In this project, we assess the strengths and shortcomings of Reynolds-averaged Navier-Stokes (RANS) models through derivations and comparison to DNS simulations of various flow classes.

Part 1 Derivation of RANS Models

Please see the attached handwritten work; much of this problem consists of derivations which are long enough to preclude typesetting in a reasonable amount of time.

Part 2 Testing of RANS Models: Turbulent Channel Flow

Problem 2.1

Consider a fully-developed turbulent channel flow. In such a flow, which of the components of $\overline{u}_i$, $\overline{S}_{ij}$, and $\overline{W}_{ij}$ are non-zero? What are $\partial/\partial t$ and $\overline{u}_i\partial/\partial x_i$? Comment on the validity of the equilibrium assumption used in Problem 1.8.

We will assume this turbulent channel flow to be either two- or three-dimensional, with the stream-wise direction denoted by x, and the span-wise direction(s) denoted by y.

Only $\overline{u}_x$ is non-zero, since this is the mean direction of flow. Any non-zero $\overline{u}_y$ would violate symmetry and, if near a wall, also the no-penetration boundary condition.

In the core flow away from the walls, all components of $\overline{S}_{ij}$ are zero, since the velocity profile has negligible gradients in all directions ($\partial/\partial x$ is identically zero in a fully-developed flow). In the near-wall region, $\overline{S}_{ij}$ does have non-zero components, because there is a large velocity gradient in the wall-normal direction. In this region, $\overline{S}_{xy}$ is non-zero; it involves the derivative of the stream-wise velocity with respect to the wall-normal direction. All other components are zero because span-wise mean velocities are zero.

In a similar manner, all components of $\overline{W}_{ij}$ are zero in the core flow. Again, this is because there exist no substantial velocity gradients in the core region. In the near-wall region, however, hairpin vortices peel off and create preferential vorticity. These structures create zero mean downstream and wall-normal vorticity due to symmetry of the hairpin, but non-zero vorticity in the direction of (wall-normal $\times$ stream-wise). Taking the channel to be three-dimensional, if we consider the flow direction to be into the page x, along the bottom wall (whose normal is in the y-direction, pointing into the channel) vorticity will be preferentially in the $-\hat{z}$-direction. Thus, near this wall, $\overline{W}_{xy}$ and $\overline{W}_{yx}$ will be non-zero. Similar arguments apply to the other walls.

Since the flow is fully-developed, the averages of flow quantities will have no change in time, making $\partial/\partial t = 0$. However, there will still be fluctuations in the flow for which $\partial/\partial t \neq 0$. For the convective derivative, we already know that spatial derivatives in the stream-wise direction must be zero, so $\overline{u}_x\partial/\partial x = 0$. Additionally, there are no mean velocities in the span-wise directions, so $\overline{u}_y = 0$. Thus all $\overline{u}_i\partial/\partial x_i = 0$.

In conclusion, the equilibrium assumption that postulates constant anisotropy is an acceptable one for average quantities. Though this is not true for fluctuating, instantaneous quantities, we will presumably be using the RANS equations for modelling, in which only average quantities are used.

Problem 2.2

Using the turbulent channel flow DNS data from Moser, Kim, and Mansour (1999), we calculate and plot the non-zero components of $\overline{u}_i$, $\overline{u_i'u_j'}$, k, ϵ, a_{ij}, and $\overline{S}_{ij}$ as functions of both y^+ and y/h. Results are displayed in Figure 1 through Figure 6

Problem 2.3

Using a_{12} and $\overline{S}_{12}$, the mean value of the eddy viscosity coefficient C_μ away from the walls ($y/h > 0.2$) is calculated to be approximately 0.086. This is the constant value that gives the closest agreement between the computational data and the closure model of

$$C_\mu = \frac{-\epsilon a_{12}}{2k\overline{S}_{12}}. \tag{1}$$

A plot of C_μ and its average value away from the wall is presented in Figure 7.

Problem 2.4

Problem 2.5

Problem 2.6

Problem 2.7

Problem 2.8

Part 3 Testing of RANS Models: Unsteady Homogeneous Flow

Problems 3.1–3.4 are derivation-heavy; they are not typeset here, but are included in the attached handwritten documents.

Problem 3.5

Assuming that $a_{12} = 0$ at $t = 0$, we numerically integrate the set of ordinary differential equations found in Problem 3.4 for the SKE and DKE models. We take $S^* \equiv Sk_0/\epsilon_0 = 3.3$, and examine the flow evolution for $(\omega/S) = \{0.01, 0.1, 0.5, 1, 10\}$. The evolution of a_{12} is plotted as a function of $S \cdot t = S^*\tau$ for each of the ω/S values and both models. We furthermore assume that $C_\mu = 0.09$ in the SKE model.

Problem 3.6

Problem 3.7

Part 4 Testing of RANS Models: Computational Fluid Dynamics Code

Problem 4.1

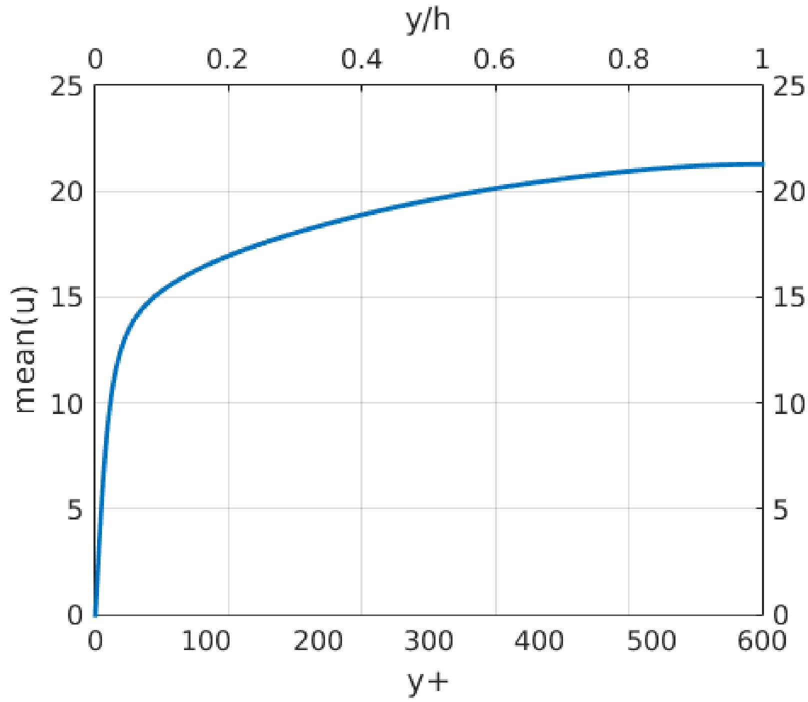

Figure 1: Mean velocity in the stream-wise direction.

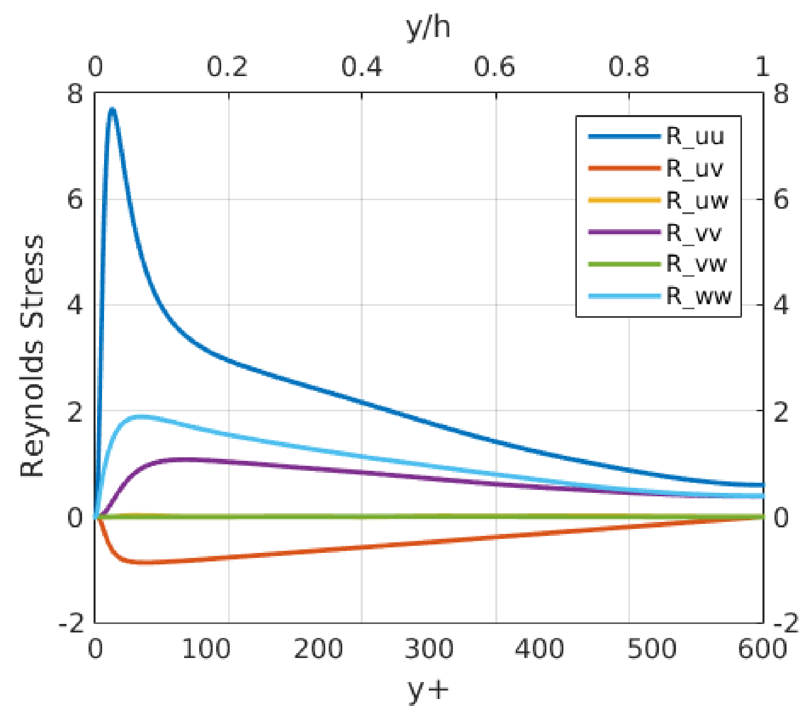

Figure 2: Components of the Reynolds stress tensor.

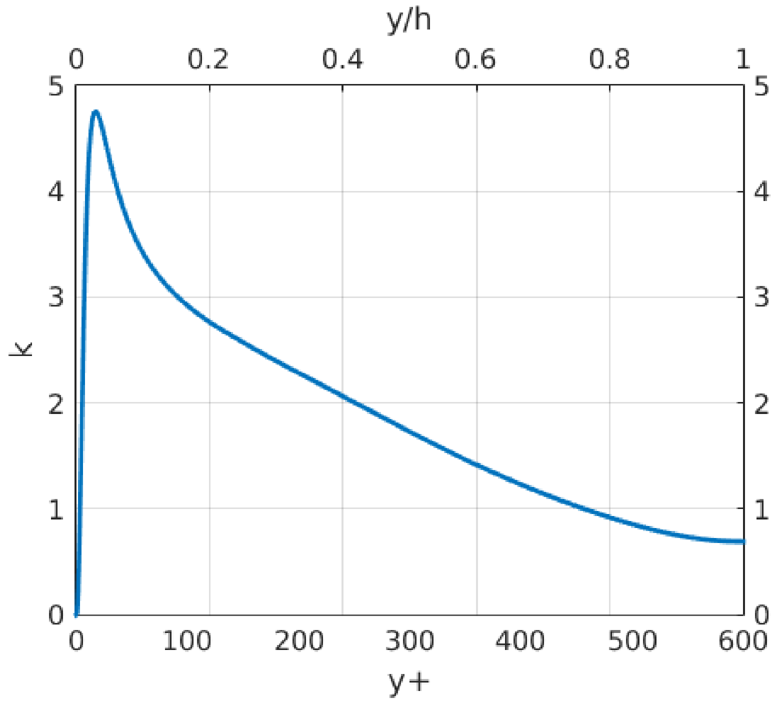

Figure 3: Turbulence intensity.

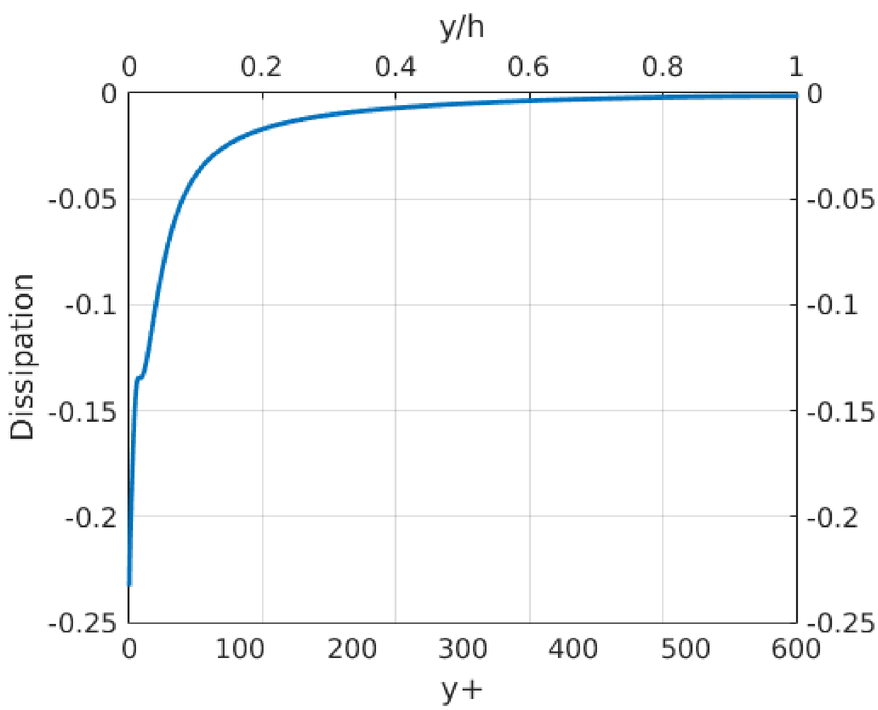

Figure 4: Turbulent kinetic energy dissipation.

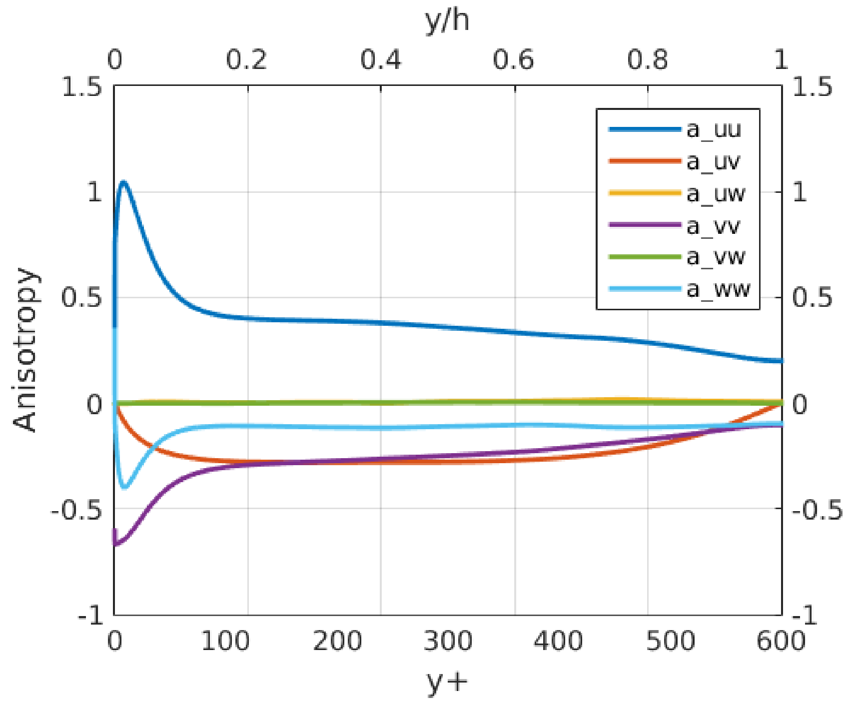

Figure 5: Components of the anisotropy tensor.

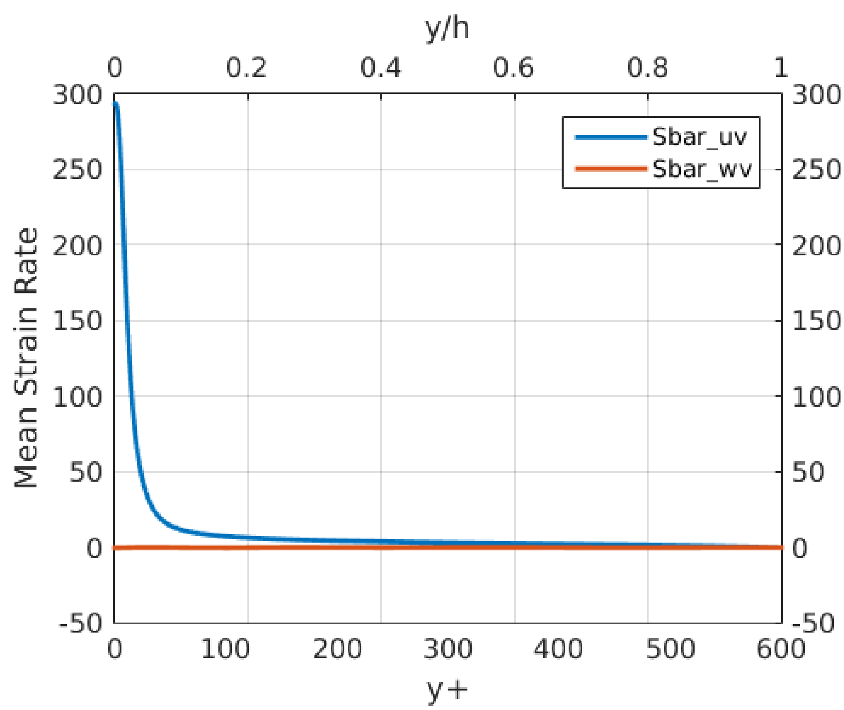

Figure 6: Components of the mean strain rate tensor; all components not shown are zero. Additionally, $\overline{S}_{wv}$ should be zero in an ideal flow because there should be no mean span-wise velocities.

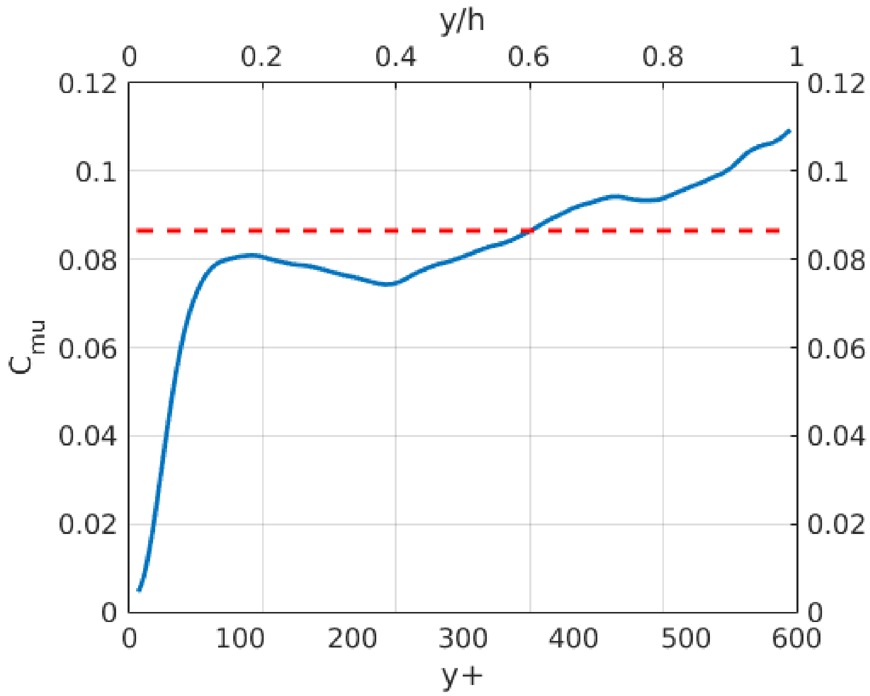

Figure 7: Eddy viscosity coefficient C_μ as a function of distance from wall, calculated using a_{12} and $\overline{S}_{12}$. The average value is 0.086 for $y/h > 0.2$.