -6.2     0.0     41.4     0.0     0.0    -0.4969  5089.5042 Rannametsa
 10031701  2010-03-17, 13:31:20      28  5089.9650    14.0     -6.6     0.0     41.4     0.0     0.0    -0.4969  5089.5029 Rannametsa
    80006  2010-03-17, 14:02:18      29  5120.2030    14.0    -19.0     0.0     40.8     0.0     0.0    -0.4999  5119.7250 ReiuGR
    80006  2010-03-17, 14:03:03      30  5120.2070    20.0    -19.2     0.0     40.8     0.0     0.0    -0.4999  5119.7287 ReiuGR
    80006  2010-03-17, 14:04:07      31  5120.2060    13.0    -19.6     0.0     40.8     0.0     0.0    -0.4999  5119.7273 ReiuGR
```

Appendix B

Global grid of tidal parameters

The interpolation of amplitude factor δ and phase lag κ of the main waves (needed for tidal correction) is based on the global grid in file `WPARM.DAT`, for more details see file header. The grid plot in Fig. B.1 showed that grid nodes (as the center of grid cells, pixels) are available over land, but in some areas (coasts, islands) the interpolation might not work due to the missing nodes. In such cases following solutions are suggested:

1) For the processing of less accurate measurements (e.g survey data) no tidal parameter file `*.tide` is interpolated. Dummy path or empty file as an input of GRREDU3 is then used which introduces default values (1.16, 0.0° for δ and κ, respectively) in the computation of tidal corrections.
2) The coordinates of coastal stations can be shifted (e.g by using temporary coordinate file) to get station's position into the grid domain for successful interpolation.
3) The values of nearest nodes can be used, eg by using "Nearest neighbor" algorithm.
4) By converting text grid file to the binary NETCDF, geotiff file for advanced interpolation with tools like GMT, QGIS etc.

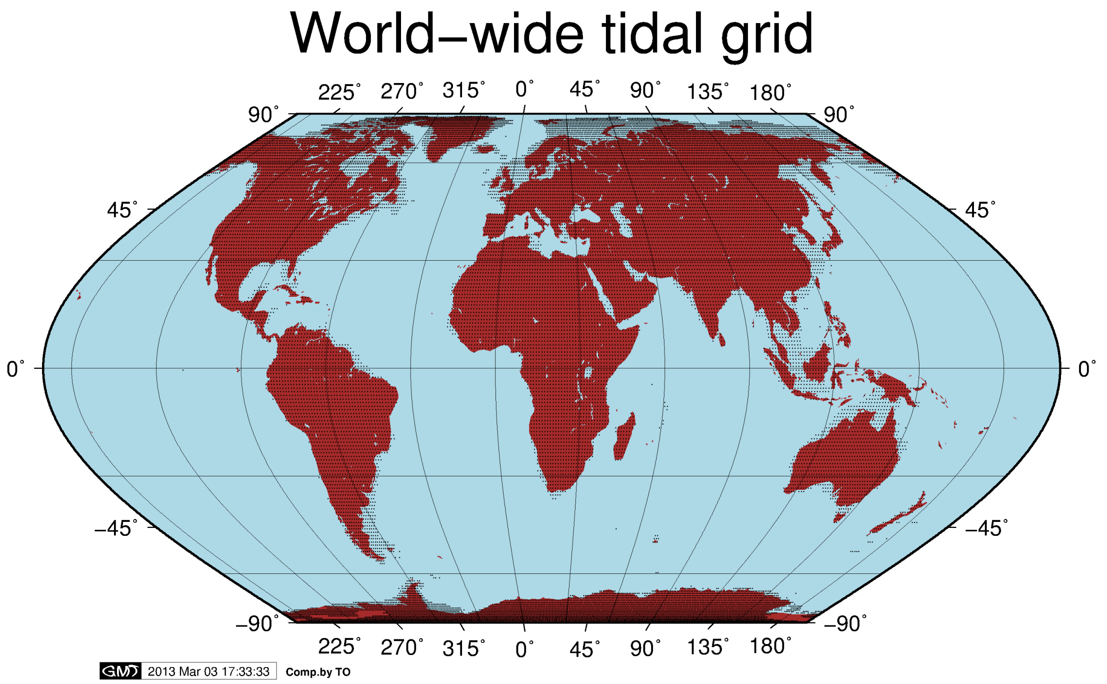

Figure B.1: The nodes with δ and κ values of global 1°x1° grid in file `WPARM.DAT`.

Appendix C

GRADJ3 input and output

C.1 Adjusted gravity result file *.grav

./Examples/2010-03-17_GulfofRiga/adj_all/GoF2010.grav

```
# Adjusted results for project: GoF2010.proj

# ======= Fixed stations and adjustment residuals (mGal) =======
# seq   stat      fix g   sigma   weight        adj g       res  statname
# no      ID             (sc_std=1/  1.0)
  1    80006  981772.1920 0.0080    9.77    981772.1920   0.0000  ReiuGR

# ======= Adjusted results with standard deviations (mGal) ======
# seq    stat     gravity     std      statname
# no       ID             (sc_std=  1.0)
   1 10031601  981757.8188  0.0144     Võiste
   2 10031604  981761.4161  0.0362     Vana1111
   3 10031701  981741.9379  0.0142     Rannametsa
   4 10031702  981732.4002  0.0387     vana-1118
   5 10031703  981757.7950  0.0510     vana-1112
   6 10031711  981762.1679  0.0323     Sunset
   7 10031712  981759.5651  0.0366     Uus2
   8 10031713  981752.4831  0.0266     Vana1121
   9 10031714  981760.9948  0.0363     Vana1116
  10 10031715  981762.6306  0.0362     Kalamees
  11 10031717  981763.2269  0.0362     Vana1109

=== Statistics of adjustment ===
 Adjustment observations:    52
 Stations:                   12
 Total unknowns:             18
 Degrees of freedom:         34
 SIGMA1 (apriori st.dev. of unit weight, mGal)    :   0.0250
 SIGMA2 (aposteriori st.dev. of unit weight, mGal):   0.0246

=== Statistical tests ===
 Confidence level:      0.950

== Critical t-value for statistical parameter testing: t-crit=  2.03
 NB! If t-stat > t-crit -> parameter significant, else with " ! " mark.

== Chi-square test of the SIGMA1 (variance factor) ==
 Statistic (norm. with dof): X^2 =  0.97
                X^2 lower value =  0.58
                X^2 upper value =  1.53  ->  test PASSED!

== Information Criteria AIC, BIC as relative measures
== to select the best adjustment (the lower is better):
 # AIC =     218.36, BIC =     255.44
```

C.2 Residual and mean residual files

Example of `*.resi`:

./Examples/2010-03-17_GulfofRiga/adj_all/GoF2010.resi

```
GoF2010.proj
 ---------------------------------------------------------------
 Critical values for parameter and residual statistical tests:
 conf.level=  0.95, t-crit=  2.03, tau-crit=  1.89

 === Residuals of the readings with tare and drift info ===
 ----------------------------------------------------------------------------------------------------------------
 station                     instr.  oID  red.obs           drift    res  st.res   red.no   res.plot    station
      ID     date,      time     ID         (mGal)   weight  |---(uGal)---|     tau-test   ( 5 uGal/div)  name
 ----------------------------------------------------------------------------------------------------------------
 DRIFT degree = 2
 DRIFT of  1 order:   -2690.5 +/-     464.4 uGal/day^1 (t-stat=  5.79)
 DRIFT of  2 order:    7735.4 +/-    1378.7 uGal/day^2 (t-stat=  5.61)
 BIAS parameter:      -244816.7 +/-   17.9 uGal
   80006  2010-03-17, 07:53:00  191     1  5527.3819   1.00      0.0   -6.6    0.4      0.6      *!       ReiuGR
   80006  2010-03-17, 07:55:00  191     2  5527.3784   1.00     -3.7   -6.8    0.4      0.6      *!       ReiuGR
10031701  2010-03-17, 09:05:00  191     3  5496.9862   1.00   -115.2   19.8    1.0      0.7       !   *   Rannametsa
10031701  2010-03-17, 09:06:00  191     4  5496.9855   1.00   -116.5   19.1    1.0      0.7       !   *   Rannametsa
10031702  2010-03-17, 10:55:00  191     5  5487.4818   0.20   -216.5 -114.8    2.4 !    0.7 X     !       vana-1118
10031702  2010-03-17, 10:57:00  191     6  5487.4799   0.20   -217.5 -113.9    2.4 !    0.7 X     !       vana-1118
10031703  2010-03-17, 12:16:00  191     7  5512.7439   0.20   -233.4    1.0    0.0      0.5       *       vana-1112
10031703  2010-03-17, 12:20:00  191     8  5512.7463   0.20   -232.9   -1.0    0.0      0.5       *       vana-1112
10031702  2010-03-17, 14:03:00  191     9  5487.2887   0.20   -180.6  114.1    2.4 !    0.7       !     X vana-1118
10031702  2010-03-17, 14:05:00  191    10  5487.2900   0.20   -178.8  114.6    2.4 !    0.7       !     X vana-1118
10031701  2010-03-17, 14:44:00  191    12  5496.9866   1.00   -137.8   -3.2    0.2      0.7      *!       Rannametsa
10031701  2010-03-17, 14:45:00  191    13  5496.9821   1.00   -136.6    2.5    0.1      0.7       *       Rannametsa
10031601  2010-03-17, 15:20:00  191    14  5512.8953   1.00    -89.8   17.0    0.8      0.7       !  *    Võiste
10031601  2010-03-17, 15:21:00  191    15  5512.8929   1.00    -88.3   20.8    1.0      0.7       !   *   Võiste
   80006  2010-03-17, 15:43:00  191    17  5527.3550   1.00    -54.1  -33.8    1.6      0.7 X     !       ReiuGR
   80006  2010-03-17, 15:44:00  191    18  5527.3515   1.00    -52.5  -28.7    1.4      0.7 *     !       ReiuGR
             TARE:    -160.7 +/-   28.5 uGal   (t-stat=  5.64)
10031601  2010-03-17, 16:07:00  191    19  5512.8484   1.00      0.0  -19.7    1.0      0.6   *   !       Võiste
10031601  2010-03-17, 16:10:00  191    20  5512.8581   1.00      5.5  -23.9    1.2      0.6  *    !       Võiste
10031701  2010-03-17, 16:40:00  191    21  5496.9886   1.00     64.0   23.3    1.2      0.6       !    *  Rannametsa
10031701  2010-03-17, 16:41:00  191    22  5496.9936   1.00     66.1   20.3    1.0      0.6       !   *   Rannametsa
Set RMS of res:     53.8 uGal
   WRMS of res:     32.4 uGal
 ----------------------------
Total(prev sets) RMS of residuals:     53.8 uGal
                WRMS of residuals:     32.4 uGal
------------------------------------------------
 DRIFT degree = 1
 DRIFT of  1 order:     -94.9 +/-      51.1 uGal/day^1 (t-stat=  1.86) !
 BIAS parameter:      -652432.3 +/-   12.6 uGal
   80006  2010-03-17, 07:49:39   36     1  5119.7627   2.00      0.0   -3.0    0.2      0.7      *!       ReiuGR
   80006  2010-03-17, 07:51:07   36     2  5119.7532   2.00     -0.1    6.4    0.4      0.7       !*      ReiuGR
   80006  2010-03-17, 07:52:15   36     3  5119.7577   2.00     -0.2    1.9    0.1      0.7       *       ReiuGR
10031711  2010-03-17, 08:26:12   36     4  5109.7402   0.17     -2.4   -7.0    0.1      0.7      *!       Sunset
10031711  2010-03-17, 08:27:18   36     5  5109.7406   0.17     -2.5   -7.5    0.1      0.7      *!       Sunset
10031712  2010-03-17, 08:57:57   36     6  5107.1295   0.25     -4.5   -1.1    0.0      0.5       *       Uus2
10031712  2010-03-17, 08:59:36   36     7  5107.1271   0.25     -4.6    1.1    0.0      0.5       *       Uus2
10031713  2010-03-17, 09:31:34   36     8  5100.0772   0.25     -6.7  -33.1    0.8      0.7 X     !       Vana1121
10031713  2010-03-17, 09:33:13   36     9  5100.0837   0.25     -6.8  -39.7    0.9      0.7 X     !       Vana1121
10031714  2010-03-17, 10:06:22   36    10  5108.5697   0.25     -9.0  -16.2    0.5      0.5    *  !       Vana1116
10031714  2010-03-17, 10:08:41   36    11  5108.5371   0.25     -9.2   16.2    0.5      0.5       !  *    Vana1116
10031715  2010-03-17, 10:29:54   36    12  5110.1916   0.25    -10.6   -3.8    0.1      0.5      *!       Kalamees
10031715  2010-03-17, 10:31:33   36    13  5110.1839   0.25    -10.7    3.8    0.1      0.5       !*      Kalamees
10031604  2010-03-17, 10:45:40   36    14  5108.9776   0.25    -11.6   -5.4    0.2      0.5      *!       Vana1111
10031604  2010-03-17, 10:47:17   36    15  5108.9667   0.25    -11.7    5.4    0.2      0.5       !*      Vana1111
10031717  2010-03-17, 11:11:12   36    16  5110.8018   0.25    -13.3  -20.4    0.6      0.5   *   !       Vana1109
10031717  2010-03-17, 11:12:51   36    17  5110.7608   0.25    -13.4   20.4    0.6      0.5       !   *   Vana1109
10031713  2010-03-17, 11:40:58   36    18  5100.0326   0.25    -15.2    2.9    0.1      0.7       !*      Vana1121
10031713  2010-03-17, 11:42:35   36    19  5099.9655   0.25    -15.4   69.9    1.6      0.7       !     X Vana1121
10031711  2010-03-17, 12:15:31   36    20  5109.7091   0.15    -17.5    9.0    0.2      0.8       ! *     Sunset
10031711  2010-03-17, 12:16:39   36    21  5109.7099   0.15    -17.6    8.1    0.1      0.8       ! *     Sunset
   80006  2010-03-17, 12:41:55   36    22  5119.7346   2.00    -19.3    5.9    0.4      0.9       !*      ReiuGR
   80006  2010-03-17, 12:43:03   36    23  5119.7443   2.00    -19.3   -3.9    0.2      0.9      *!       ReiuGR
   80006  2010-03-17, 12:44:36   36    24  5119.7339   2.00    -19.4    6.4    0.4      0.9       !*      ReiuGR
```

```
10031601  2010-03-17, 13:03:59   36    25  5105.3625   2.00    -20.7    3.3    0.2      0.6      !*     Võiste
10031601  2010-03-17, 13:05:05   36    26  5105.3661   2.00    -20.8   -0.4    0.0      0.6      *      Võiste
10031701  2010-03-17, 13:30:15   36    27  5089.5042   2.00    -22.4  -21.1    1.5      0.7   *  !      Rannametsa
10031701  2010-03-17, 13:31:20   36    28  5089.5029   2.00    -22.5  -19.8    1.4      0.7   *  !      Rannametsa
   80006  2010-03-17, 14:02:18   36    29  5119.7250   2.00    -24.6   10.2    0.7      0.8      ! *    ReiuGR
   80006  2010-03-17, 14:03:03   36    30  5119.7287   2.00    -24.6    6.4    0.4      0.8      !*     ReiuGR
   80006  2010-03-17, 14:04:07   36    31  5119.7273   2.00    -24.7    7.7    0.5      0.8      ! *    ReiuGR
Set RMS of res:     18.4 uGal
   WRMS of res:     12.5 uGal
 ----------------------------
Total(prev sets) RMS of residuals:     18.4 uGal
                WRMS of residuals:     12.5 uGal
------------------------------------------------
```

Example of *.resi.mean:

./Examples/2010-03-17_GulfofRiga/adj_all/GoF2010.resi.mean

```
GoF2010.proj
 --------------------------------------------------------------
 Critical values for parameter and residual statistical tests:
 conf.level=  0.95, t-crit=  2.03, tau-crit=  1.89

 === AVERAGED readings, residuals etc with tare and drift info ===
 -----------------------------------------------------------------------------------------------------------------
 station                      instr. seqno  oID  red.obs             drift   res  st.res   red.no   res.plot    station
      ID     date,     time      ID               (mGal)   weight  |---(uGal)---|     tau-test   ( 5 uGal/div)  name
 -----------------------------------------------------------------------------------------------------------------
 DRIFT degree = 2
 DRIFT of  1 order:   -2690.5 +/-     464.4 uGal/day^1 (t-stat=  5.79)
 DRIFT of  2 order:    7735.4 +/-    1378.7 uGal/day^2 (t-stat=  5.61)
 BIAS parameter:    -244816.7 +/-   17.9 uGal
   80006  2010-03-17, 07:54:00  191     1     1  5527.3802   1.00     -1.9   -6.7    0.4      0.6      *!       ReiuGR
10031701  2010-03-17, 09:05:30  191     2     3  5496.9859   1.00   -115.9   19.5    1.0      0.7       !   *  Rannametsa
10031702  2010-03-17, 10:56:00  191     3     5  5487.4808   0.20   -217.0 -114.4    2.4 !    0.7 X     !       vana-1118
10031703  2010-03-17, 12:18:00  191     4     7  5512.7451   0.20   -233.1    0.0    0.0      0.5       *       vana-1112
10031702  2010-03-17, 14:04:00  191     5     9  5487.2894   0.20   -179.7  114.4    2.4 !    0.7       !     X vana-1118
10031701  2010-03-17, 14:44:30  191     6    12  5496.9844   1.00   -137.2   -0.4    0.1      0.7       *       Rannametsa
10031601  2010-03-17, 15:20:30  191     7    14  5512.8941   1.00    -89.1   18.9    0.9      0.7       !   *  Võiste
   80006  2010-03-17, 15:43:30  191     8    17  5527.3533   1.00    -53.3  -31.2    1.5      0.7 *     !       ReiuGR
           TARE:    -160.7 +/-   28.5 uGal   (t-stat=  5.64)
10031601  2010-03-17, 16:08:30  191     9    19  5512.8533   1.00      2.7  -21.8    1.1      0.6    *  !       Võiste
10031701  2010-03-17, 16:40:30  191    10    21  5496.9911   1.00     65.1   21.8    1.1      0.6       !   *  Rannametsa
Set RMS of res:     53.8 uGal
   WRMS of res:     32.4 uGal
 ----------------------------
Total(prev sets) RMS of residuals:     53.8 uGal
                WRMS of residuals:     32.4 uGal
------------------------------------------------
 DRIFT degree = 1
 DRIFT of  1 order:     -94.9 +/-      51.1 uGal/day^1 (t-stat=  1.86) !
 BIAS parameter:    -652432.3 +/-   12.6 uGal
   80006  2010-03-17, 07:51:00   36     1     1  5119.7579   2.00     -0.1    1.8    0.3      0.7       *       ReiuGR
10031711  2010-03-17, 08:26:45   36     2     4  5109.7404   0.17     -2.4   -7.3    0.1      0.7      *!       Sunset
10031712  2010-03-17, 08:58:47   36     3     6  5107.1283   0.25     -4.6    0.0    0.0      0.5       *       Uus2
10031713  2010-03-17, 09:32:23   36     4     8  5100.0804   0.25     -6.8  -36.4    0.9      0.7 X     !       Vana1121
10031714  2010-03-17, 10:07:32   36     5    10  5108.5534   0.25     -9.1    0.0    0.5      0.5       *       Vana1116
10031715  2010-03-17, 10:30:43   36     6    12  5110.1878   0.25    -10.6    0.0    0.1      0.5       *       Kalamees
10031604  2010-03-17, 10:46:29   36     7    14  5108.9721   0.25    -11.7    0.0    0.2      0.5       *       Vana1111
10031717  2010-03-17, 11:12:01   36     8    16  5110.7813   0.25    -13.3    0.0    0.6      0.5       *       Vana1109
10031713  2010-03-17, 11:41:46   36     9    18  5099.9991   0.25    -15.3   36.4    0.9      0.7       !     X Vana1121
10031711  2010-03-17, 12:16:05   36    10    20  5109.7095   0.15    -17.6    8.5    0.2      0.8       ! *     Sunset
   80006  2010-03-17, 12:43:11   36    11    22  5119.7376   2.00    -19.3    2.8    0.3      0.9       !*      ReiuGR
10031601  2010-03-17, 13:04:32   36    12    25  5105.3643   2.00    -20.8    1.4    0.1      0.6       *       Võiste
10031701  2010-03-17, 13:30:48   36    13    27  5089.5036   2.00    -22.5  -20.4    1.4      0.7    *  !       Rannametsa
   80006  2010-03-17, 14:03:09   36    14    29  5119.7270   2.00    -24.6    8.1    0.5      0.8       ! *     ReiuGR
Set RMS of res:     18.4 uGal
   WRMS of res:     12.5 uGal
 ----------------------------
Total(prev sets) RMS of residuals:     18.4 uGal
                WRMS of residuals:     12.5 uGal
------------------------------------------------
```

C.3 Tie files with observed and adjusted ties

Example file of observed and sorted ties `*.ties.sort` containing observed gravity differences `dg` between occupied stations, corrected for drift `dD` and residuals `dv`:

./Examples/2010-03-17_GulfofRiga/adj_all/GoF2010.ties.sort

```
    21 ties
    from    /   to       inst.no  date         time      dt(hr) dg(mGal) dD(uGal) dg+dD     dv   dg+dD+dv
Rannametsa   vana-1118      191  2010-03-17, 09:05:30  1.842    -9.5050 101.1   -9.4039-133.8   -9.5377
Rannametsa   vana-1118      191  2010-03-17, 14:04:00  0.675    -9.6950  42.5   -9.6525 114.7   -9.5377
ReiuGR       Rannametsa      36  2010-03-17, 13:30:48  0.539   -30.2234  -2.1  -30.2256 -28.5  -30.2541
ReiuGR       Rannametsa     191  2010-03-17, 07:54:00  1.192   -30.3943 114.0  -30.2803  26.2  -30.2541
ReiuGR       Sunset          36  2010-03-17, 07:51:00  0.596   -10.0175   2.4  -10.0151  -9.0  -10.0241
ReiuGR       Sunset          36  2010-03-17, 12:16:05  0.452   -10.0281  -1.8  -10.0299   5.7  -10.0241
ReiuGR       Võiste          36  2010-03-17, 12:43:11  0.356   -14.3733   1.4  -14.3719  -1.3  -14.3732
ReiuGR       Võiste         191  2010-03-17, 15:20:30  0.383   -14.4592  35.8  -14.4234  50.1  -14.3732
Sunset       Uus2            36  2010-03-17, 08:26:45  0.534    -2.6121   2.1   -2.6100   7.3   -2.6027
Sunset       Vana1121        36  2010-03-17, 11:41:46  0.572    -9.7104  -2.3   -9.7127  27.9   -9.6848
Uus2         Vana1121        36  2010-03-17, 08:58:47  0.560    -7.0479   2.2   -7.0456 -36.4   -7.0821
Võiste       Rannametsa      36  2010-03-17, 13:04:32  0.438   -15.8607   1.7  -15.8590 -21.9  -15.8809
Võiste       Rannametsa     191  2010-03-17, 14:44:30  0.600   -15.9097  48.1  -15.8617 -19.3  -15.8809
Võiste       Rannametsa     191  2010-03-17, 16:08:30  0.533   -15.8622 -62.3  -15.9245  43.6  -15.8809
vana-1118    vana-1112      191  2010-03-17, 10:56:00  1.367    25.2643  16.2   25.2804 114.4   25.3948
vana-1118    vana-1112      191  2010-03-17, 12:18:00  1.767    25.4558  53.4   25.5092-114.4   25.3948
Vana1111     Kalamees        36  2010-03-17, 10:30:43  0.262     1.2156  -1.0    1.2146   0.0    1.2146
Vana1111     Vana1109        36  2010-03-17, 10:46:29  0.426     1.8092   1.7    1.8108   0.0    1.8108
Vana1116     Kalamees        36  2010-03-17, 10:07:32  0.387     1.6344   1.5    1.6359   0.0    1.6359
Vana1121     Vana1109        36  2010-03-17, 11:12:01  0.496    10.7823  -2.0   10.7803 -36.4   10.7439
Vana1121     Vana1116        36  2010-03-17, 09:32:23  0.586     8.4730   2.3    8.4753  36.4    8.5117
```

Example file of adjusted ties `*.ties.adj` containing adjusted gravity differences between all observed stations and their uncertainties (based on the full covariance matrix of adjusted results):

./Examples/2010-03-17_GulfofRiga/adj_all/GoF2010.ties.adj

```
  from     /      to         adj.dg(mGal)+/- std(uGal)
ReiuGR      --> Võiste         -14.3732 +/-   12.1
ReiuGR      --> Vana1111       -10.7759 +/-   35.3
ReiuGR      --> Rannametsa     -30.2541 +/-   11.8
ReiuGR      --> vana-1118      -39.7918 +/-   37.9
ReiuGR      --> vana-1112      -14.3970 +/-   50.4
ReiuGR      --> Sunset         -10.0241 +/-   31.4
ReiuGR      --> Uus2           -12.6269 +/-   35.7
ReiuGR      --> Vana1121       -19.7089 +/-   25.4
ReiuGR      --> Vana1116       -11.1972 +/-   35.4
ReiuGR      --> Kalamees        -9.5614 +/-   35.4
ReiuGR      --> Vana1109        -8.9651 +/-   35.3

Võiste      --> Vana1111         3.5973 +/-   36.7
Võiste      --> Rannametsa     -15.8809 +/-   12.0
Võiste      --> vana-1118      -25.4186 +/-   35.9
Võiste      --> vana-1112       -0.0238 +/-   48.4 !
Võiste      --> Sunset           4.3491 +/-   33.0
Võiste      --> Uus2             1.7464 +/-   37.4
Võiste      --> Vana1121        -5.3357 +/-   27.2
Võiste      --> Vana1116         3.1760 +/-   36.9
Võiste      --> Kalamees         4.8119 +/-   36.7
Võiste      --> Vana1109         5.4081 +/-   36.6

Vana1111    --> Rannametsa     -19.4782 +/-   36.6
Vana1111    --> vana-1118      -29.0159 +/-   51.1
Vana1111    --> vana-1112       -3.6211 +/-   60.8
```

```
Vana1111      --> Sunset           0.7518 +/-   46.4
Vana1111      --> Uus2            -1.8509 +/-   49.4
Vana1111      --> Vana1121        -8.9330 +/-   42.7
Vana1111      --> Vana1116        -0.4213 +/-   49.3
Vana1111      --> Kalamees         1.2146 +/-   49.3
Vana1111      --> Vana1109         1.8108 +/-   49.3

Rannametsa    --> vana-1118       -9.5377 +/-   34.8
Rannametsa    --> vana-1112       15.8571 +/-   47.4
Rannametsa    --> Sunset          20.2300 +/-   33.0
Rannametsa    --> Uus2            17.6273 +/-   37.4
Rannametsa    --> Vana1121        10.5452 +/-   27.2
Rannametsa    --> Vana1116        19.0569 +/-   36.9
Rannametsa    --> Kalamees        20.6928 +/-   36.7
Rannametsa    --> Vana1109        21.2890 +/-   36.5

vana-1118     --> vana-1112       25.3948 +/-   48.1
vana-1118     --> Sunset          29.7677 +/-   48.6
vana-1118     --> Uus2            27.1650 +/-   51.7
vana-1118     --> Vana1121        20.0829 +/-   44.8
vana-1118     --> Vana1116        28.5946 +/-   51.3
vana-1118     --> Kalamees        30.2305 +/-   51.2
vana-1118     --> Vana1109        30.8267 +/-   51.0

vana-1112     --> Sunset           4.3729 +/-   58.8
vana-1112     --> Uus2             1.7702 +/-   61.5
vana-1112     --> Vana1121        -5.3119 +/-   55.7
vana-1112     --> Vana1116         3.1998 +/-   61.0
vana-1112     --> Kalamees         4.8357 +/-   60.9
vana-1112     --> Vana1109         5.4320 +/-   60.7

Sunset        --> Uus2            -2.6027 +/-   46.5
Sunset        --> Vana1121        -9.6848 +/-   39.4
Sunset        --> Vana1116        -1.1731 +/-   46.4
Sunset        --> Kalamees         0.4628 +/-   46.4
Sunset        --> Vana1109         1.0591 +/-   46.5

Uus2          --> Vana1121        -7.0821 +/-   42.8
Uus2          --> Vana1116         1.4296 +/-   49.3
Uus2          --> Kalamees         3.0655 +/-   49.4
Uus2          --> Vana1109         3.6618 +/-   49.5

Vana1121      --> Vana1116         8.5117 +/-   42.7
Vana1121      --> Kalamees        10.1476 +/-   42.7
Vana1121      --> Vana1109        10.7439 +/-   42.7

Vana1116      --> Kalamees         1.6359 +/-   49.3
Vana1116      --> Vana1109         2.2322 +/-   49.3

Kalamees      --> Vana1109         0.5963 +/-   49.3
```

Appendix D

The uncertainty of calibration parameters

As a result of the adjustment the parameters $\Delta\alpha, \Delta\beta$ with variances $\mathbf{D}(\Delta\alpha), \mathbf{D}(\Delta\beta)$ are estimated. Now the variances of functions with random variables

$$\begin{aligned}\Delta A &= \sqrt{\Delta\alpha^2 + \Delta\beta^2},\\ \Delta\varphi &= \arctan\left(\frac{\Delta\alpha}{\Delta\beta}\right)\end{aligned} \tag{D.1}$$

are derived using error propagation.

In general let $Y = \theta\left(X_1, X_2, \ldots, X_n\right)$ be the function of random variables. By approximating the second moment of the function to a first-order Taylor series expansion (wikipedia.org/wiki/Propagation_of_uncertainty) the variance of Y is

$$\mathbf{D}Y \approx \sum_{k=1}^{n}\left(\frac{\partial\theta}{\partial x_k}\right)^2 \mathbf{D}X_k + 2\sum_{k<j}\left(\frac{\partial\theta}{\partial x_k}\right)\left(\frac{\partial\theta}{\partial x_j}\right)\mathrm{Cov}\left(X_k, X_j\right). \tag{D.2}$$

Accordingly, the variance of $\Delta A = \theta_1\left(\Delta\alpha, \Delta\beta\right))$ is found by

$$\begin{aligned}\mathbf{D}(\Delta A) \;\approx\; & \left(\frac{\partial\theta_1}{\partial(\Delta\alpha)}\right)^2\mathbf{D}(\Delta\alpha) + \left(\frac{\partial\theta_1}{\partial(\Delta\beta)}\right)^2\mathbf{D}(\Delta\beta) + \\ & 2\left(\frac{\partial^2\theta_1}{\partial(\Delta\alpha)\partial(\Delta\beta)}\right)\mathrm{Cov}\left(\Delta\alpha, \Delta\beta\right).\end{aligned} \tag{D.3}$$

Analogously the variance of $\Delta\varphi = \theta_2\left(\Delta\alpha, \Delta\beta\right)$ can be derived.

Now partial derivatives from (D.1) are found for $\mathbf{D}(\Delta A)$

$$\begin{aligned}\frac{\partial\left(\Delta A\right)}{\partial(\Delta\alpha)} &= \frac{\Delta\alpha}{\sqrt{\Delta\alpha^2 + \Delta\beta^2}} = \frac{\Delta\alpha}{\Delta A},\\ \frac{\partial\left(\Delta A\right)}{\partial(\Delta\beta)} &= \frac{\Delta\beta}{\sqrt{\Delta\alpha^2 + \Delta\beta^2}} = \frac{\Delta\beta}{\Delta A},\\ \frac{\partial^2\left(\Delta A\right)}{\partial(\Delta\alpha)\partial(\Delta\beta)} &= -\frac{\Delta\alpha\Delta\beta}{\left(\sqrt{\Delta\alpha^2 + \Delta\beta^2}\right)^3} = -\frac{\Delta\alpha\Delta\beta}{\Delta A^3},\end{aligned} \tag{D.4}$$

and for $\mathbf{D}(\Delta\varphi)$

$$\begin{aligned}
\frac{\partial\left(\Delta\varphi\right)}{\partial(\Delta\alpha)} &= \frac{\Delta\beta}{\Delta\alpha^2+\Delta\beta^2} = \frac{\Delta\beta}{\Delta A^2}, \\
\frac{\partial\left(\Delta\varphi\right)}{\partial(\Delta\beta)} &= -\frac{\Delta\alpha}{\Delta\alpha^2+\Delta\beta^2} = -\frac{\Delta\alpha}{\Delta A^2}, \\
\frac{\partial^2\left(\Delta\varphi\right)}{\partial(\Delta\alpha)\partial(\Delta\beta)} &= \frac{\Delta\alpha^2-\Delta\beta^2}{\left(\Delta\alpha^2+\Delta\beta^2\right)^2} = \frac{\Delta\alpha^2-\Delta\beta^2}{\Delta A^4}.
\end{aligned} \tag{D.5}$$

Now the variances of $\Delta A, \Delta\varphi$ are given by

$$\mathbf{D}(\Delta A) = \frac{\left|\mathbf{D}(\Delta\alpha)\Delta\alpha^2+\mathbf{D}(\Delta\beta)\Delta\beta^2+2\Delta\alpha\Delta\beta\mathrm{Cov}\left(\Delta\alpha,\Delta\beta\right)\right|}{\Delta A^2}, \tag{D.6}$$

and

$$\mathbf{D}(\Delta\varphi) = \frac{\left|\mathbf{D}(\Delta\alpha)\Delta\beta^2+\mathbf{D}(\Delta\beta)\Delta\alpha^2-2\Delta\alpha\Delta\beta\mathrm{Cov}\left(\Delta\alpha,\Delta\beta\right)\right|}{\Delta A^4}. \tag{D.7}$$

Appendix E

WZZ2 output files

E.1 WZZ2 output file

/Examples/Haanja_2008-2019_VGG/Haanja_2008-2019_RCR_d2_Wzz21.out

```
File: Haanja_2008-2019_RCR_d2_Wzz21.out
Comp.time: 2019-07-09, 18:19:02.46
------------------------------------------------------
 *** Wzz2.1 (2019-02-16) ***
 Insert the names (without spaces!) of input files (bodyfile, obsfile):
 Input files: body.inp            , Haanja_2008-2019.inp
 Result file: Haanja_2008-2019.wzz
 Detailed result file: Haanja_2008-2019.vgg
 Insert order of polynom (max. 3):
 Inserted order: 2
 The weighted LSQ according uncertainties in obs file (T/F) and a priori st.dev of unit weight:
 The weights included: T , stdev=  1.

 Insert attracting bodies (max no 5) dim and rho:
# R1
 Tyyp( 1)=R
 (R)ectangular inserted
 Dim vec of body (no 1):
 X: -0.6  0.6
 Y: -0.6  0.6
 Z: -0.021  0.979
 rho=  1.
Weight of body:     1.4 kg

# C1
 Tyyp( 2)=C
 (C)ylinder inserted
 Dim vec of body (no 2):
 X:  0.98
 Y:  2.22
 Z:  1.
 rho=  1.
Weight of body:     1.0 kg

# R1
 Tyyp( 3)=R
 (R)ectangular inserted
 Dim vec of body (no 3):
```

```
 X: -0.11  0.11
 Y: -0.11  0.11
 Z: -0.021  0.011
 rho= -1966.
Weight of body:    -3.0 kg

 No of bodies: 3

 Numbers of adjustment process:
 total no. of unknowns:              3
 dim. of symm. covariance matrix:    6
 dim. of extended normal matrix:     9
 No of fixed readings: 1
 No of observations: 11
 No of all obs: 12
 No of unknowns: 3
 dof: 9
 - solution OK, max loss of digits:  2.1

Lower triangular of symmetric covariance matrix (Cx) of adjusted parameters with dim(   3,   3):
         15.481094
         -1.268375          7.825086
          0.868719         -5.640089          4.096797
 ---------------------------------------
 Input data, adjusted results and residuals:
 Id     obs g/dg   weight    h1     h2      adj g/dg      res    wres  res.plot( 0.50 uGal/div)
  1 981678514.00     0.07  0.000  1.200 981678514.00     0.00    0.00       *
  2      -328.60     0.64  0.154  1.289      -326.68     1.92    1.54       !   *
  3      -304.20     1.60  0.142  1.192      -303.22     0.98    1.24       ! *
  4      -304.60     1.88  0.145  1.195      -303.18     1.42    1.95       !  *
  5      -218.90     0.94  0.141  0.889      -218.05     0.85    0.83       ! *
  7      -217.30     0.67  0.143  0.891      -218.03    -0.73   -0.60      *!
  8      -332.00     2.30  0.142  1.293      -331.31     0.69    1.04       !*
 10      -320.30     2.16  0.164  1.278      -320.83    -0.53   -0.79      *!
 11      -176.60     0.74  0.167  0.776      -178.22    -1.62   -1.40    *  !
 12      -319.00     2.52  0.167  1.282      -320.91    -1.91   -3.04   *   !
 13      -187.80     0.51  0.159  0.807      -189.36    -1.56   -1.11    *  !
 14      -312.00     1.42  0.159  1.241      -312.00     0.00    0.01       *
 ---------------------------------------
 Statistics of residuals:
RMS=     1.380
STDEV=   1.594
 ---------------------------------------
g(0) and its stdev= 981678860.14   6.27
b(1),stdev=   -302.44      4.46
b(2),stdev=      9.78      3.23
corr(12)=  -0.99614
 -------------------------------------------------------
   h        g(h)          VG*       Gravity change due to..
  (m)      (uGal)       (uGal/m) ..constVG  ..noncVG  ..attraction of body #
                                         (uGal)          1       2       3
 -------------------------------------------------------
 0.000   981678860.14   -281.14      0.00      0.00    0.00    0.00    0.00
 0.050   981678846.05   -283.72    -15.12      1.03    0.00    0.00    1.01
 0.100   981678831.71   -289.85    -30.24      1.81    0.00    0.00    1.72
 0.150   981678817.10   -293.70    -45.37      2.33    0.00    0.00    2.11
 0.200   981678802.37   -295.27    -60.49      2.72   -0.01    0.00    2.33
 0.250   981678787.60   -295.62    -75.61      3.06   -0.01    0.00    2.46
```

```
 0.300   981678772.82   -295.36    -90.73      3.41   -0.01    0.00    2.54
 0.350   981678758.06   -294.80   -105.86      3.78   -0.01    0.00    2.59
 0.400   981678743.34   -294.07   -120.98      4.18   -0.01    0.00    2.62
 0.450   981678728.66   -293.25   -136.10      4.62   -0.01    0.00    2.65
 0.500   981678714.02   -292.38   -151.22      5.10   -0.01    0.00    2.67
 0.550   981678699.42   -291.47   -166.34      5.62   -0.01    0.00    2.68
 0.600   981678684.87   -290.54   -181.47      6.19   -0.01    0.00    2.69
 0.650   981678670.37   -289.60   -196.59      6.81   -0.01    0.00    2.70
 0.700   981678655.91   -288.65   -211.71      7.48   -0.01    0.00    2.70
 0.750   981678641.50   -287.69   -226.83      8.19   -0.01    0.00    2.71
 0.800   981678627.14   -286.73   -241.95      8.95   -0.01    0.00    2.71
 0.850   981678612.83   -285.77   -257.08      9.76   -0.02    0.00    2.72
 0.900   981678598.56   -284.80   -272.20     10.62   -0.02    0.00    2.72
 0.950   981678584.35   -283.83   -287.32     11.53   -0.02    0.00    2.72
 1.000   981678570.18   -282.86   -302.44     12.48   -0.02    0.00    2.72
 1.050   981678556.06   -281.88   -317.57     13.49   -0.02    0.00    2.73
 1.100   981678541.99   -280.91   -332.69     14.54   -0.02    0.00    2.73
 1.150   981678527.97   -279.94   -347.81     15.64   -0.02    0.00    2.73
 1.200   981678514.00   -278.96   -362.93     16.79   -0.02    0.00    2.73
 1.250   981678500.08   -277.98   -378.05     17.99   -0.02    0.00    2.73
 1.300   981678486.20   -277.01   -393.18     19.24   -0.02    0.00    2.73
 1.350   981678472.38   -276.03   -408.30     20.53   -0.02    0.00    2.73
 1.400   981678458.60   -275.06   -423.42     21.88   -0.02    0.00    2.73
 1.450   981678444.87   -274.08   -438.54     23.27   -0.02    0.00    2.74
 1.500   981678431.19   -273.10   -453.67     24.71   -0.02    0.00    2.74
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* NB! Actual resolution of VG computation: 0.0010 m

 Program Wzz2 OK
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