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Implementation and Validation of the Spalart-Allmaras Curvature Correction in PHASTA

1 Introduction

It is well-known that the presence of rotation and streamline curvature (RC) substantially alter the physics of turbulent shear flows. Bradshaw [1] notes that these changes are "surprisingly large," in that they are usually "an order of magnitude more important than normal pressure gradients and other explicit terms" in the RANS equations for curved flows. This leads to significant effects on shear stresses and other quantities when the stream-wise radius of curvature is as small as one-hundredth of the shear layer thickness [1]. For aerodynamicists in particular, RC phenomena have high impact on boundary layer development, turbulent mixing, and heat transfer in applications ranging from flow over high-camber airfoils to rapidly-rotating turbomachinery blades.

For computational studies to effectively guide developments in design areas dominated by RC-effects, it is imperative that the turbulence models employed capture these effects in some way. Reynolds stress transport (RST) models commonly held superior to simpler eddy-viscosity models, because RC-terms appear explicitly in the Reynolds transport equation. Despite their accuracy, full RST models are much more costly, and adding an RC-correction term to the latter class of models would be a boon for workflows that require rapid design iteration.

The Spalart-Allmaras (SA) one equation turbulence model captures important features of aerodynamic flows involving complex geometry and adverse pressure gradients well, and is thus one of the most appropriate eddy-viscosity models for such studies [4]. However, the original model neglects the effects of streamline curvature and rotation. To better capture these effects, Shur et al. [3] introduce an RC-correction, resulting in the SARC model. They validate their correction against experimental and DNS data of a number of canonical wall-bounded turbulent shear flows:

- one-dimensional, fully-developed flow in a plane rotating channel,
- one-dimensional, fully-developed flow in a curved channel,
- two-dimensional flow in a channel with a U-turn, and
- three-dimensional flow in a channel of rectangular cross-section with a 90°streamwise bend.

In all cases, the authors demonstrate substantial improvements of the SARC model over the standard SA model and in most cases the Menter two-equation shear stress transport (M-SST). These conclusions are based on predictions of mean velocity and wall shear stress distributions.

In the present project, the Shur et al. [3] RC-correction is added to the existing SA model in PHASTA, the Parallel Hierarchic Adaptive Transient Analysis CFD code developed and maintained by Prof. Kenneth E. Jansen's group at the University of Colorado at Boulder. Our group focuses on aerodynamic flow control, which in many cases simplifies to "the pursuit of bent streamlines". Thus, the ability to run RC-corrected RANS simulations will be a welcome addition to our research capabilities. To test the implementation's correctness, we simulate the 90°-bend case listed above, verify it using both the SA and SARC data of Shur et al. [3], and validate it against the experimental data of Kim and Patel [2].

With the motivation clear, the remainder of this paper discusses the philosophy underpinning the mathematics of the RC-correction; the changes made to PHASTA during implementation; and the validation procedures employed, including geometry construction, meshing, and comparison to the published data mentioned above.

2 Model Equations

2.1 Spalart-Allmaras

As the standard Spalart-Allmaras (SA) one-equation model [4] is our point of departure for the RC-correction, a brief overview is apropos.

The SA model was developed as a middle-ground between algebraic and two-equation models. It sought to address algebraic models' shortcomings in massively-separated flows, while retaining some advantages of two-equation models and forgoing their additional computational complexity. It is tuned specifically for aerodynamic flows, which can exhibit substantial separation and involve complex geometries. Its derivation starts from a blank slate; production, transport, and diffusion terms are constructed from scratch using dimensional and invariance arguments applied to four canonical flows.

Fundamentally, the SA model solves a transport equation for the pseudo-eddy viscosity $\tilde{\nu}$, which is calibrated to behave properly within the log layer, and then scales it to the canonical eddy viscosity ν_T in a manner consistent with the viscous sublayer. PHASTA's current version of the SA model omits the original reference's trip term[1], and chooses the vorticity magnitude as the scalar norm of the deformation tensor. The model, as implemented, can be written in full detail as

$$\frac{\partial\tilde{\nu}}{\partial t} + u_j\frac{\partial\tilde{\nu}}{\partial x_j} = c_{b1}\tilde{\Omega}\tilde{\nu} - c_{w1}f_w\left(\frac{\tilde{\nu}}{d}\right)^2 + \frac{1}{\sigma}\left[\frac{\partial}{\partial x_j}\left((\nu+\tilde{\nu})\frac{\partial\tilde{\nu}}{\partial x_j}\right) + c_{b2}\frac{\partial\tilde{\nu}}{\partial x_j}\frac{\partial\tilde{\nu}}{\partial x_j}\right] , \tag{1}$$

$$\begin{aligned}
&\nu_T = \tilde{\nu}f_{v1} , && f_{v1} = \frac{\chi^3}{\chi^3 + c_{v1}^3} , && \chi = \frac{\tilde{\nu}}{\nu} , \\
&\tilde{\Omega} = \Omega + \frac{\tilde{\nu}}{\kappa^2 d^2}f_{v2} , && f_{v2} = 1 - \frac{\chi}{1+\chi f_{v1}} , && \Omega = \sqrt{2\omega_{ij}\omega_{ij}} , \\
&\omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) , && f_w = g\left[\frac{1+c_{w3}^6}{g^6 + c_{w3}^6}\right]^{1/6} , && g = r + c_{w2}(r^6 - r) , \\
& && && r = \min\left[\frac{\tilde{\nu}}{\tilde{S}\kappa^2 d^2}, 10\right] ,
\end{aligned} \tag{2}$$

$$\begin{aligned}
&c_{b1} = 0.1355 , && c_{b2} = 0.622 , && \sigma = 2/3 , && \kappa = 0.41 , \\
&c_{v1} = 7.1 , && c_{w2} = 0.3 , && c_{w3} = 2 , && c_{w1} = \frac{c_{b1}}{\kappa^2} + \frac{1+c_{b2}}{\sigma} .
\end{aligned} \tag{3}$$

2.2 Shur Curvature-Correction

Developed by Shur et al. [3], the method of accounting for streamline curvature in the standard SA model is relatively simple. In our PHASTA implementation, we assume a stationary reference frame. Thus $\Omega'_m = 0$ in the referenced paper, and Coriolos terms vanish. With this assumption, the only modification required

[1] Technically, PHASTA's version of the SA model corresponds to SA-noft2 on the NASA Turbulence Modelling Resource website.

to the standard SA-noft2 model is to multiply the production term $c_{b1}\tilde{S}\tilde{\nu}$ in (1) by a rotation function f_{r1}, where

$$f_{r1} = (1 + c_{r1})\frac{2r^*}{1 + r^*}\left[1 - c_{r3}\arctan(c_{r2}\tilde{r})\right] - c_{r1}\ , \tag{4}$$

$$\begin{aligned} r^* &= S/\Omega\ , & \tilde{r} &= \frac{2\omega_{ik}S_{jk}}{D^4}\left(\frac{DS_{ij}}{Dt}\right)\ , & S_{ij} &= \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\ , \\ S^2 &= 2S_{ij}S_{ij}\ , & \Omega^2 &= 2\omega_{ij}\omega_{ij}\ , & D^2 &= (S^2 + \Omega^2)/2 \end{aligned} \tag{5}$$

$$c_{r1} = 1.0\ , \quad c_{r2} = 12\ , \quad c_{r3} = 1.0\ . \tag{6}$$

3 PHASTA Implementation

The SA-noft2 model has already been implemented in PHASTA, so we must only calculate and pre-multiply the production term by f_{r1}. The material derivative of the strain tensor, DS_{ij}/Dt, in (5) makes this process slightly more complicated than initially expected. Here, we describe the process used to compute f_{r1} conceptually and in code.

3.1 Mathematics

We can write the proper definition of DS_{ij}/Dt as

$$\frac{DS_{ij}}{Dt} \equiv \left(\frac{\partial}{\partial t} + u_k\frac{\partial}{\partial x_k}\right)S_{ij} = \left(\frac{\partial}{\partial t} + u_k\frac{\partial}{\partial x_k}\right)\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\ , \tag{7}$$

from which it is clear that we need to compute two kinds of terms within the code: the temporal derivatives and spatial gradients of the velocity gradient tensor $\partial u_i/\partial x_j$.

For the time term, we can interchange the order of differentiation due to the continuum assumption,

$$\frac{\partial}{\partial t}\frac{\partial u_i}{\partial x_j} = \frac{\partial}{\partial x_j}\frac{\partial u_i}{\partial t} = \frac{\partial}{\partial x_j}a_i\ , \tag{8}$$

where a_i is the fluid acceleration, which is already computed during each flow solve. The spatial derivatives of a_i can then be computed using shape function gradients in the manner standard to finite element analysis. That is, (math here from FEM)......

4 Model Validation

Appendix A: PHASTA Code

All changes discussed here pertain to the incompressible code, including files common to both when appropriate.

In `input_fform.f`, the entry `RANS-SARC` is added to represent this turbulence model in `input.config`.

References

[1] P. Bradshaw. Effects of streamline curvature on turbulent flow. Technical report, NATO AGARD, 1973.

[2] W. J. Kim and V. C. Patel. Origin and decay of longitudinal vortices in developing flow in a curved rectangular duct. *Journal of Fluids Engineering*, 116(1), 1994.

[3] M. L. Shur, M. K. Strelets, A. K. Travin, and P. R. Spalart. Turbulence modeling in rotating and curved channels: Assessing the Spalart-Shur correction. *AIAA Journal*, 38(5), 2000.

[4] P. R. Spalart and S. R. Allmaras. A one-equation turbulence model for aerodynamic flows. *AIAA 30th Aerospace Sciences Meeting*, 1992.