1 Introduction

The equations that govern open channel flow at an inclination of θ degrees with the horizontal can be transformed into a Poisson equation by scaling the streamwise velocity u as

$$U(Y,Z) = \frac{u(Y,Z)}{L^2 \rho g \sin(\theta/\mu)} , \tag{1}$$

where L is the length of the square channel, ρ is the fluid density, g is gravitational acceleration, and μ is the fluid's dynamic viscosity. The cross-sectional dimensions in y and z are also normalized by $Y = y/L$ and $Z = Z/L$. Through these scaling procedures, the governing equations map onto a unit square as

$$U_{,YY} + U_{,ZZ} = 0 , \tag{2}$$

$$\begin{aligned} U(0,Z) &= 0 , \\ U_{,Y}(1,Z) &= 0 , \\ U(Y,0) = U(Y,1) &= 0 . \end{aligned} \tag{3}$$

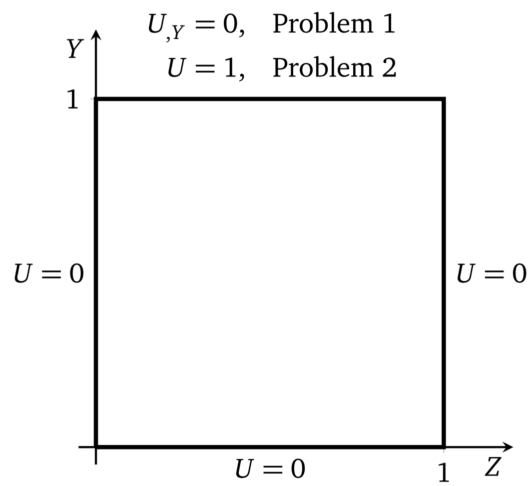

Figure 1: Boundary value problem for Homework 6. No-slip conditions are imposed at three side walls. For Problem 2, the upper boundary is a moving wall.

1.1 Problem 1

Numerically integrate (2) with the stated boundary conditions (3) using the ADI method with $N = 101$ grid points in each direction. Implement LU decomposition to solve the tridiagonal systems, and determine convergence by a reduction of the original error by three orders of magnitude.

Plot the contours of U in the Y-Z plane at convergence.

1.2 Problem 2

Change the upper boundary condition to represent a solid lid moving at a constant velocity with

$$U_{,Y}(1,Z) = 0 \quad \longrightarrow \quad U(1,Z) = 1 . \tag{4}$$

Solve this problem using the SOR method. As in Problem 1, define convergence as a reduction by three orders of magnitude of the initial error.

Obtain the best estimate for the acceleration parameter ω by numerical experimentation. That is, plot the number of iterations required for convergence as a function of ω, and determine the value of ω that minimizes this function. How does this value of ω compare to the theoretical value?

Plot the contours of U in the Y-Z plane at convergence, and compare the results to Problem 1.

2 Methodology

2.1 Problem 1

2.2 Problem 2

3 Results

3.1 Problem 1

3.2 Problem 2

4 Discussion

4.1 Problem 1

4.2 Problem 2

5 References

No external references were used other than the course notes for this assignment.

Appendix: MATLAB Code

The following code listings generate all figures presented in this homework assignment.