

1 INTRODUCTION

Consider the non-linear, inviscid Burgers equation for $u(x, t)$,

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0, \quad (1)$$

with the initial conditions

$$\begin{aligned} u(x, 0) &= 10 & 0 \leq x \leq 30, \\ u(x, 0) &= 0 & 30 \leq x \leq 40. \end{aligned} \quad (2)$$

Use the following finite-difference approximations to numerically integrate this equation using appropriate Dirichlet or Neumann BCs on an x -grid with $\Delta x = 0.2$:

1. MacCormack explicit method
2. Beam and Warming implicit method

Note that the second method may require the incorporation of a smoothing operator added directly to the finite difference formula. Using a fourth-order artificial viscosity, optimize the coefficient of this operator for minimum amplitude errors,

$$D_\epsilon = -\epsilon(\Delta x)^4 \frac{\partial^4 u}{\partial x^4}, \quad (3)$$

where the negative sign ensures that positive dissipation is produced. Using central differences, we obtain

$$\epsilon(\Delta x)^4 \frac{\partial^4 u}{\partial x^4} = u_{i-2} - 4u_{i-1} + 6u_i - 4u_{i+1} + u_{i+2}. \quad (4)$$

The coefficient ϵ generally obeys $0 \leq \epsilon \leq 1/8$, with a preferred value of $\epsilon = 0.1$.

For both methods, plot the solutions at intervals of about two time units up to about $t = 8$ time units. Obtain solutions for Courant numbers of $C = \{\frac{3}{4}, 1, \frac{5}{4}\}$. Comment on the stability of the scheme and dispersive/dissipative errors.

2 METHODOLOGY

3 RESULTS

4 DISCUSSION

5 REFERENCES

No external references were used other than the course notes for this assignment.

APPENDIX: MATLAB CODE

The following code listings generate all figures presented in this homework assignment.

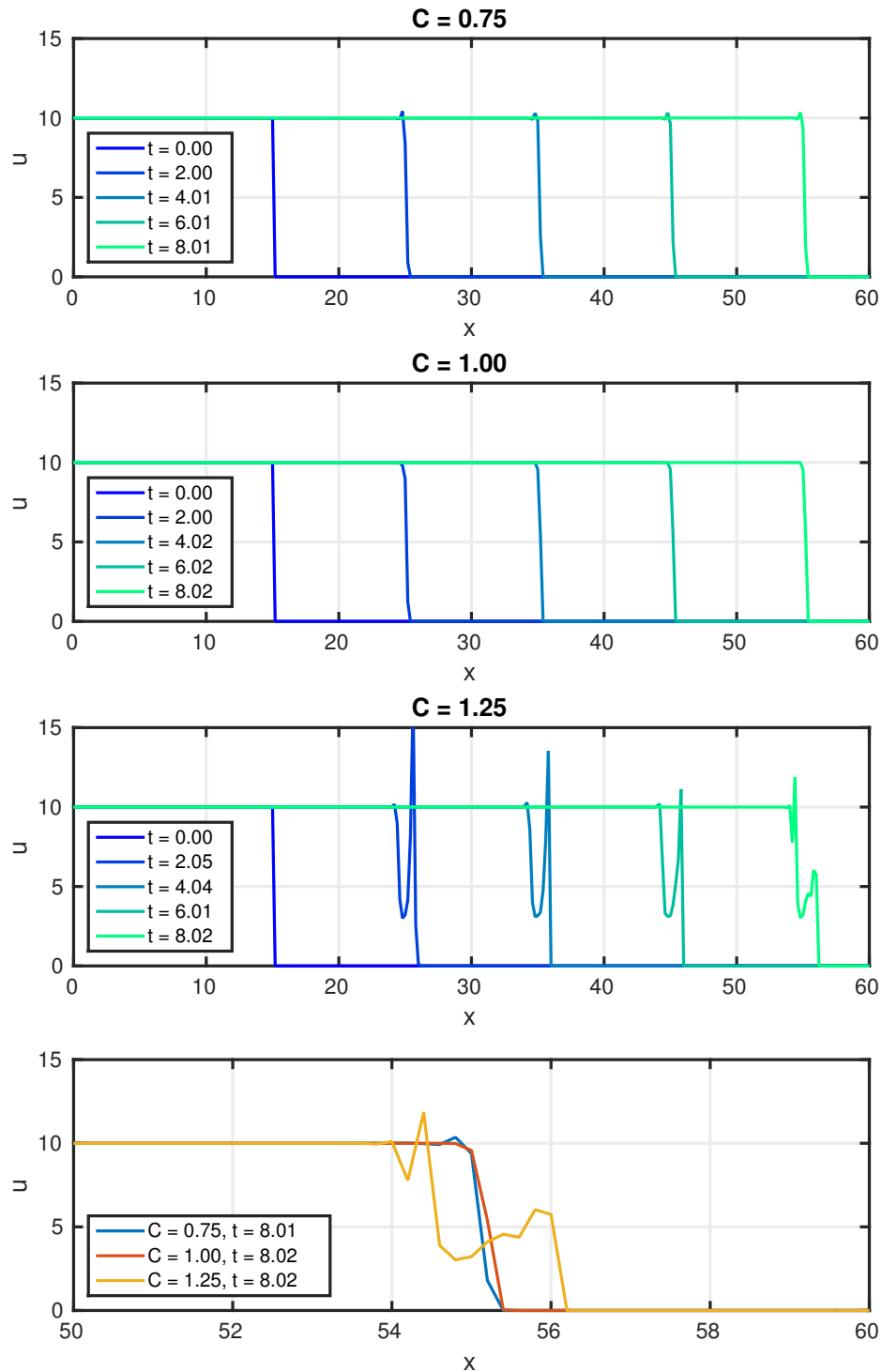


Figure 1: MacCormack method solution for different Courant numbers, and comparison at time $t \sim 8$. The Courant number is seen to substantially affect the accuracy of the results.

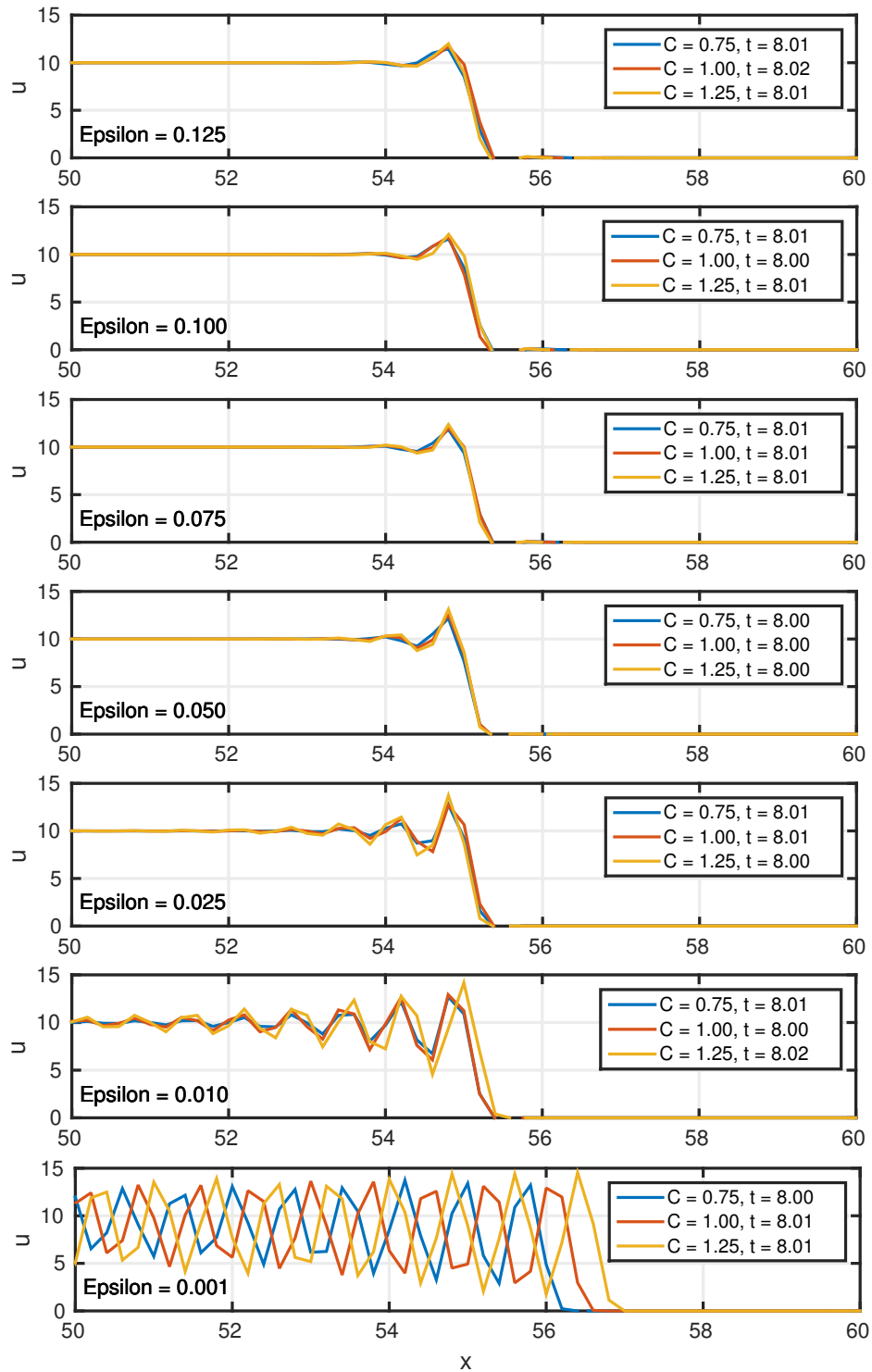


Figure 2: Beam and Warming solution for different values of the dissipation term's coefficient ϵ . Solutions at time $t \sim 8$ are shown for different Courant numbers in each plot. The solution diverged for values of ϵ even a few percent above 0.125. A value of $\epsilon = 0.1$ appears to be sufficient.

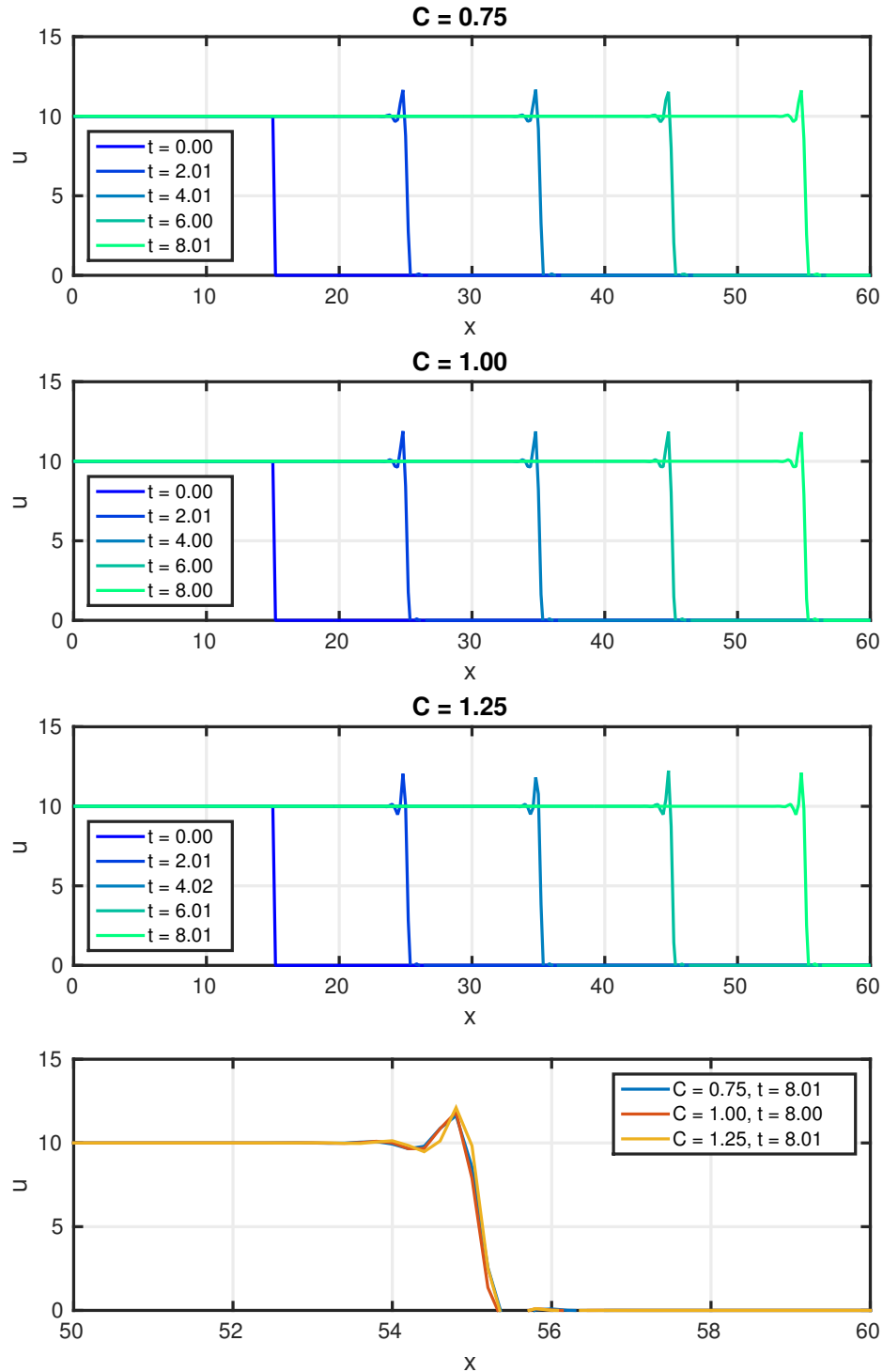


Figure 3: Beam and Warming method solution for three Courant numbers, and comparison at time $t \sim 8$. The dissipation coefficient is $\epsilon = 0.1$. Little difference is observed between different Courant numbers.