

1 INTRODUCTION

Consider the linear convection-diffusion equation

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \frac{1}{\text{Pr Re}} \frac{\partial^2 T}{\partial y^2}, \quad v = \sin(\pi y), \quad (1)$$

subject to the initial conditions

$$\begin{aligned} \text{(a)} \quad T(y, t = 0) &= \cos(2\pi y) \sin(\pi y) \\ \text{(b)} \quad T(y, t = 0) &= \cos(2\pi y), \end{aligned} \quad (2)$$

and parameters

$$\begin{aligned} \text{Re} &= 1 && \text{(Reynolds number, molten glass)} \\ \text{Pr} &= 25 && \text{(Prandtl number, molten glass)} \\ \Delta t &= 0.001 && \text{(Time step)} \\ L_y &= 2 && \text{(Domain } y\text{-length)} \\ N &= 2^n + 1 && \text{(Number of } y\text{-points, where } n = 5, 6). \end{aligned} \quad (3)$$

1.1 PROBLEM 1

Use the Fourier pseudo-spectral method to numerically integrate (1) with the given parameters. Use the Euler explicit method for time advancement. Higher resolution with $n = 6$ will improve the accuracy of the method for the initial condition (a). Plot T as a function of time t at $t = \{0.2, 2, 5, 10\}$.

1.2 PROBLEM 2

Use the FTCS Euler explicit method with second-order finite differences for the same computation, and compare results to the Fourier pseudo-spectral method using the same mesh resolution.

2 METHODOLOGY

2.1 PROBLEM 1

We re-write (1) as

$$\frac{\partial T}{\partial t} = \frac{1}{\text{Pr Re}} \frac{\partial^2 T}{\partial y^2} - v \frac{\partial T}{\partial y}, \quad v = \sin(\pi y). \quad (4)$$

The first and second spatial derivatives of T are calculated by taking the Fourier transform of T , multiplying the Fourier coefficients by ik_n and $-k_n^2$, respectively, and then taking the inverse Fourier transform. With values of $\partial T / \partial t$ known at all grid points now, values of T at the next time step are calculated using the explicit Euler method,

$$T^{n+1} = T^n + \frac{\partial T}{\partial t}. \quad (5)$$

3 RESULTS

4 DISCUSSION

5 REFERENCES

No external references were used other than the course notes for this assignment.

APPENDIX: MATLAB CODE

The following code listings generate all figures presented in this homework assignment.