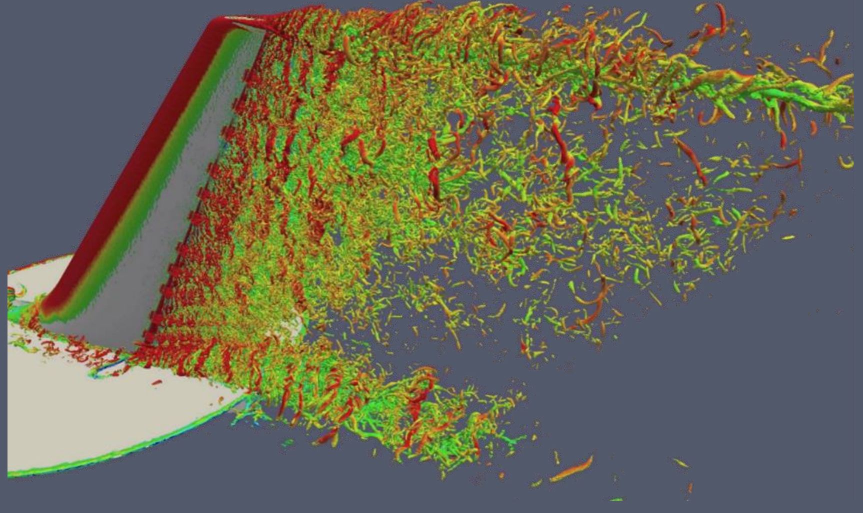
Bi-Fidelity Modeling of Geometric Impact on NACA Airfoil Performance

Ryan W. Skinner[1]

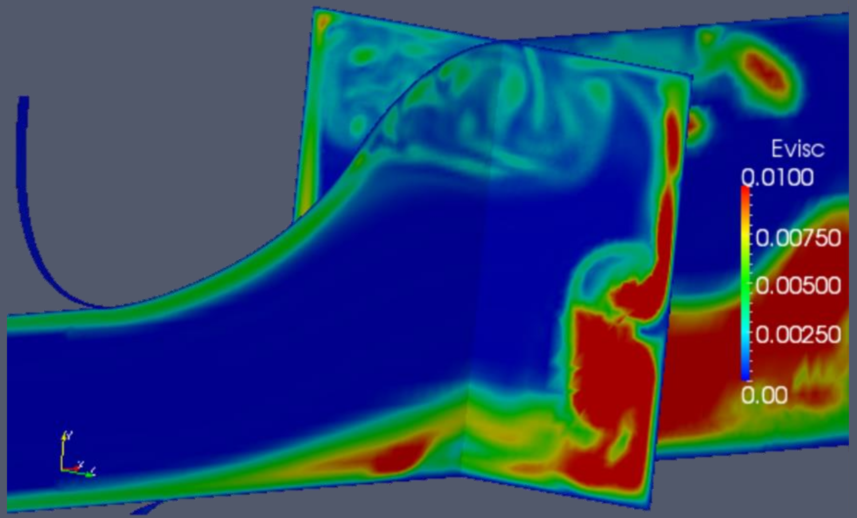
[1]University of Colorado, Boulder CO

Motivation

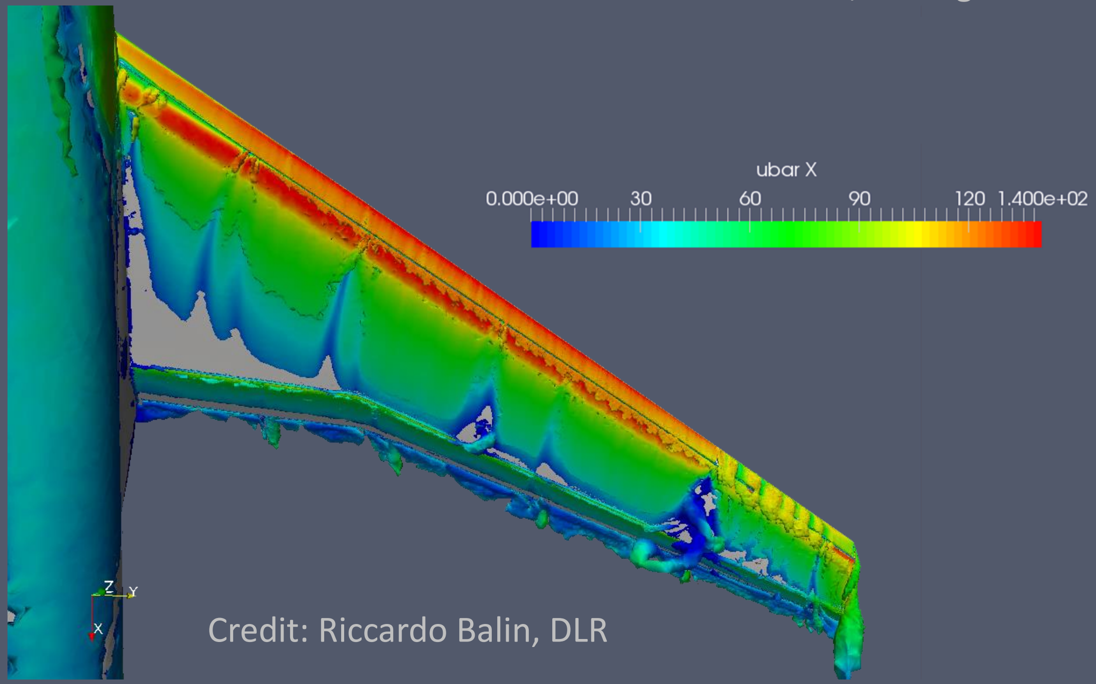
- Simulation of unsteady turbulent flows
- Prohibitively expensive for
  - Design optimization (DO)
  - Non-intrusive UQ
- Can we speed up <u>accurate</u> DO/UQ?



Credit: Prof. Kenneth E. Jansen, Boeing



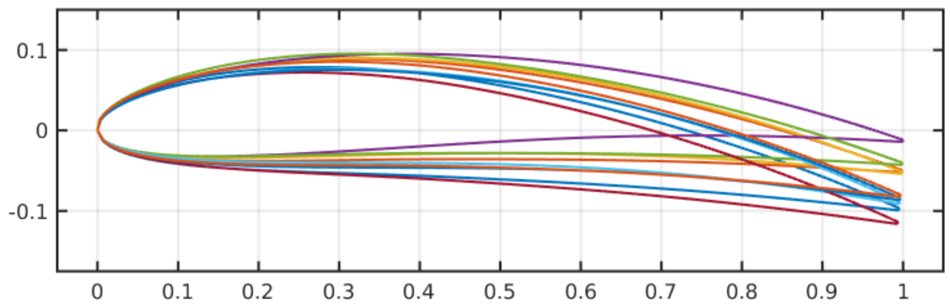
Credit: Ryan W. Skinner, Northrop Grumman



Credit: Riccardo Balin, DLR

Test Problem

- Before addressing bi-fidelity modeling, helpful to consider an example
- NACA Airfoil
  - Camber (max *m*, location *p*)
  - Thickness (max *t*)
  - Chord (*c*)
  - Angle of attack (α)



Create random geometries

$$m \sim U[0.0, 0.1]$$
$$p \sim U[0.3, 0.6]$$
$$t \sim U[0.05, 0.2]$$
$$c = 1.0$$
$$\alpha \sim U[0, 7]$$

These are the random inputs to our system; form the $\mathbf{y}^{(i)}$'s

Test Problem

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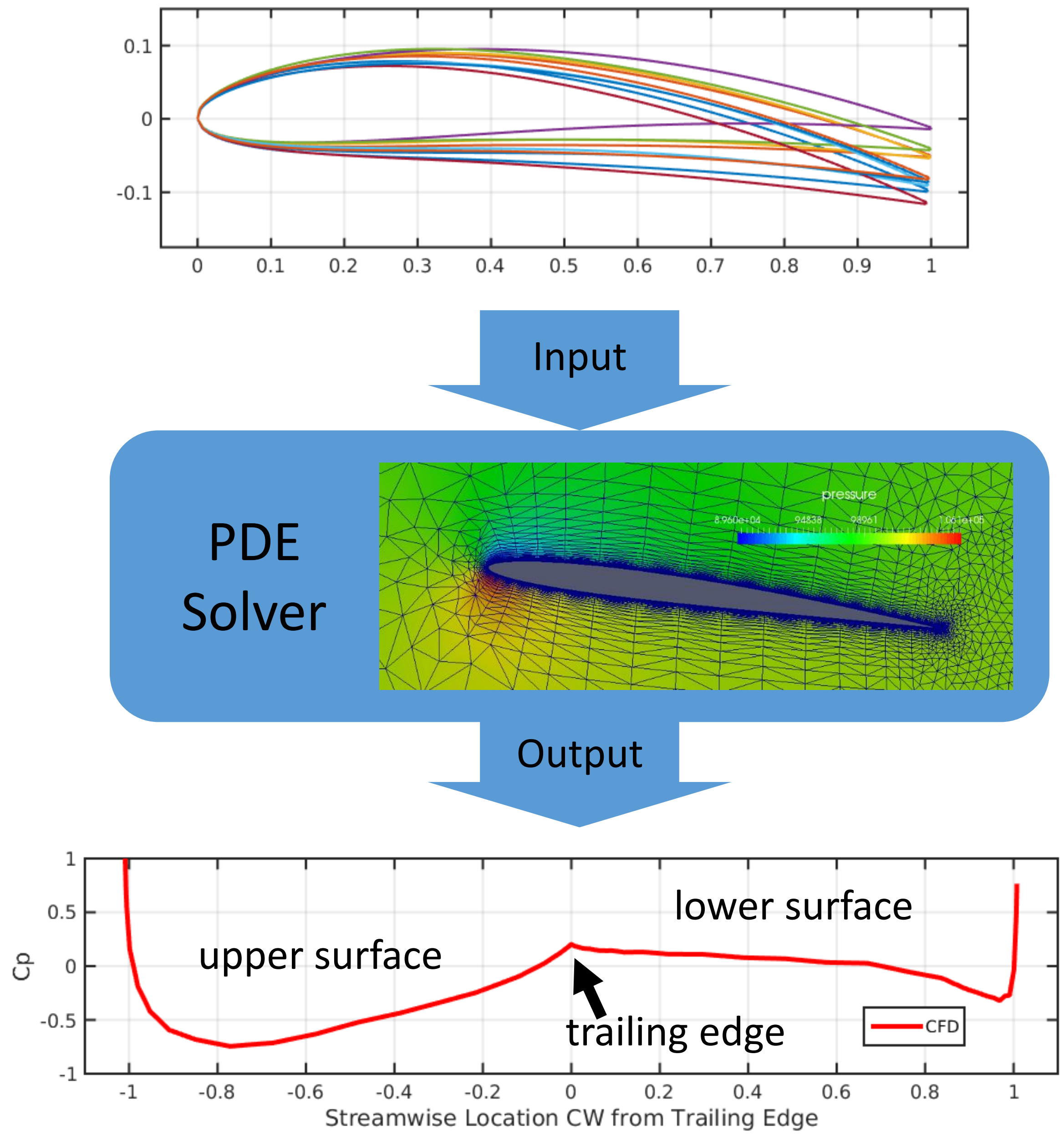
- NACA Airfoil
  - Camber (max m, location p)
  - Thickness (max t)
  - Chord (c)
  - Angle of attack (α)
- Plot coefficient of pressure around the surface



Bi-Fidelity Modeling

- Can reduce cost of mapping design space to performance objective
  - (for low-rank systems)

- Low-Fidelity (LF)
  - Run lots of simulations
  - Cheap but not accurate

- High-Fidelity (HF)
  - Run a select few
  - Expensive but accurate

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Combine to accelerate accurate DO/UQ

(complex, unsteady aerodynamic systems)

Both models:

- *Re* = *3* million, incompressible
- Spalart-Allmaras RANS
- Time step 1e-2 sec
- 1000 runs of each for the <u>same</u> random geometries

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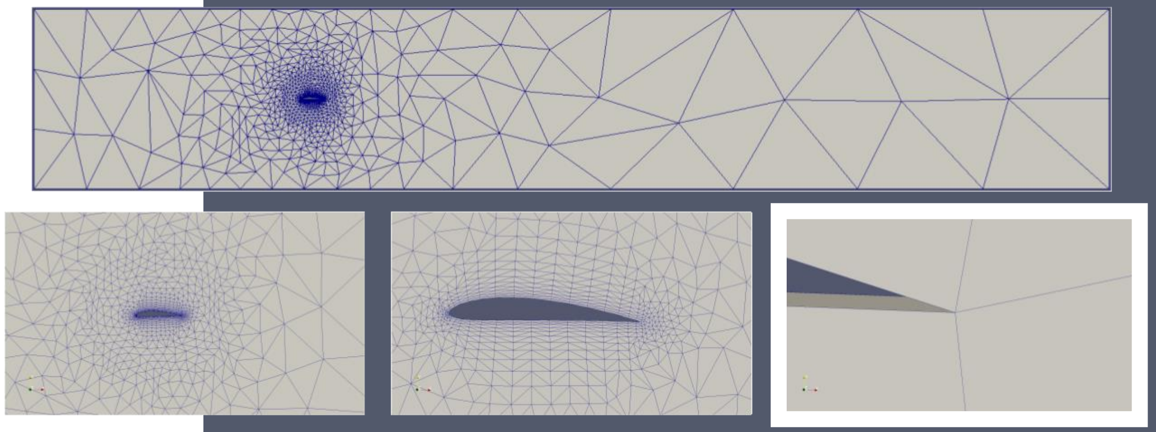
LF

1 run = 0.9 seconds

Coarse mesh

Boundary layers <u>not</u> resolved

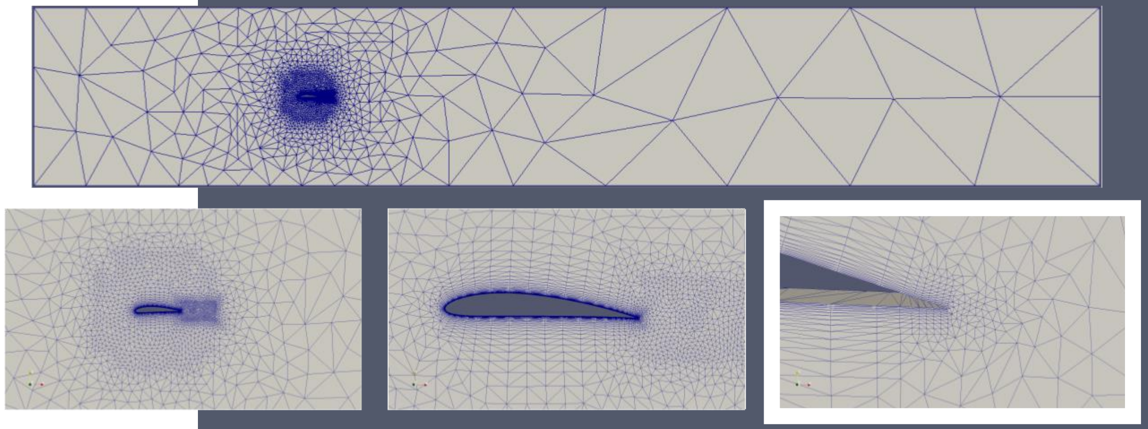


HF

1 run = 90 seconds

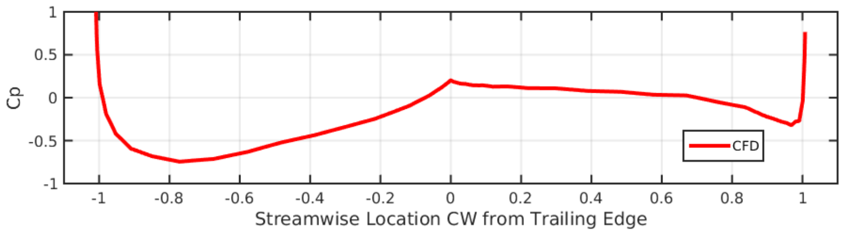
Fine mesh

Boundary layers <u>are</u> resolved



Bi-Fidelity Modeling

one realization



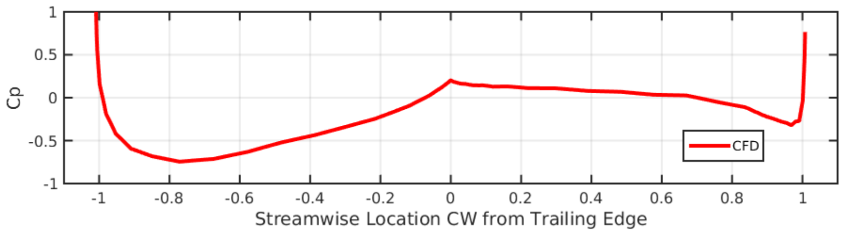
1) Build low-fidelity realization matrix $\boldsymbol{U}^L \equiv \begin{bmatrix} | & | & & | \\ \boldsymbol{u}^L(\boldsymbol{y}^{(1)}) & \boldsymbol{u}^L(\boldsymbol{y}^{(2)}) & \cdots & \boldsymbol{u}^L(\boldsymbol{y}^{(K)}) \\ | & | & & | \end{bmatrix}_{m \times K}$

1000 columns
b/c
1000 runs of
random geometries

72 rows
b/c
72 points along
airfoil surface

Bi-Fidelity Modeling

one realization



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Small subset of columns / $\mathbf{y}^{(i)}$'s

2) Interpolating decomposition (ID)

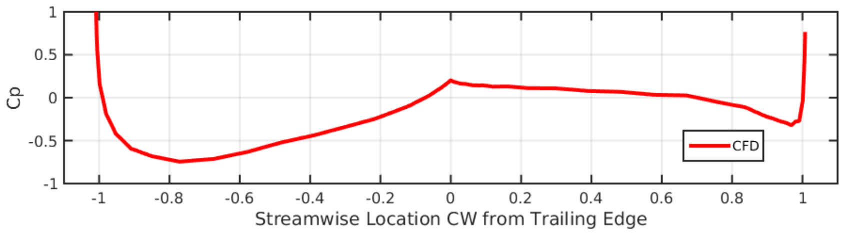
$$\boldsymbol{U}^L \approx \underbrace{\begin{bmatrix} | & | & & | \\ \boldsymbol{u}^L(\boldsymbol{y}^{(1)}) & \boldsymbol{u}^L(\boldsymbol{y}^{(2)}) & \cdots & \boldsymbol{u}^L(\boldsymbol{y}^{(r)}) \\ | & | & & | \end{bmatrix}_{m\times r}} \quad \left[\lambda_{ij}^L\right]_{r\times K}$$

LF coefficient matrix

$$\boldsymbol{U}^L \approx \boldsymbol{U}_C^L \boldsymbol{\Lambda}^L$$

Bi-Fidelity Modeling

one realization



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Results from high-fidelity model run for **y**$^{(i)}$'s deemed important by LF ID

LF coefficient matrix

LF coefficient matrix

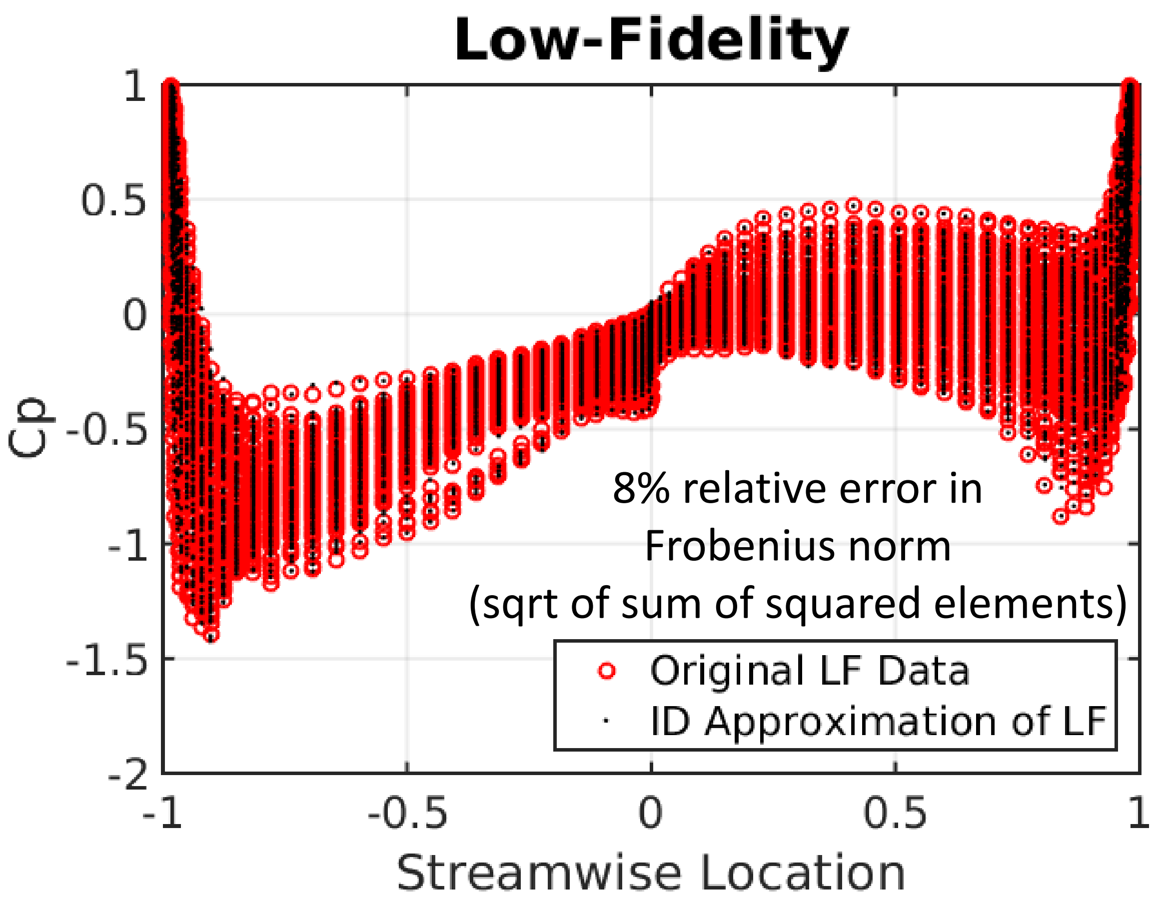
3) Apply LF coefficient matrix to HF results

$$\hat{U}^H = U^H_C \Lambda^L$$

$$U^L \approx U^L_C \Lambda^L$$

Results

Red: 1000 LF runs (cheap)
Black: ID approximation using 8 LF runs

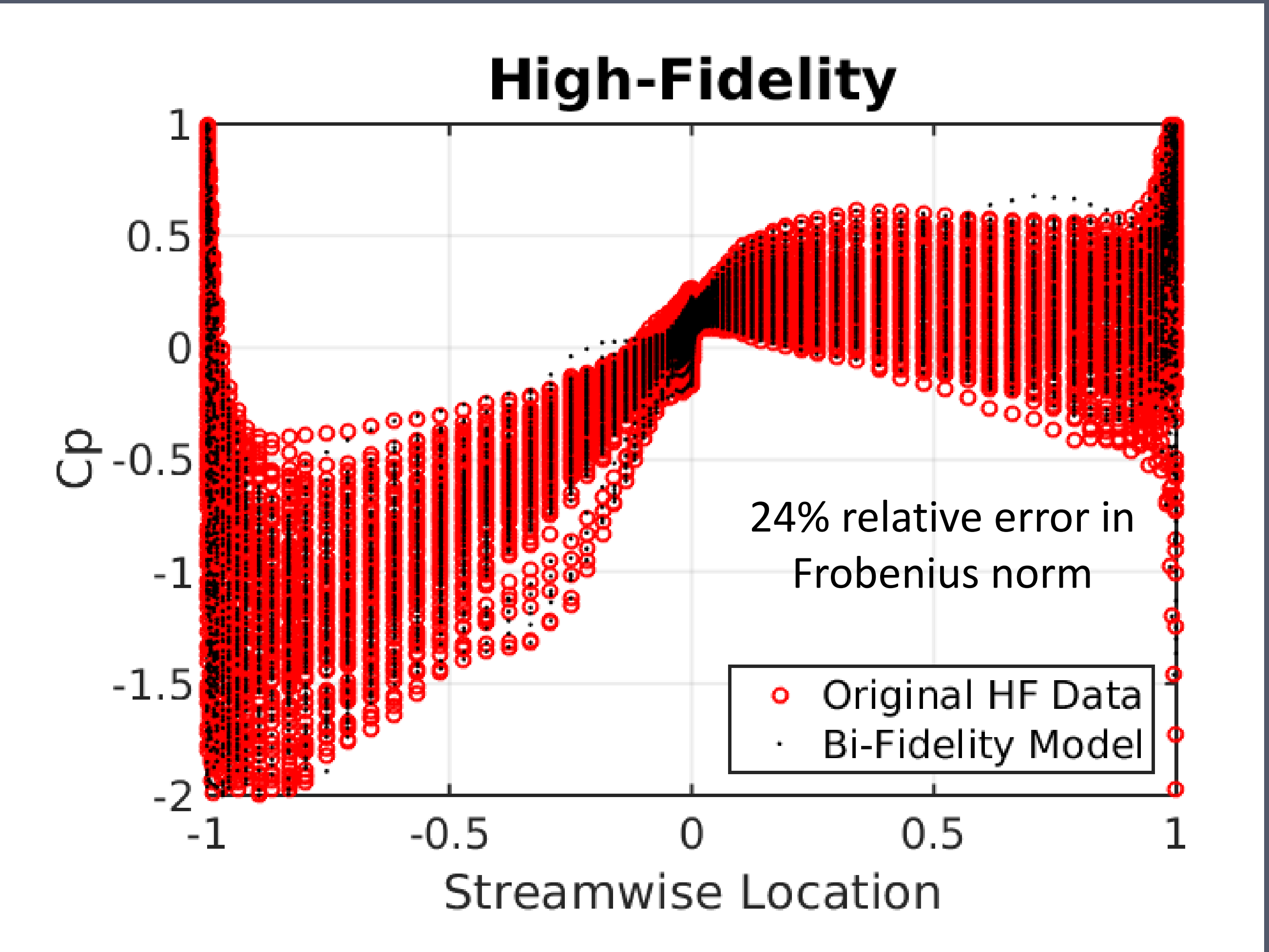


- Interpolating decomposition does well with just 8 LF runs
  - (LF system is sparse)
- It is designed to get this part right

$$\boldsymbol{U}^L \approx \boldsymbol{U}_C^L \boldsymbol{\Lambda}^L$$

Results

Red: 1000 HF runs (expensive)
Black: Bi-fidelity model employing 8 HF runs



- Bi-fidelity model is pretty good!
- Not designed explicitly to get this part right
- Large reduction in cost
  - Only need 1000 LF runs + 8 HF runs to generate bi-fidelity model

$$\boxed{\hat{\boldsymbol{U}}^H = \boldsymbol{U}_C^H \boldsymbol{\Lambda}^L}$$

Discussion

- Bi-fidelity model performs well on this system
  - Reduce cost of HF runs 100x (1000 → 8 runs)
- Surprising because LF model does not resolve boundary layers <u>at all</u>
  - First point off wall: $y^+ = 40{,}000$
  - Recommended $y^+ = 1$
- HF mesh still needs work
  - Validation case of NACA 4412 airfoil at $\alpha = 0°$ disagrees with Cp plots
  - Higher resolution in leading edge and shear layer