ASEN 5417 Numerical Solution of ODEs and PDEs

Fall 2015

Homework Set #1

Date: August 30, 2015

Due: September 8, 2015

(a) Using Simpson's integration, write a computer program to show the following equality:

$$\int_0^{2\pi} \frac{dx}{1 + a\cos x} = \frac{2\pi}{\sqrt{1-a^2}}, \quad (\mathrm{a}^2 < 1)$$

Obtain π from

$$\pi = 4\tan^{-1}(1)$$

$$a = 1/2.$$

Use (*i*) 10 equal spaced parts and (*ii*) 100 equal spaced parts. Compare your results for both parts with the right hand side of the equation. Calculate the relative error for each case. We define the relative error as

$$e_r = \frac{|f_e - f_c|}{f_e}$$

where, f_e is the exact value and f_c is the computed value. Repeat the problem by doubling the grid points for problem (a) and use Richardson interpolation to calculate the value of the integral.

(b) Numerically show that

$$(i)\ \int_0^\infty \frac{\sin^4 x}{x^4}dx = \frac{\pi}{3}, \quad \text{(ii)}\ \int_0^\infty \frac{\text{sinx}}{\sqrt{\text{x}}}dx = \sqrt{\frac{\pi}{2}}$$

In each case include the lower boundary and devise a method to eliminate the singularity at x=0. The upper boundary should be placed at a reasonably large finite value. Find what that value will be. Does the periodicity of the integrands impose any conditions in the way the limits will be prescribed for improved accuracy? Do the problem using the Trapezium method and Richardson extrapolation . Compute the relative error in each case. What is the minimum number of intervals you would need to use for a relative error of less that 10^{-4}?

(c) Estimate

$$I = \int_1^3 \int_1^2 f(x, y)dxdy$$

(i) For $f(x,y,) = xy(1+x)$

(ii) For $f(x,y) = x^2y^3(1+x)$

using Simpson's rule. You can do this problem by hand with $h=dx=1/2$, $k=dy=1$. Comment on the accuracy of the solution comparing with the exact solution.