1 Introduction

Consider the non-linear, inviscid Burgers equation for $u(x,t)$,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0\,, \tag{1}$$

with the initial conditions

$$\begin{aligned} u(x,0) &= 10 \qquad 0 \le x \le 30\,, \\ u(x,0) &= 0 \qquad 30 \le x \le 40\,. \end{aligned} \tag{2}$$

Use the following finite-difference approximations to numerically integrate this equation using appropriate Dirichlet or Neumann BCs on an x-grid with $\Delta x = 0.2$:

1. MacCormack explicit method
2. Beam and Warming implicit method

Note that the second method may require the incorporation of a smoothing operator added directly to the finite difference formula. Using a fourth-order artificial viscosity, optimize the coefficient of this operator for minimum amplitude errors,

$$D_\epsilon = -\epsilon(\Delta x)^4 \frac{\partial^4 V}{\partial x^4}\,, \tag{3}$$

where the negative sign ensures that positive dissipation is produced. Using central differences, we obtain

$$\epsilon(\Delta x)^4 \frac{\partial^4 V}{\partial x^4} = V_{i-2} - 4V_{i-1} + 6V_i - 4Vi + 1 + Vi + 2\,. \tag{4}$$

The coefficient ϵ generally obeys $0 \le \epsilon \le 1/8$, with a preferred value of $\epsilon = 0.1$.

For both methods, plot the solutions at intervals of about two time units up to about $t = 8$ time units. Obtain solutions for Courant numbers of $C = \{\frac{3}{4}, 1, \frac{5}{4}\}$. Comment on the stability of the scheme and dispersive/dissipative errors.

2 Methodology

3 Results

4 Discussion

5 References

No external references were used other than the course notes for this assignment.

Appendix: MATLAB Code

The following code listings generate all figures presented in this homework assignment.