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Impact of Geometric Uncertainty on a NACA 4412 Airfoil

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The shape of a 4-digit NACA xyzz airfoil is specified by three geometric parameters, all non-dimensionalized with respect to the chord length c:

- $m = \mathrm{x}/100$ is the maximum camber
- $p = \mathrm{y}/10$ is the location of maximum camber
- $t = \mathrm{zz}/100$ is the maximum thickness

For a physical coordinate $x \in [0, c]$, the thickness of a symmetric airfoil is given by

$$y_t = 5tc\left[0.2969\sqrt{\frac{x}{c}} + (-0.1260)\left(\frac{x}{c}\right) + (-0.3516)\left(\frac{x}{c}\right)^2 + 0.2843\left(\frac{x}{c}\right)^3 + (-0.1015)\left(\frac{x}{c}\right)^4\right] \tag{1}$$

The coordinate pairs of points on the upper (x_U, y_U) and lower (x_L, y_L) surface are simply $x_U = x_L = x$, $y_U = y_t$, and $y_L = -y_t$. A symmetric airfoil corresponds to the NACA 00zz series. One example is NACA 0012, shown in Figure 1.

Cambered NACA airfoils define thickness perpendicular to the camber line, which can be defined as

$$y_c = \begin{cases} m\frac{x}{p^2}\left(2p - \frac{x}{c}\right) & 0 \le x \le pc \\ m\frac{c-x}{(1-p)^2}\left(1 - 2p + \frac{x}{c}\right) & pc \le x \le c \end{cases} \tag{2}$$

The upper and lower coordinate pairs become

$$\begin{aligned} x_U &= x - y_t \sin\theta \qquad & y_U &= y_c + y_t \cos\theta \\ x_L &= x + y_t \sin\theta \qquad & y_L &= y_c - y_t \cos\theta \end{aligned} \tag{3}$$

where

$$\theta = \arctan\left(\frac{dy_c}{dx}\right) \quad \text{and} \quad \frac{dy_c}{dx} = \begin{cases} \frac{2m}{p^2}\left(p - \frac{x}{c}\right) & 0 \le x \le pc \\ \frac{2m}{(1-p)^2}\left(p - \frac{x}{c}\right) & pc \le x \le c \end{cases} \tag{4}$$

Because cambered profiles are generated from their symmetric counterparts, we can analytically determine the displacements necessary to deform any symmetric NACA mesh into an arbitrary 4-digit NACA mesh. This results in substantial efficiency gains. In our case, we create a CAD model of a NACA 0012 airfoil, mesh it, and partition it once. Then, in the initialization stage of a new simulation, we generate a stochastic NACA realization parameterized by random variables $\mathbf{m}$, $\mathbf{p}$, and $\mathbf{t}$. Each mesh node on the airfoil surface is prescribed a displacement based on its x-coordinate, which is then passed to a linear elastic structural solver. Numerical solution of the unsteady incompressible Navier-Stokes equations proceeds on the deformed mesh.

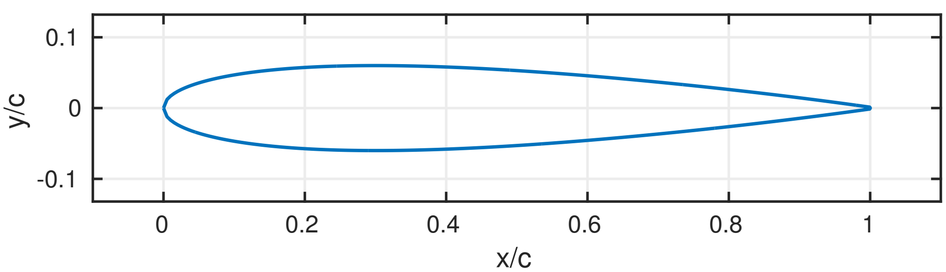

Figure 1: Surface of a NACA 0012 airfoil.