Implementation and Validation of the Spalart-Allmaras Curvature Correction in PHASTA

1 Introduction

It is well-known that the presence of rotation and streamline curvature (RC) substantially alters the physics of turbulent shear flows. Bradshaw [1] notes that these changes are "surprisingly large," in that they are usually "an order of magnitude more important than normal pressure gradients and other explicit terms" in the RANS equations for curved flows. This can lead to significant effects on shear stresses and other quantities when the stream-wise radius of curvature is as large as one hundred times the shear layer thickness [1]. For aerodynamicists in particular, RC phenomena have high impact on boundary layer development, turbulent mixing, and heat transfer in applications ranging from flow over high-camber airfoils to rapidly-rotating turbomachinery blades.

For computational studies to effectively guide developments in design areas dominated by RC-effects, it is imperative that the turbulence models employed capture their influence in some way. Reynolds stress transport (RST) models are sometimes viewed as superior to simpler eddy-viscosity models, because RC terms appear explicitly in the Reynolds transport equation. However, this explicit influence is limited to the production term, and there appears to be little consensus on how rotation and curvature influence the other diffusion and destruction terms [5]. Despite the philosophical advantages that may exist, the accuracy of full RST models comes at substantial computational cost. Accordingly, adding an RC-correction term to the latter class of models would be a boon for workflows that require rapid design iteration.

The Spalart-Allmaras (SA) one equation turbulence model captures important features of aerodynamic flows involving complex geometry and adverse pressure gradients well, and is thus one of the most appropriate eddy-viscosity models for such studies [4]. However, the original model neglects the effects of streamline curvature and rotation. To remedy this, Shur et al. [3] develop an RC-correction term that scales eddy viscosity production, giving rise to the SARC model. They validate their correction against experimental and DNS data of a number of canonical wall-bounded turbulent shear flows:

- one-dimensional, fully-developed flow in a plane rotating channel,
- one-dimensional, fully-developed flow in a curved channel,
- two-dimensional flow in a channel with a U-turn, and
- three-dimensional flow in a channel of rectangular cross-section with a 90°streamwise bend.

In all cases, the authors demonstrate substantial improvements of the SARC model over the standard SA model and in most cases the Menter two-equation shear stress transport (M-SST). These conclusions are based on predictions of mean velocity and wall shear stress distributions.

In the present project, the Shur et al. [3] curvature-correction (sans rotation terms) is added to the existing SA model in PHASTA, the Parallel Hierarchic Adaptive Stabilized Transient Analysis CFD code developed and maintained by Prof. Kenneth E. Jansen's group at the University of Colorado at Boulder. Our group focuses on aerodynamic flow control, which in many cases simplifies to "the pursuit of bent streamlines". Of particular interest to the author is the increased accuracy curvature-correction could bring to performance predictions of flow control strategies in an aggressive subsonic diffuser. Other applications include unsteady separation in flows over high-lift wing and tail configurations. Thus, the ability to run curvature-corrected RANS simulations will be a welcome addition to our research capabilities. To test our implementation's correctness, we simulate the 90°-bend case listed above, verify it using both the SA and SARC data of Shur et al. [3], and validate it against the experimental data of Kim and Patel [2].

With the motivation clear, the remainder of this paper discusses the philosophy underpinning the mathematics of the RC-correction; the changes made to PHASTA during implementation; and the validation procedures employed, including geometry construction, meshing, and comparison to the published data mentioned above.

2 Model Equations

2.1 Spalart-Allmaras

As the standard Spalart-Allmaras (SA) one-equation model [4] is our point of departure for the RC-correction, a brief overview is apropos.

The SA model was developed as a middle-ground between algebraic and two-equation models. It sought to address algebraic models' shortcomings in massively-separated flows, while retaining some advantages of two-equation models and forgoing their additional computational complexity. It is tuned specifically for aerodynamic flows, which can exhibit substantial separation and involve complex geometries. Its derivation starts from a blank slate; production, transport, and diffusion terms are constructed from scratch using dimensional and invariance arguments applied to four canonical flows.

Fundamentally, the SA model solves a transport equation for the pseudo-eddy viscosity $\tilde{\nu}$, which is calibrated to behave properly within the log layer, and then scales it to the canonical eddy viscosity ν_T in a manner consistent with the viscous sublayer. PHASTA's current version of the SA model omits the original reference's trip term[1], and chooses the vorticity magnitude as the scalar norm of the deformation tensor. The model, as implemented, can be written in full detail as

$$\frac{\partial \tilde{\nu}}{\partial t} + u_j \frac{\partial \tilde{\nu}}{\partial x_j} = c_{b1}\tilde{\Omega}\tilde{\nu} - c_{w1} f_w \left(\frac{\tilde{\nu}}{d}\right)^2 + \frac{1}{\sigma}\left[\frac{\partial}{\partial x_j}\left((\nu+\tilde{\nu})\frac{\partial \tilde{\nu}}{\partial x_j}\right) + c_{b2}\frac{\partial \tilde{\nu}}{\partial x_j}\frac{\partial \tilde{\nu}}{\partial x_j}\right] , \tag{1}$$

$$\begin{aligned}
&\nu_T = \tilde{\nu} f_{v1} && f_{v1} = \frac{\chi^3}{\chi^3 + c_{v1}^3} && \chi = \frac{\tilde{\nu}}{\nu} \\
&\tilde{\Omega} = \Omega + \frac{\tilde{\nu}}{\kappa^2 d^2} f_{v2} && f_{v2} = 1 - \frac{\chi}{1+\chi f_{v1}} && \Omega = \sqrt{2\omega_{ij}\omega_{ij}} \\
&\omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) && f_w = g\left[\frac{1+c_{w3}^6}{g^6 + c_{w3}^6}\right]^{1/6} && g = r + c_{w2}(r^6 - r) \\
& && && r = \min\left[\frac{\tilde{\nu}}{\tilde{S}\kappa^2 d^2}, 10\right] ,
\end{aligned} \tag{2}$$

$$\begin{aligned}
&c_{b1} = 0.1355 && c_{b2} = 0.622 && \sigma = 2/3 && \kappa = 0.41 \\
&c_{v1} = 7.1 && c_{w2} = 0.3 && c_{w3} = 2 && c_{w1} = \frac{c_{b1}}{\kappa^2} + \frac{1+c_{b2}}{\sigma} .
\end{aligned} \tag{3}$$

Because this model has been extensively summarized in prior work for this course, further exposition will be left to Spalart and Allmaras [4], and we will proceed with a discussion of the curvature correction.

[1]Technically, PHASTA's version of the SA model corresponds to SA-noft2 on the NASA Turbulence Modelling Resource website.

2.2 SPALART-SHUR CURVATURE-CORRECTION

The key concepts underpinning the RC-correction were proposed by Spalart and Shur [5], and are understandably similar to those used in developing the SA model due to Spalart's hand in both formulations. Our examination of these concepts is paraphrased and condensed in large part from the aforementioned source. As a precursor, we note that descriptions of the 'strength' of rotation and curvature are made relative to the shear rate and inverse shear-layer thickness, respectively.

To develop or critique any RC-correction, it is of paramount importance that one understand the mechanisms by which rotation and curvature influence turbulent flows. Spalart and Shur [5] consider to two extreme cases for their model: (a) thin shear flows with weak rotation or weak curvature, and (b) homogeneous rotating shear flows, in which strong rotation eliminates the anisotropic Reynolds stresses ($\tau'_{ij} = -\overline{u'_i u'_j} \rightarrow 0$ for $i \neq j$). Intuitive reasoning based on the former suggests a non-dimensional measure of RC-effects, which the authors determine is adequate to employed in the latter case.

For analysis, we assume a statistically-stationary two-dimensional shear flow with mean velocity $\overline{u}$ in the x direction and positive $\partial\overline{u}/\partial y$. We seek to understand the behavior of the Reynolds shear stress $-\overline{u'v'} > 0$, in order to clarify its effects on the eddy-viscosity-based SA model. To correct the Reynolds stress transport equation (RSTE) for reference frame rotation, one must add the term

$$2\Omega_k\left(\overline{u'_j u'_m}\epsilon_{ikm} + \overline{u'_i u'_m}\epsilon_{jkm}\right) \quad \rightarrow \quad 2\Omega\left(\overline{u'^2} - \overline{v'^2}\right), \tag{4}$$

where Ω is the rotation rate about the z-axis, and the simplification applies to our two-dimensional shear flow. Since the stream-wise fluctuations are greater than their span-wise counterparts, the Coriolis term increases production of shear stress when $\Omega > 0$, and vice versa. Curvature produces a similar term in the RSTE. We conclude that the means by which Reynolds stresses (and through the Boussinesq hypothesis, eddy viscosity ν_T) are enhanced or diminished in RC-present flows are indeed subtle. Furthermore, to maintain the original SA model's strengths, any indicator of rotation and curvature must be invariant under Galilean transformations.

Since $\overline{u'^2} > \overline{v'^2}$ is the defining characteristic of our turbulent shear flow, it is natural to question what it implies for Galilean invariants.

Developed by Shur et al. [3], the method of accounting for streamline curvature in the standard SA model is relatively simple. In our PHASTA implementation, we assume a stationary reference frame. Thus $\Omega'_m = 0$ in the referenced paper, and Coriolos terms vanish. With this assumption, the only modification required to the standard SA-noft2 model is to multiply the production term $c_{b1}\tilde{S}\tilde{\nu}$ in (1) by a rotation function f_{r1}, where

$$f_{r1} = (1 + c_{r1})\frac{2r^*}{1 + r^*}\left[1 - c_{r3}\arctan(c_{r2}\tilde{r})\right] - c_{r1} \tag{5}$$

$$\begin{aligned} r^* &= S/\Omega & \tilde{r} &= \frac{2\omega_{ik}S_{jk}}{D^4}\left(\frac{DS_{ij}}{Dt}\right) & S_{ij} &= \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \\ S^2 &= 2S_{ij}S_{ij} & \Omega^2 &= 2\omega_{ij}\omega_{ij} & D^2 &= (S^2 + \Omega^2)/2 \end{aligned} \tag{6}$$

$$c_{r1} = 1.0 \qquad c_{r2} = 12 \qquad c_{r3} = 1.0\,. \tag{7}$$

3 PHASTA Implementation

The SA-noft2 model has already been implemented in PHASTA, so we must only calculate and pre-multiply the production term by f_{r1}. The material derivative of the strain tensor, DS_{ij}/Dt, in (6) makes this process slightly more complicated than initially expected. Here, we describe the process used to compute f_{r1} conceptually and in code.

3.1 Mathematics

We can write the proper definition of DS_{ij}/Dt as

$$\frac{DS_{ij}}{Dt} \equiv \left(\frac{\partial}{\partial t} + u_k \frac{\partial}{\partial x_k}\right) S_{ij} = \left(\frac{\partial}{\partial t} + u_k \frac{\partial}{\partial x_k}\right) \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) , \tag{8}$$

from which it is clear that we need to compute two kinds of terms within the code: the temporal derivatives and spatial gradients of the velocity gradient tensor $\partial u_i/\partial x_j$.

For the time term, we can interchange the order of differentiation due to the continuum assumption,

$$\frac{\partial}{\partial t}\frac{\partial u_i}{\partial x_j} = \frac{\partial}{\partial x_j}\frac{\partial u_i}{\partial t} = \frac{\partial}{\partial x_j} a_i , \tag{9}$$

where a_i is the fluid acceleration, which is already computed during each flow solve. The spatial derivatives of a_i can then be computed using shape function gradients in the manner standard to finite element analysis. That is, (math here from FEM)......

4 Model Validation

Appendix A: PHASTA Code

All changes discussed here pertain to the incompressible code, including files common to both when appropriate.

In `input_fform.f`, the entry `RANS-SARC` is added to represent this turbulence model in `input.config`.

References

[1] P. Bradshaw. Effects of streamline curvature on turbulent flow. Technical report, NATO AGARD, 1973.

[2] W. J. Kim and V. C. Patel. Origin and decay of longitudinal vortices in developing flow in a curved rectangular duct. *Journal of Fluids Engineering*, 116(1), 1994.

[3] M. L. Shur, M. K. Strelets, A. K. Travin, and P. R. Spalart. Turbulence modeling in rotating and curved channels: Assessing the Spalart-Shur correction. *AIAA Journal*, 38(5), 2000.

[4] P. R. Spalart and S. R. Allmaras. A one-equation turbulence model for aerodynamic flows. *AIAA 30th Aerospace Sciences Meeting*, 1992.

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