1 Introduction

Consider the linear convection-diffusion equation

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \frac{1}{\mathrm{Pr}\,\mathrm{Re}} \frac{\partial^2 T}{\partial y^2}\,, \qquad v = \sin(\pi y)\,, \tag{1}$$

subject to the initial conditions

$$\begin{aligned} &\text{(a)} \quad T(y, t=0) = \cos(2\pi y)\sin(\pi y) \\ &\text{(b)} \quad T(y, t=0) = \cos(2\pi y)\,, \end{aligned} \tag{2}$$

and parameters

$$\begin{aligned} \mathrm{Re} &= 1 && \text{(Reynolds number, molten glass)} \\ \mathrm{Pr} &= 25 && \text{(Prandtl number, molten glass)} \\ \Delta t &= 0.001 && \text{(Time step)} \\ L_y &= 2 && \text{(Domain } y\text{-length)} \\ N &= 2^n + 1 && \text{(Number of } y\text{-points, where } n = 5, 6)\,. \end{aligned} \tag{3}$$

1.1 Problem 1

Use the Fourier pseudo-spectral method to numerically integrate (1) with the given parameters. Use the Euler explicit method for time advancement. Higher resolution with $n = 6$ will improve the accuracy of the method for the initial condition (a). Plot T as a function of time t at $t = \{0.2, 2, 5, 10\}$.

1.2 Problem 2

Use the FTCS Euler explicit method with second-order finite differences for the same computation, and compare results to the Fourier pseudo-spectral method using the same mesh resolution.

2 Methodology

2.1 Problem 1

We re-write (1) as

$$\frac{\partial T}{\partial t} = \frac{1}{\mathrm{Pr}\,\mathrm{Re}} \frac{\partial^2 T}{\partial y^2} - v \frac{\partial T}{\partial y}\,, \qquad v = \sin(\pi y)\,. \tag{4}$$

The first and second spatial derivatives of T are calculated by taking the Fourier transform of T, multiplying the Fourier coefficients by ik_n and $-k_n^2$, respectively, and then taking the inverse Fourier transform. With values of $\partial T/\partial t$ known at all grid points now, values of T at the next time step are calculated using the explicit Euler method,

$$T^{n+1} = T^n + \Delta t \frac{\partial T}{\partial t}\,. \tag{5}$$

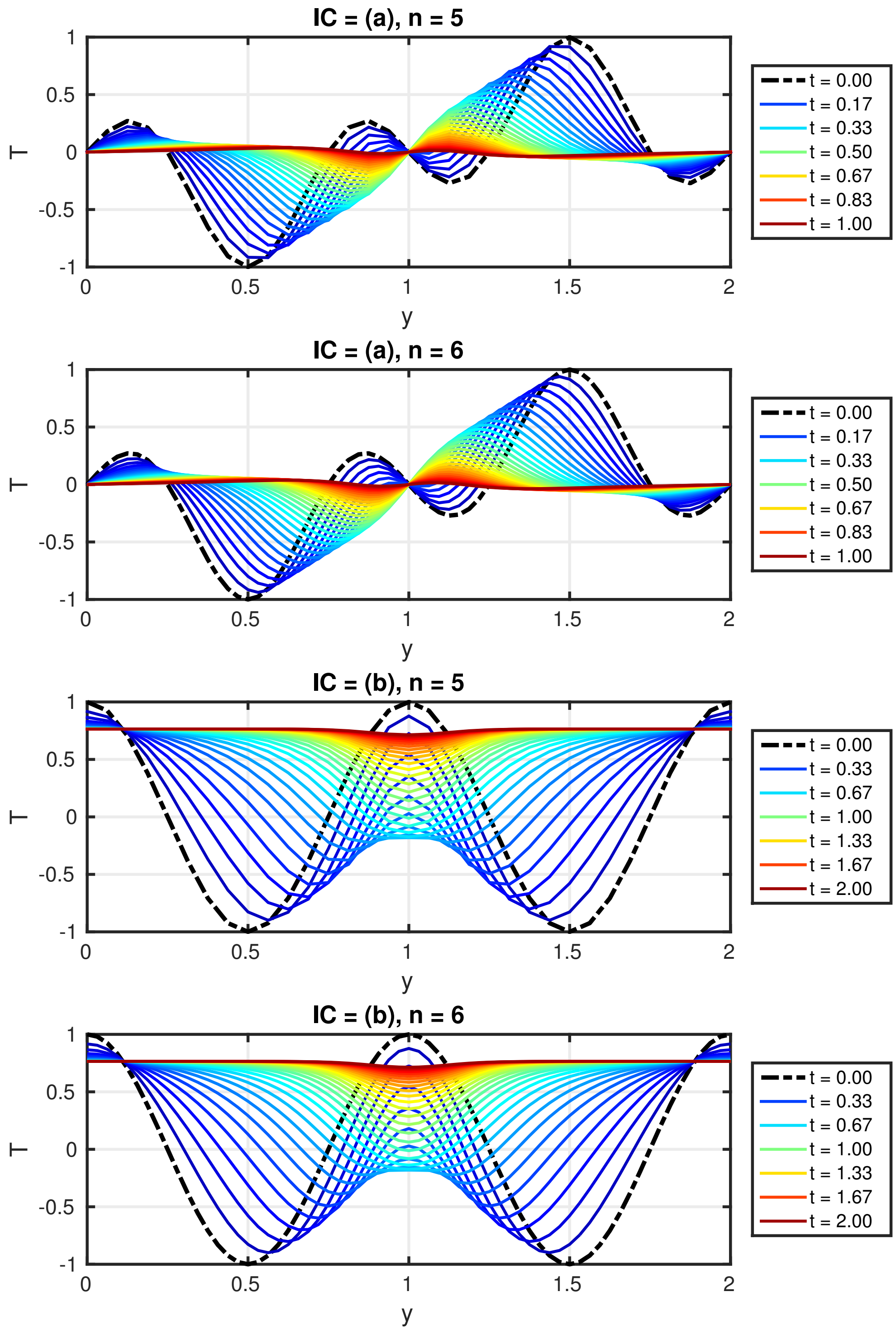

Figure 1: Solutions to the linear convection-diffusion equation using the Fourier pseudo-spectral method for different initial conditions and mesh resolution parameters. Most interesting behavior occurs when (a) $t < 1$ and (b) $t < 2$. Trends can be extrapolated to future times $t = 2, 5, 10$, which are not shown.

3 Results

4 Discussion

5 References

No external references were used other than the course notes for this assignment.

Appendix: MATLAB Code

The following code listings generate all figures presented in this homework assignment.