OLABISI ONABANJO UNIVERSITY
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FACULTY OF ENGINEERING
DEPARTMENT OF MECHANICAL ENGINEERING

FEG 501: ENGINEERING ECONOMY

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DEFINITION AND SCOPE OF ENGINEERING ECONOMICS

- **THE ENGINEERING PROCESS**

The engineering process employed from time a particular need is recognised until it is satisfied may be divided into number of phases that include the following:

1. **Determination of objectives**

2. **Identification of strategic factors**

THE ENGINEERING PROCESS CONT'D

3. **Determination of means**

4. **Evaluation of engineering proposals**

5. **Decision making**

MEANING AND DEFINITION OF ENGINEERING ECONOMICS

- Investment on technology should be justified on economics as well as technical grounds.
- Projects selected should be such that meet the immediate as well as future needs of the establishment.
- The technologies chosen should be such that will not be obsolete in a short while; although they must necessarily be appropriate.

- In engineering, problem solving involves both physical and economic aspects.
- Engineers have concerned themselves too long with resolving the physical problems forgetting that any useful idea must of necessity be economically viable for it to be worthwhile.
- Engineering economics deals with the concepts and techniques of analysis useful in evaluating the worth of systems, products and services in relation to their cost.

- ***Engineering economics* involves formulating, estimating and evaluating economics outcomes when choices and alternatives are available.**
- ***This involves using specific mathematical relationship to compare the cash flows of the different alternatives.***

SOME FUNDAMENTAL CONCEPTS OF ECONOMICS

a) **Concept of Value and Utility**

b) **Concept of Consumer and Producer goods**

c) **Economic Aspects of Exchange**

- **Mutual Benefit in Exchange**

INTEREST FORMULAE AND THEIR APPLICATIONS

- **TIME VALUE OF MONEY**: If someone elects to invest his money today (e.g. in a bank or savings and loan association), by tomorrow he will have accumulated money than he had originally invested. This accumulation of money is called ***time value of money***.

a) **Factors**: Inflation or deflation, Risk and Cost of money represented by

Interest, Interest Rate and Minimum Attractive Rate of Return

- The manifestation of the time value of money is called **interest**.

- When interest is expressed as a percentage of the original amount per unit time; it is called **interest rate**

- The ***effective interest rate*** is the annual interest rate that yields the same annual return as the return obtained from an interest rate specified for a compounding period shorter than a year.

- The ***nominal interest rate*** is the interest rate that yields the same annual return as the return obtained from compounding periods per year.

- **Minimum Attractive Rate of Return (MARR)** is one of the most important parameters in engineering economics.
- The MARR is an interest rate that specifies a rate of return on investments that is just acceptable to the investor.
- **Reasons**: ***Project risks***, ***Investment opportunities***, ***Limited capital, Rate of return at other companies and Tax structure***

Simple interest is one of the ways by which capital change in value over time. Total interest earned or charged is directly proportional to the initial amount of deposit or loan (principal), interest rate and number of time periods of commitment. Mathematically,

$$\text{Simple interest, } I = P.n.i$$

Compound interest

- The standard assumption is that interest is computed on the current balance which includes the accrued interest that has been paid plus the principal amount (deposit/loan).
- In computing compound interest, interest earned or charged for a period is based on remaining principal plus accumulated interest [unpaid interest] at the beginning of period. Mathematically expressed as:

$$\text{Compound interest, } I_n = P(1+i)^n - P$$

The Concept of Equivalence

- This means that different sums of money at different times can be equal in economic value.
- Equivalence is defined with respect to some given interest rate e.g. if interest rate is 6% per year, ₦100 today will be ₦106 after one year i.e.

$$\begin{aligned}\text{Amount accrued} &= 100 + 100(0.06) \\ &= 100(1 + 0.06) \\ &= 100(1.06) \\ &= ₦106\end{aligned}$$

Cash Flows

- Cash flow is the stream of monetary [Naira] values – cost [inputs] and benefits [outputs] – resulting from a project/investment.
- Engineering projects generally have economic consequences that occur over an extended period of time.
- Each project is described as cash receipts or disbursements [expenses] at different points in time.

- The expenses and receipts due to engineering projects usually fall into one of the following categories:
  - ***First cost***
  - ***Operations and maintenance***
  - ***Salvage value***
  - ***Revenues***
  - ***Overhaul***

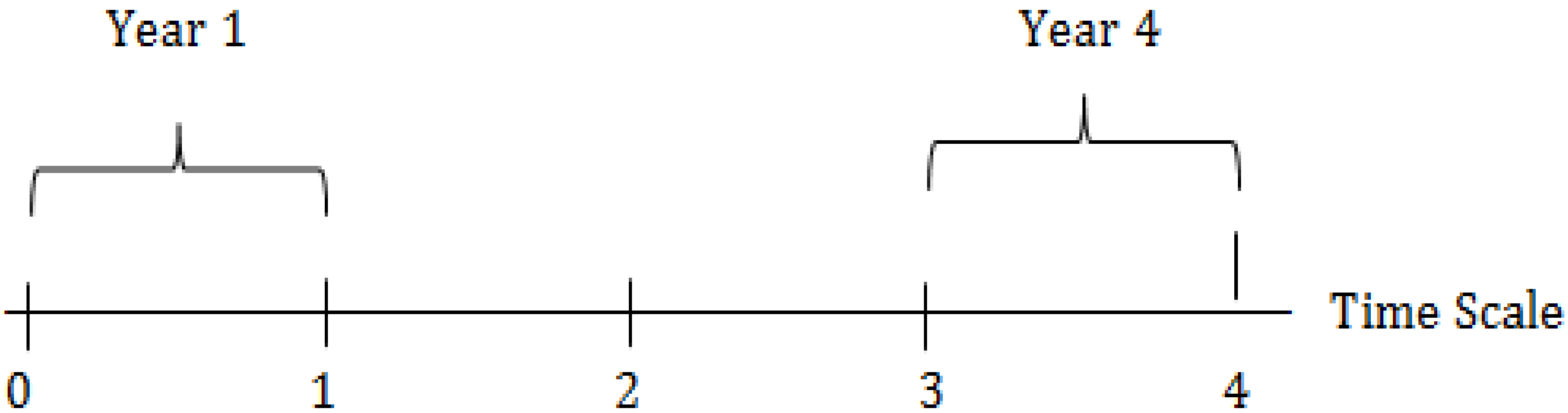
Cash Flow Diagrams [CFD]

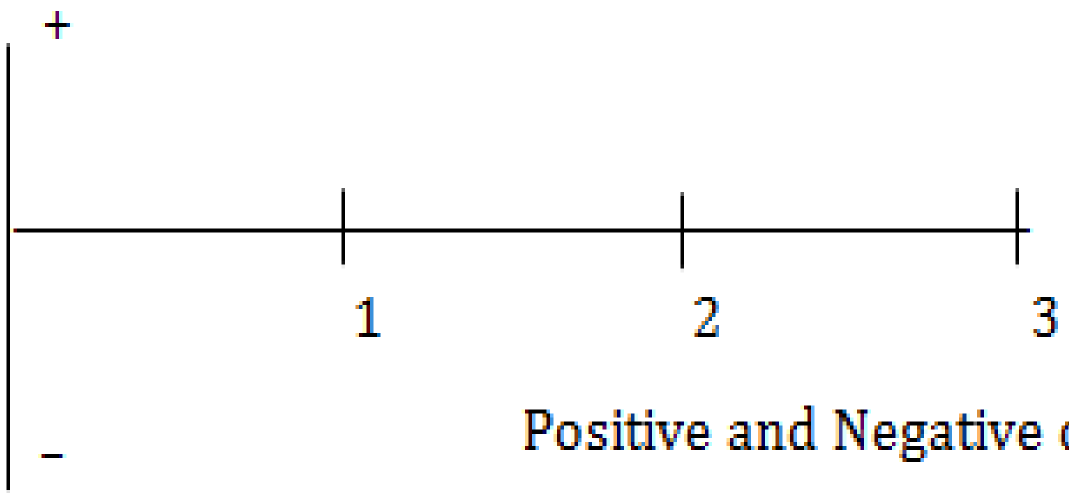
- This is the graphical representation of cash flows drawn on a time scale.

- In a CFD, we have three **timing of cash flows** namely ***beginning of period, time 0 and end of period cash flows***.

- **Beginnings of period** cash flows are: rent, lease and insurance payment; **end-of-period cash flows** are O & M, salvages, revenues, overhauls while the designation of **time 0** is arbitrary.

One person's cash flow outflow [represented as a negative value] is another person's inflow [represented as a positive value]





Positive and Negative cash flow

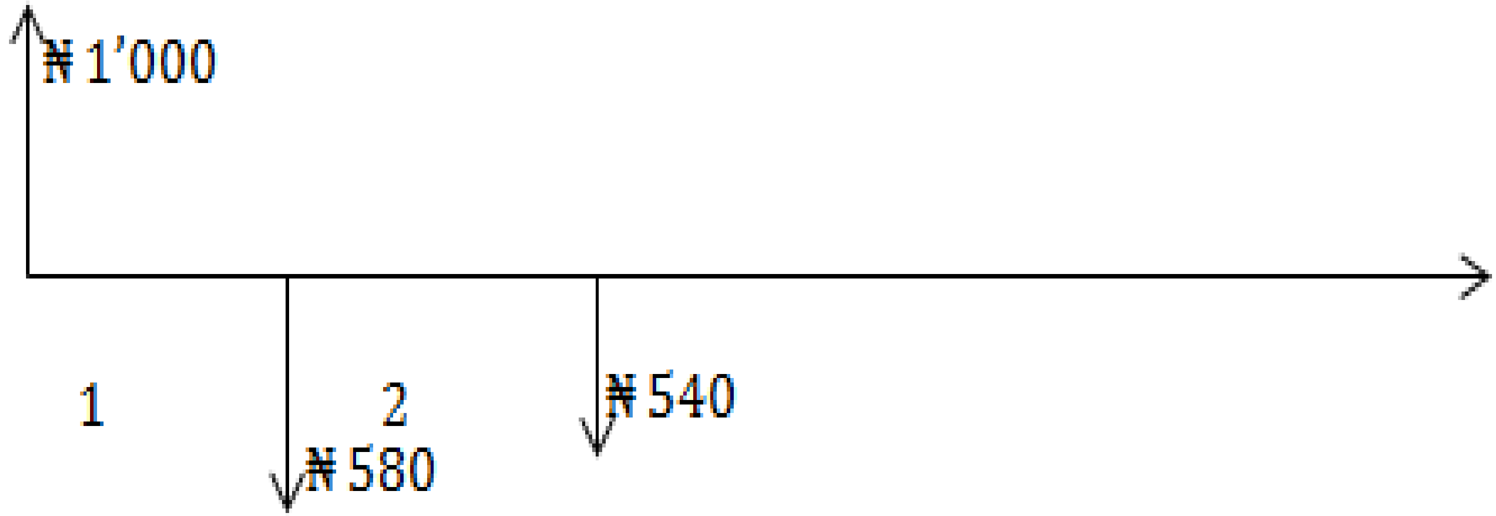
Example 2.1

- A man borrowed ₦1000 from a bank at 8% interest. Two end-of-year payment: at the end of the first year, he will repay half of the principal plus the interest that is due. At the end he second year, he will repay the remaining half plus the interest for the second year. Represent this on a CFD.

Solution

Cash flow for this problem is:

EOY	Cash flow
0	+ ₦1'000
1	- ₦ 580 (-₦ 500 – ₦ 80)
2	- ₦ 540 (-₦ 500 – ₦ 40)



Comparison of Alternative Projects

- An alternative is a stand-alone solution to a given situation.
- Various forms of alternatives exist in virtually everything we do, namely:

a. Selection of method of transportation we use to get to work/school every day.

b. Buying a house or renting one.

- Evaluation criterion is the basis for judging the alternatives of accomplishing a given objective. In this text, currency designation; ***NAIRA and DOLLAR*** is used as the basis for comparison. Typically evaluation criterion includes:

a. Lowest overall cost

b. Highest overall revenue, etc.

INTEREST FORMULAE FOR DISCRETE CASH FLOW AND DISCRETE COMPOUNDING

- Find below the terminology and notation symbols used in this section.

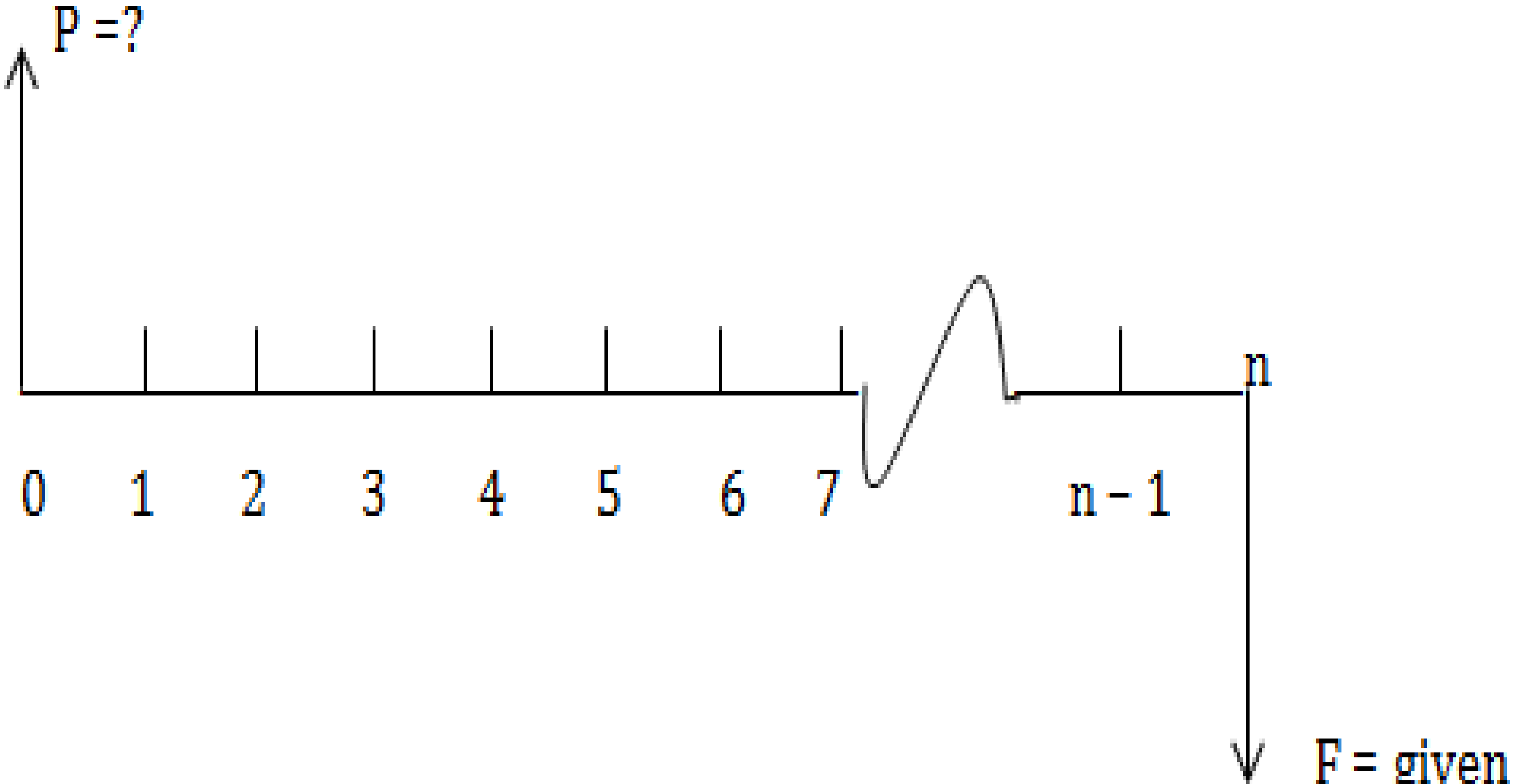
i = effective interest rate per compounding period

n = number of compounding periods

P = present sum of money (present value) i.e. *equivalent* value of cash flows at a reference point in time called the present

- F = future sum of money (future value) i.e. *equivalent* value of cash flows at a reference point in the time called the future
- A = end-of-period cash flows i.e. in a uniform series of payments continuing for a specified time, starting at the end of the first period and continuing to the end of the last period. It is refers to as ***annuity***.

Interest Formulae relating Present and Future worth of Single Cash Flows



- This determines the amount of money accumulated (F) after n years from a single investment (P) when interest is compounded one time per year .i.e.

$$F_1 = P + P\acute{\iota}$$
$$= P\,(1 + \acute{\iota})$$

@ the end of year 2;

$$F_2 = F_1 + F_1\acute{\iota}$$
$$= P\,(1 + \acute{\iota}) + P\,(1 + \acute{\iota})\,\acute{\iota}$$
$$= P\,(1 + \acute{\iota})^{2}$$

@ the end of year 3;

$$F_3 = P(1 + \acute{\imath})^3$$

- By mathematical Induction, after n years,

$$F_n = P(1 + \acute{\imath})^n$$

express as $F = P\,(F|P, \acute{\imath}\,\%, n)$

- The factor, $(1 + \acute{\imath})^n$ is refers to as ***single payment compound amount factor.***

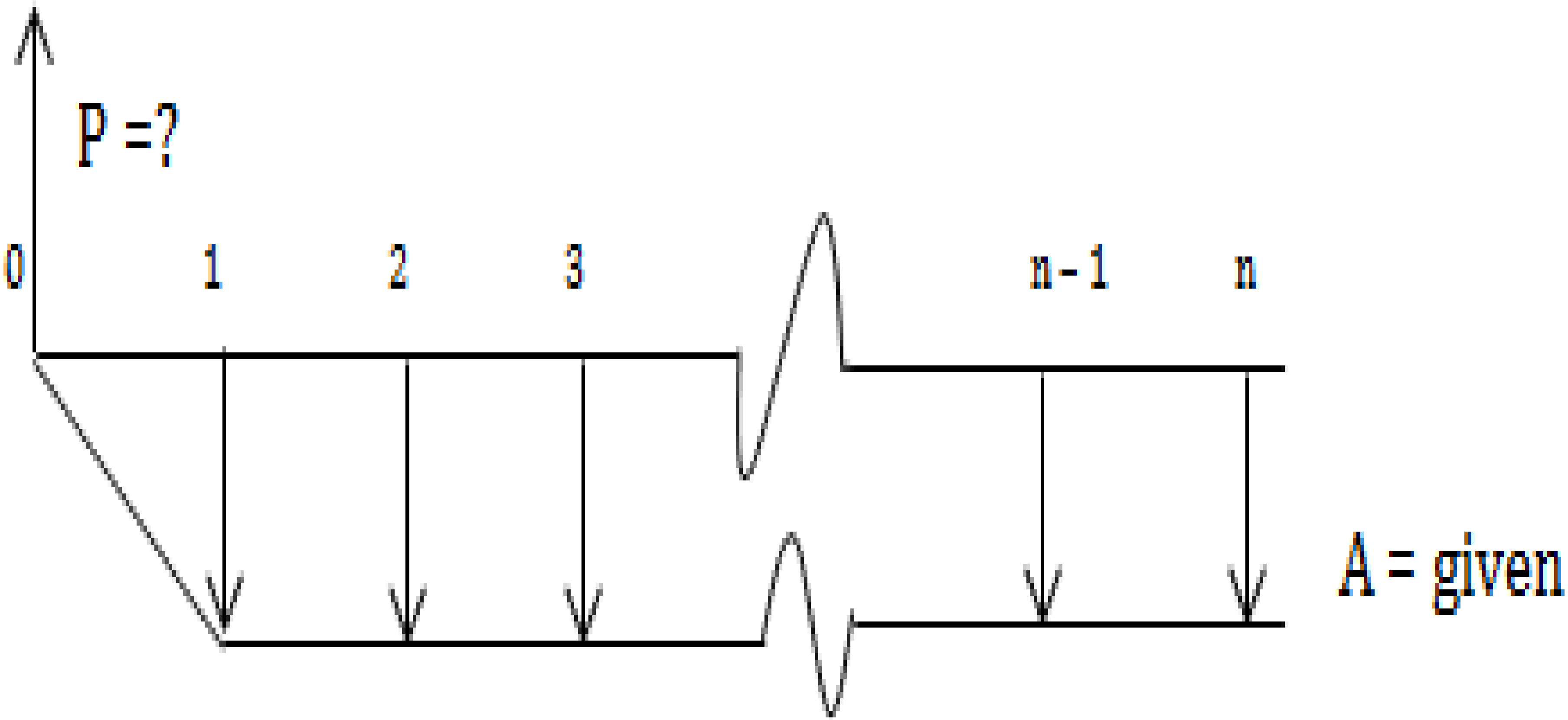
- Similarly,

$$P = \quad F\left(\frac{1}{(1+i)^n}\right)$$

express as P = F (P|F, 1%, n)

- Where the factor; $\frac{1}{(1+i)^n}$ is refers to as ***single – payment present worth factor.***

Interest Formulae relating Uniform Series (Annuity) to its Present and Future Worth



A = uniform series (annuity, a constant series amount)

- The present worth of the uniform or constant series amount is determined by considering each A value as a future worth F in the case of single payment present worth factor and then summing all present worth value i.e.

$$P = A\left[\frac{1}{(1+i)}\right] + A\left[\frac{1}{(1+i)^2}\right] + \cdots + A\left[\frac{1}{(1+i)^{n-1}}\right] + A\left[\frac{1}{(1+i)^n}\right]$$

$$= \quad A\left[\frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \cdots + \frac{1}{(1+i)^{n-1}} + \frac{1}{(1+i)^n}\right] - \text{------} -(1)$$

- Divide both sides by$(1 + i)$, we have:

$$\frac{P}{(1+i)} = A\left[\frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \cdots + \frac{1}{(1+i)^n} + \frac{1}{(1+i)^{n+1}}\right] ----(2)$$

- Subtract equation (1) from equation (2), we have:

$$\frac{P}{(1+i)} - P = A\left[-\frac{1}{1+i} + \frac{1}{(1+i)^{n+1}}\right]$$

- Simplifying, we have:

$$P = A\left(\frac{(1+i)^n - 1}{i(1+i)^n}\right)$$

$$express\ as\ P = A(P|A, \Omega\ \%, n).$$

- $\frac{(1+i)^n-1}{i(1+i)^n} = \boldsymbol{uniform\ series\ present\ worth\ factor}.$

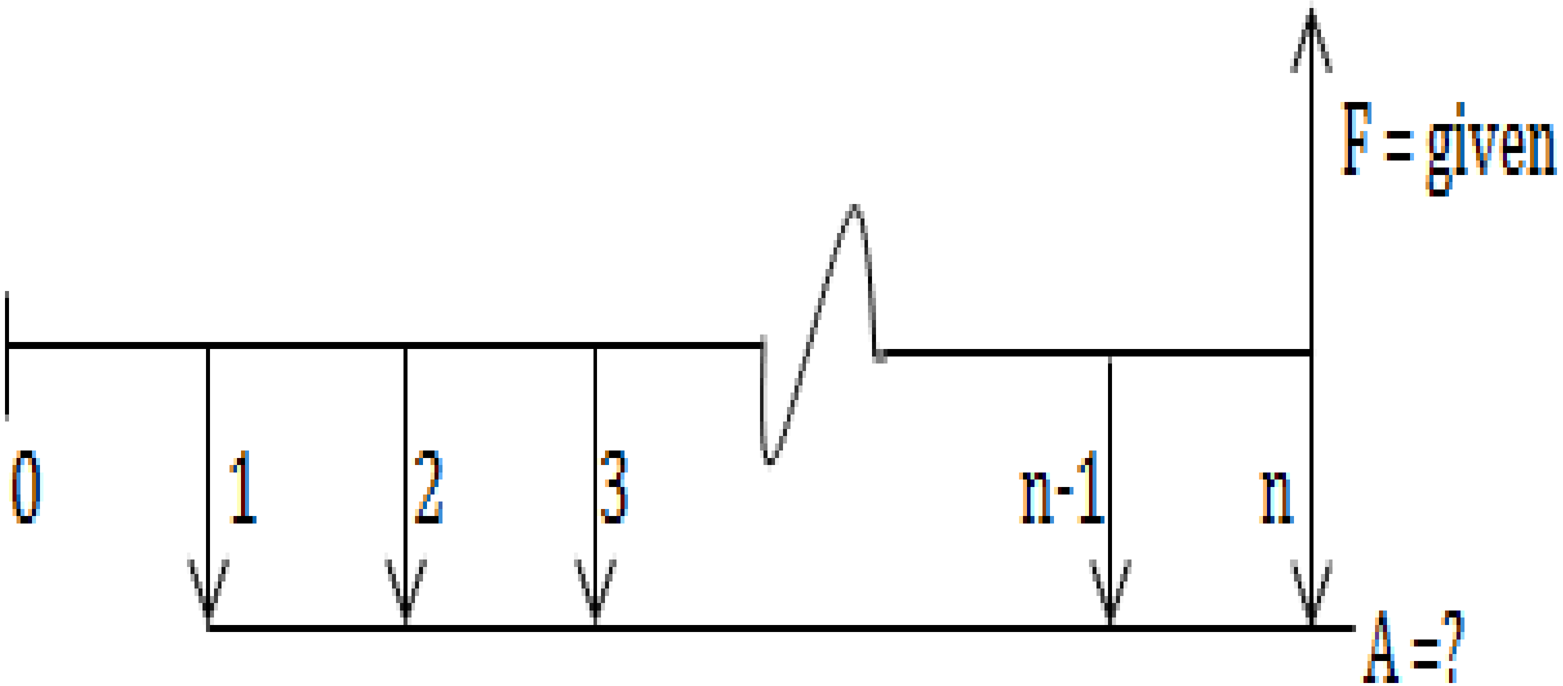
- By rearranging, we have

$$A = P\left(\frac{i(1+i)^n}{(1+i)^n - 1}\right)$$

$$express\ as\ A = P(A|P, i\ \%, n)$$

- Where the factor; $\frac{i(1+i)^n}{(1+i)^n-1}$ is refers to as $c\boldsymbol{apital\ recovery\ factor}$

Sinking Fund Factor



The figure above depicts the cash flow diagram relating Uniform series and Future worth

From $$A = P(A/P,\ i\,\%, n).$$

$$= P\left(\frac{i(1+i)^n}{(1+i)^n - 1}\right)$$

$$= \frac{F}{\cancel{(1+i)^n}}\left(\frac{i\cancel{(1+i)^n}}{(1+i)^n - 1}\right)$$

$$A = F\left(\frac{i}{(1+i)^n - 1}\right) \mathit{express\ as}$$

$$A = F(A|F, i\,\%, n)$$

$$\frac{i}{(1+i)^n - 1} = \boldsymbol{sinking\ fund\ factor}.$$

- By re-arranging, we have:

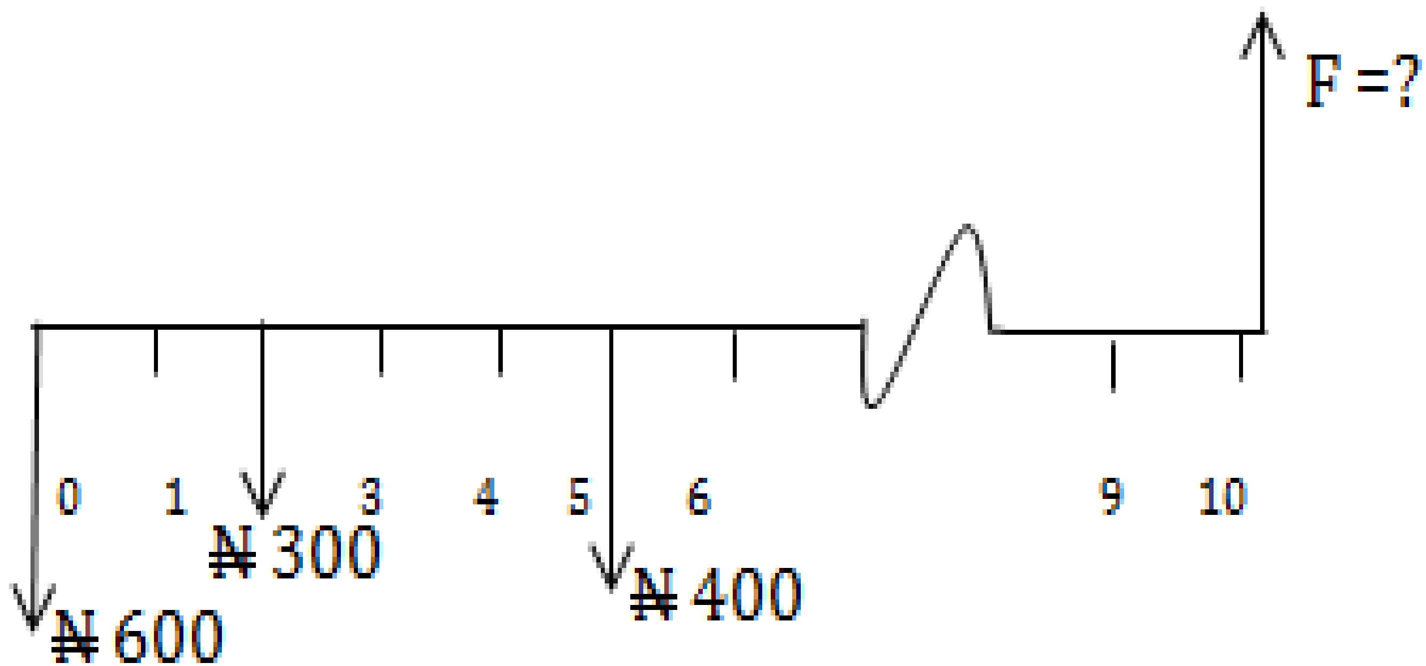
$$F = A\left(\frac{(1+i)^n - 1}{i}\right)$$

$$F = A(F|A, i\,\%, n)$$

- Where the factor; $\frac{(1+i)^n - 1}{i}$ is refers to as $\boldsymbol{uniform\ series\ compound\ Amount factor}$

Example2.2: If a woman deposit 600 now, ₦ 300 two years from now and ₦ 400 five years after, how much will she have in her account 10 years after, if the interest rate is 5%?

Solution:



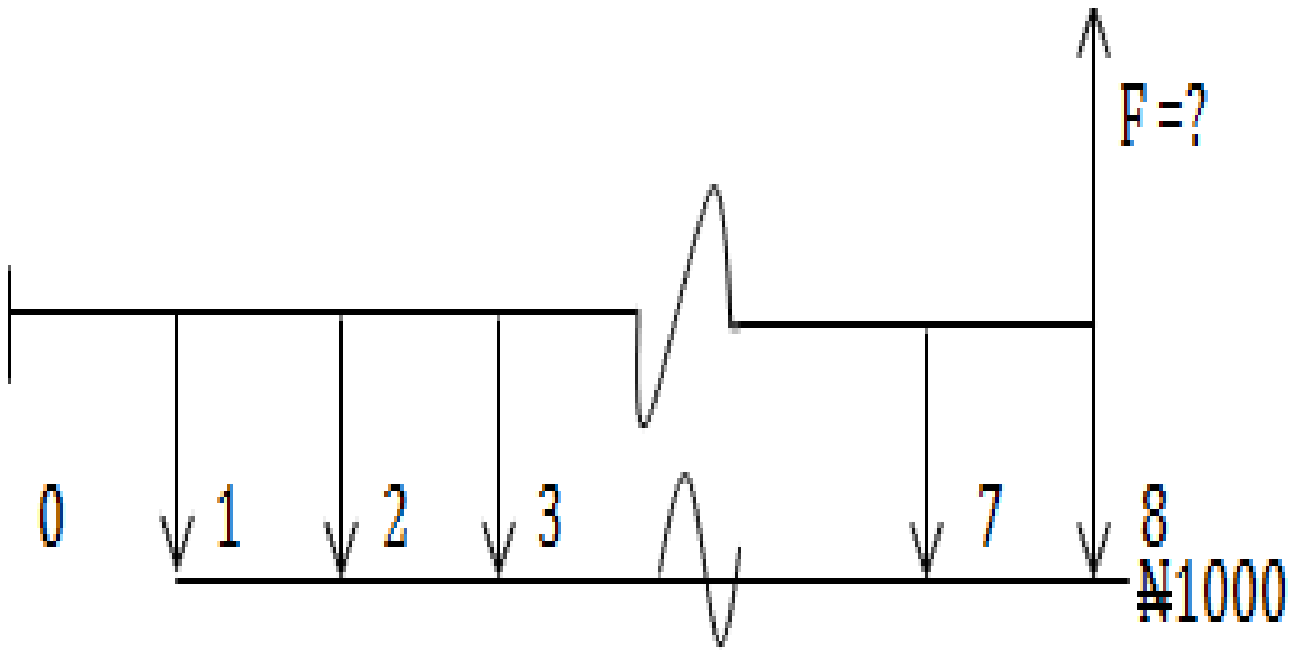
$$F = 600(F_1|P_1, 5\%, 10) + 300(F_2|P_2, 5\%, 8) + 400(F_3|P_3, 5\%, 5)$$

$$= 600(1.6289) + 300(1.4775) + 400(1.2763)$$

$$= ₦1,931.11$$

Example 2.3: How much money would a man have in his account after 8years if he deposited 1000 per year at 5%.

Solution:



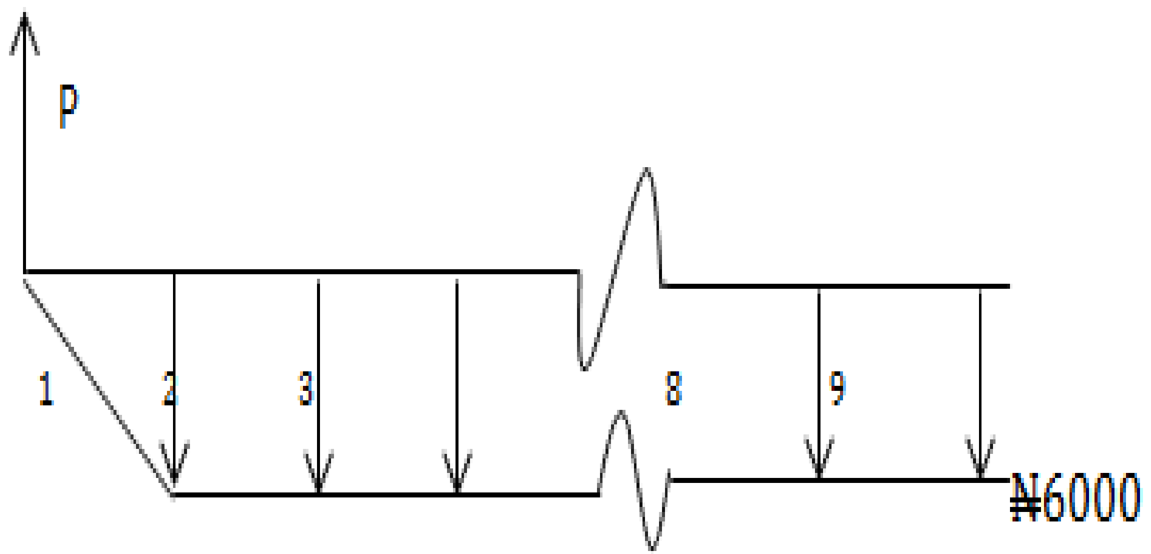
$$F = A\,(F|A, 5\%, 8)$$

$$= 1000(9.5491)$$

$$= ₦\,9{,}549.10$$

Example 2.4: How much money would you be willing to pay now for a note that will yield ₦6000 per year for nine years at 20%?

Solution:

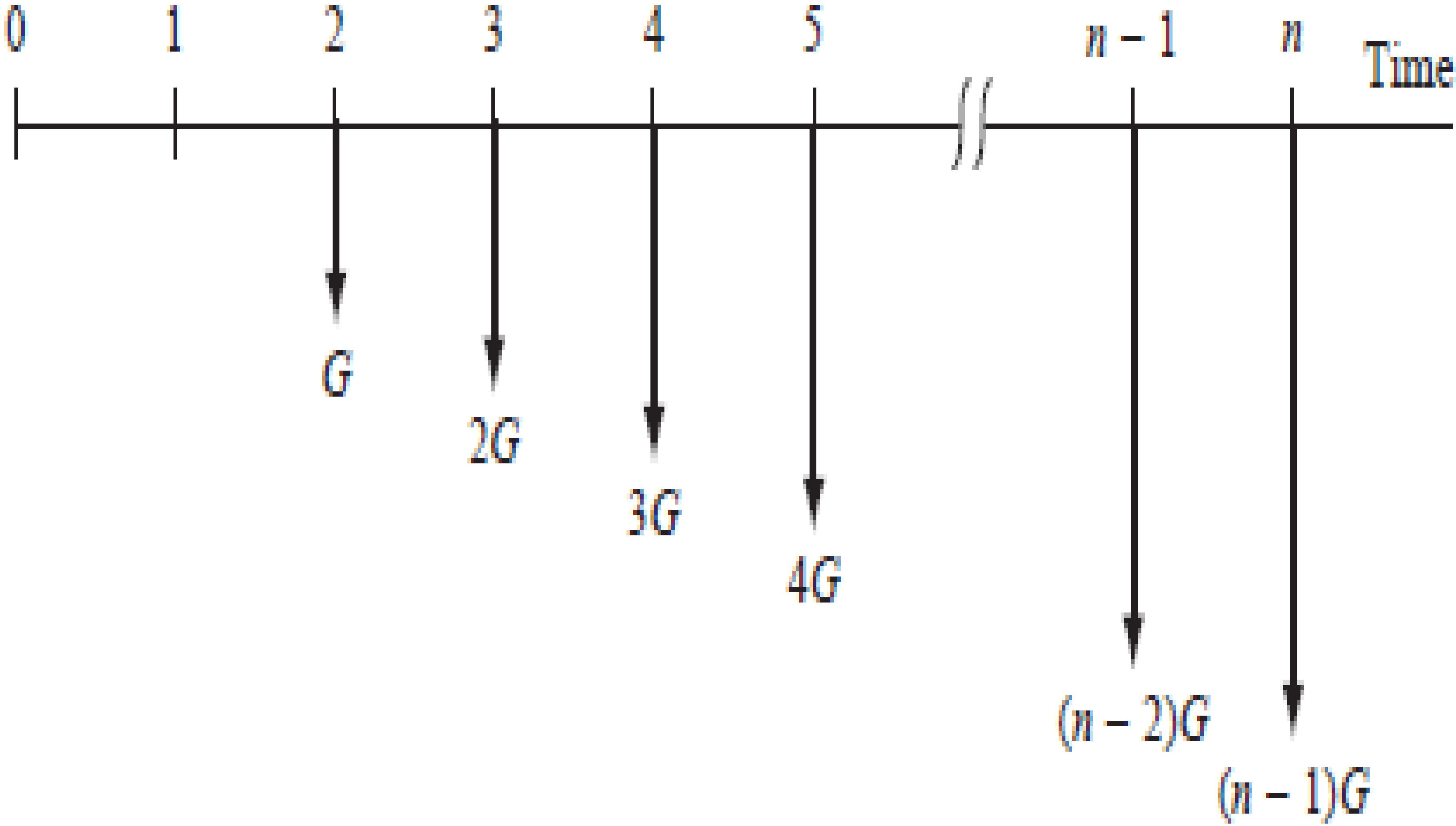


$$P = A\,(P|A, 20\%, 9)$$

$$= 6000(4.0310)$$

$$\cong ₦\,24,200$$

Arithmetic Gradient Factors (P/G and A/G)



- An **arithmetic gradient** series is a cash flow series that either increases or decreases by a **constant amount** each period.
- The amount of the change is called the **gradient**.
- Define the symbols G for gradient and CF_n for cash flow in year n as follows.

$$CF_n = \text{base amount} \pm (n - 1)\, G$$

G= constant arithmetic change in cash flows from one time period to the next; G may be positive or negative.

- **Example2.5:** Olabisi Onabanjo University has initiated a logo-licensing program with the clothier Yomakie Inc. Estimated fees (revenues) are $80,000 for the first year with uniform increases to a total of $200,000 by the end of year 9. Determine the gradient and construct a cash flow diagram that identifies the base amount and the gradient series (**Leland Blank and Anthony Tarquin, 2012)**.

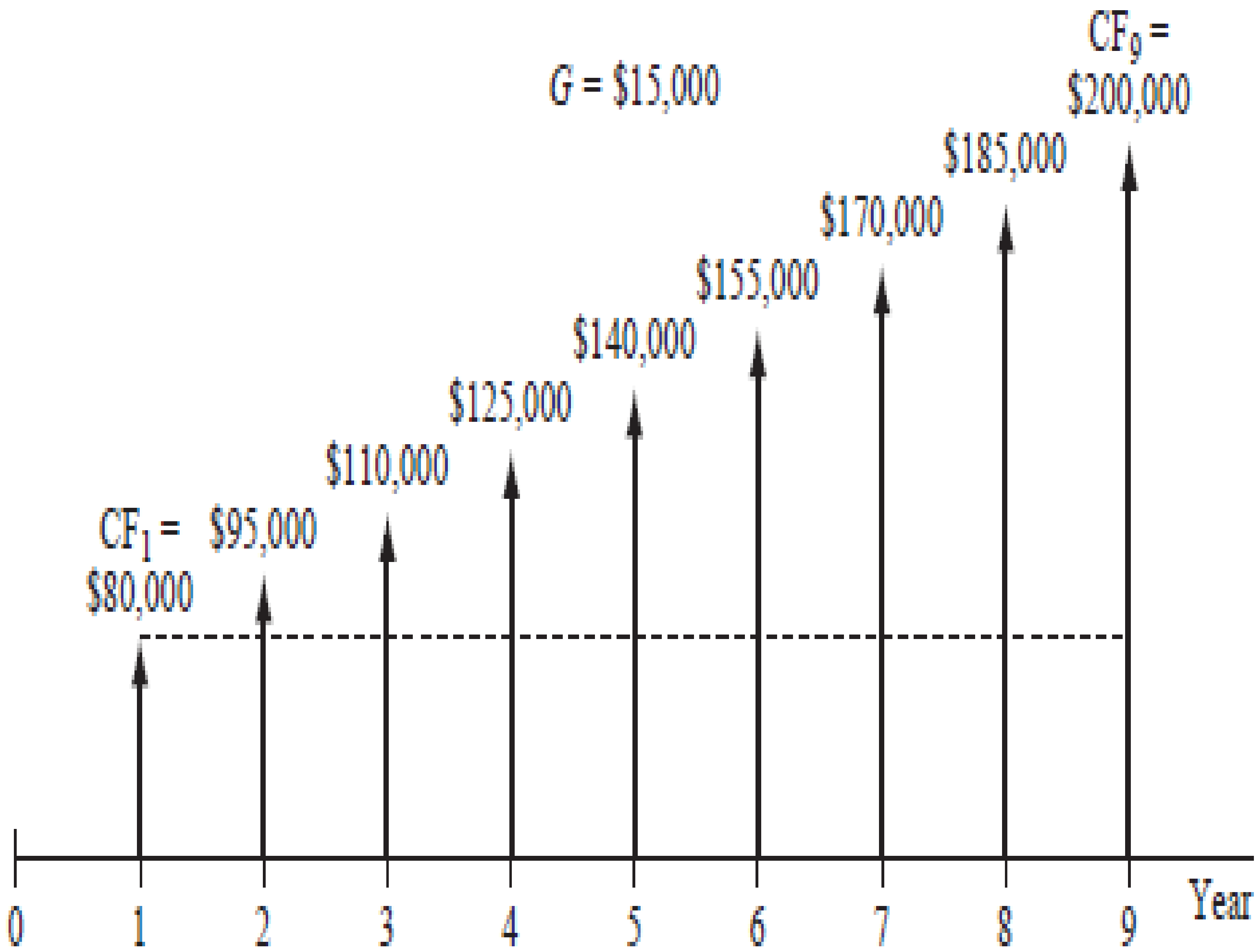
Solution:

The year 1 base amount is CF_1 = \$80,000 and the total amount increase over 9 years is

CF_9 - CF_1 = 200,000 – 80,000 = \$120,000

Equation for arithmetic gradient is given below:

$$G= \frac{CF_9 - CF_1}{n-1} = \frac{120000}{9-1} = \$15{,}000\ per\ year$$



G = $15,000
CF1 = $80,000
$95,000
$110,000
$125,000
$140,000
$155,000
$170,000
$185,000
CF9 = $200,000
0
1
2
3
4
5
6
7
8
9
Year

- The cash flow diagram shows the base amount of \$80,000 in years 1 through 9 and the \$15,000 gradient starting in year 2 and continuing through year 9.
- The total present worth P_T for a series that includes a base amount A and conventional arithmetic gradient must determine the present worth of both the base amount defined by A and the arithmetic gradient series defined by G. The addition of the two results in P_T:

$$P_T = P/A \pm P/G$$

- The corresponding equivalent annual worth A_T is the sum of the base amount series defined by A and the annual worth of the arithmetic gradient series defined by G, that is:

$$A_T = \mathrm{A} \pm A/G$$

- The P/G and A/G factors determine the present worth and annual series of the ***gradient amount.***

Example 2.6: Neighbouring parishes of Anglican Churches in Ago-Iwoye have agreed to pool road tax resources already designated for bridge refurbishment. At a recent meeting, the engineers estimated that a total of $500,000 will be deposited at the end of next year into an account for the repair of old and safety-questionable bridges throughout the area. Further, they estimate that the deposits will increase by $100,000 per year for only 9 years thereafter, and then cease. Determine the equivalent (a) present worth and

(b) annual series amounts, if public funds are earned at a rate of 5% per year (**Leland Blank and Anthony Tarquin, 2012).**

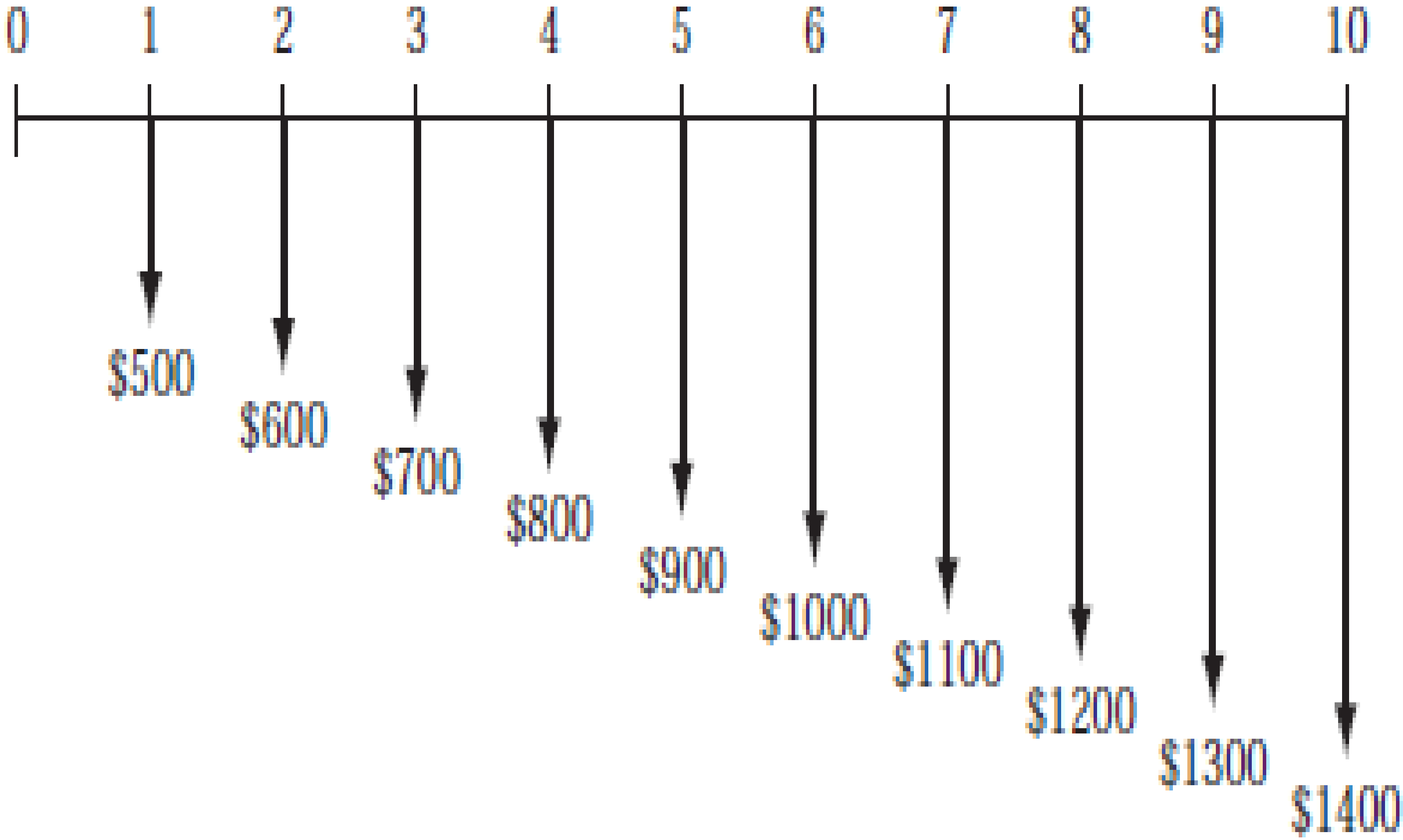
Solution

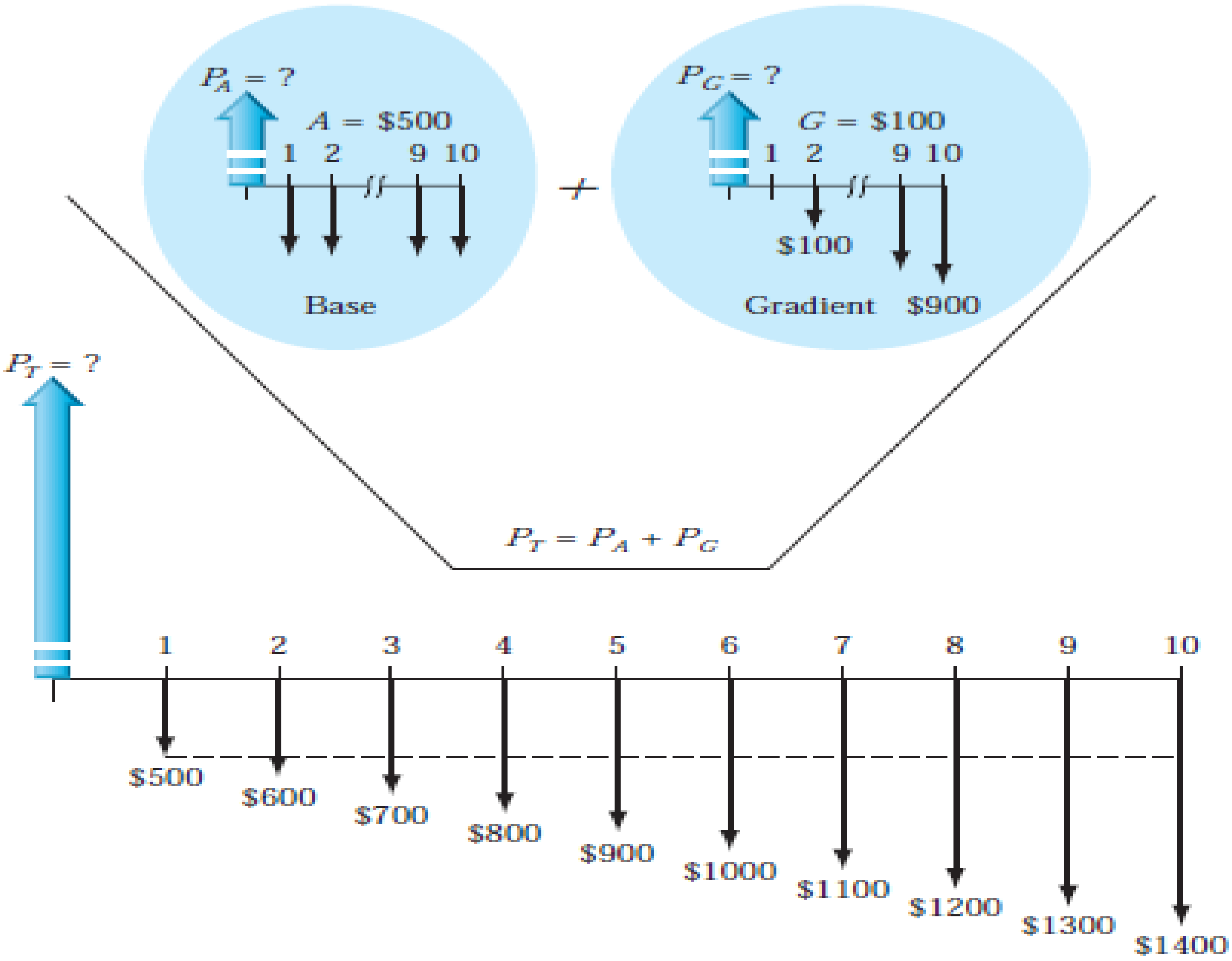
a. From the cash flow diagrams shown below; we have a base amount of \$500,000 and a gradient amount of \$100,000. So the Present worth amount is given below:

$$P_T = -500000(P/A,5\%,10) - 100000(P/G,5\%,10)$$

$$= -500000(7.7217) - 100000(31.6520)$$

$$= -\$7,026,050$$


0
1
2
3
4
5
6
7
8
9
10
$500
$600
$700
$800
$900
$1000
$1100
$1200
$1300
$1400



$P_A = ?$
$A = \$500$
1 2 9 10
Base
+
$P_G = ?$
$G = \$100$
1 2 9 10
$100
Gradient $900
$P_T = ?$
$P_T = P_A + P_G$
1 2 3 4 5 6 7 8 9 10
$500 $600 $700 $800 $900 $1000 $1100 $1200 $1300 $1400

b. The base amount is the annuity of $500,000 while the gradient amount remains $100,000. So, the annual worth of the above cash flow diagram is given below:

$$A_T = -500000 - 100000(A/G,5\%,10)$$

$$= -500000 - 100000(4.0991)$$

$$= -\$909,910 \text{ per year}$$

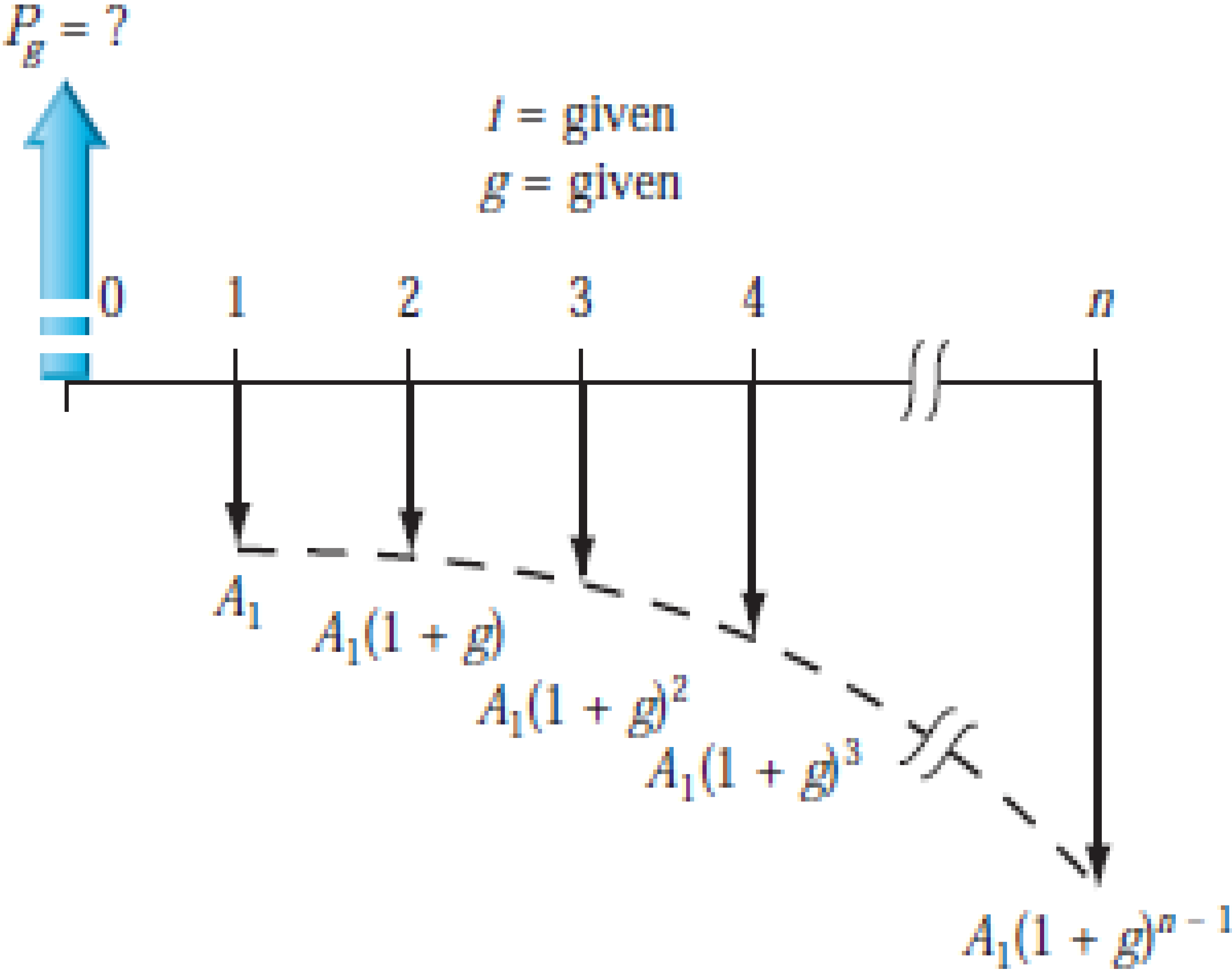
Geometric Gradient Series Factors

- A **geometric gradient** series is a cash flow series that either increases or decreases by a **constant percentage** each period. The uniform change is called the **rate of change**.

- g = **constant rate of change**, in decimal form, by which cash flow values increase or decrease from one period to the next. The gradient g can be +ve or -ve.

- A_1 = **initial cash flow in year 1** of the geometric series

- P_g = **present worth** of the entire geometric gradient series, including the initial amount A_1


$P_g = ?$
i = given
g = given
0
1
2
3
4
n
A_1
$A_1(1+g)$
$A_1(1+g)^2$
$A_1(1+g)^3$
$A_1(1+g)^{n-1}$

$$P_g = \frac{A_1}{(1+i)} + \frac{A_1(1+g)}{(1+i)^2} + \frac{A_1(1+g)^2}{(1+i)^3} + \frac{A_3(1+g)^3}{(1+i)^4} + \cdots + \frac{A_1(1+g)^{n-1}}{(1+i)^n}$$

$$P_g = A_1\left(\frac{1}{(1+i)} + \frac{(1+g)}{(1+i)^2} + \frac{(1+g)^2}{(1+i)^3} + \frac{(1+g)^3}{(1+i)^4} + \cdots + \frac{(1+g)^{n-1}}{(1+i)^n}\right) \dots\dots\dots\dots 1$$

- Multiply both sides by $\frac{(1+g)}{(1+i)}$, we have:

$$P_g\left(\frac{(1+g)}{(1+i)}\right) = A_1\left(\frac{(1+g)}{(1+i)^2} + \frac{(1+g)^2}{(1+i)^3} + \frac{(1+g)^3}{(1+i)^4} + \cdots + \frac{(1+g)^{n-1}}{(1+i)^n} + \frac{(1+g)^n}{(1+i)^{n+1}}\right) \dots\dots 2$$

- Subtract equation 1 from equation 2, we have:

$$P_g\left(\frac{(1+g)}{(1+i)}\right) - P_g = A_1\left(\frac{(1+g)^n}{(1+i)^{n+1}} - \frac{1}{(1+i)}\right)$$

$$P_g\left(\frac{(1+g)}{(1+i)} - 1\right) = A_1\left(\frac{(1+g)^n}{(1+i)^{n+1}} - \frac{1}{(1+i)}\right)$$

- Simplifying, we have:

$$P_g = \frac{1 - \frac{(1+g)^n}{(1+i)}}{(i-g)}, \qquad g \neq i$$

- $P_g = A(P/G, g, i, n)$

- If $g = i, from\ equation\ 1\ above\ P_g$ becomes:

$$P_g = A_1\left(\frac{1}{(1+i)} + \frac{1}{(1+i)} + \frac{1}{(1+i)} + \frac{1}{(1+i)} + \cdots + \frac{1}{(1+i)}\right)$$

- $P_g = \frac{An}{(1+i)}$

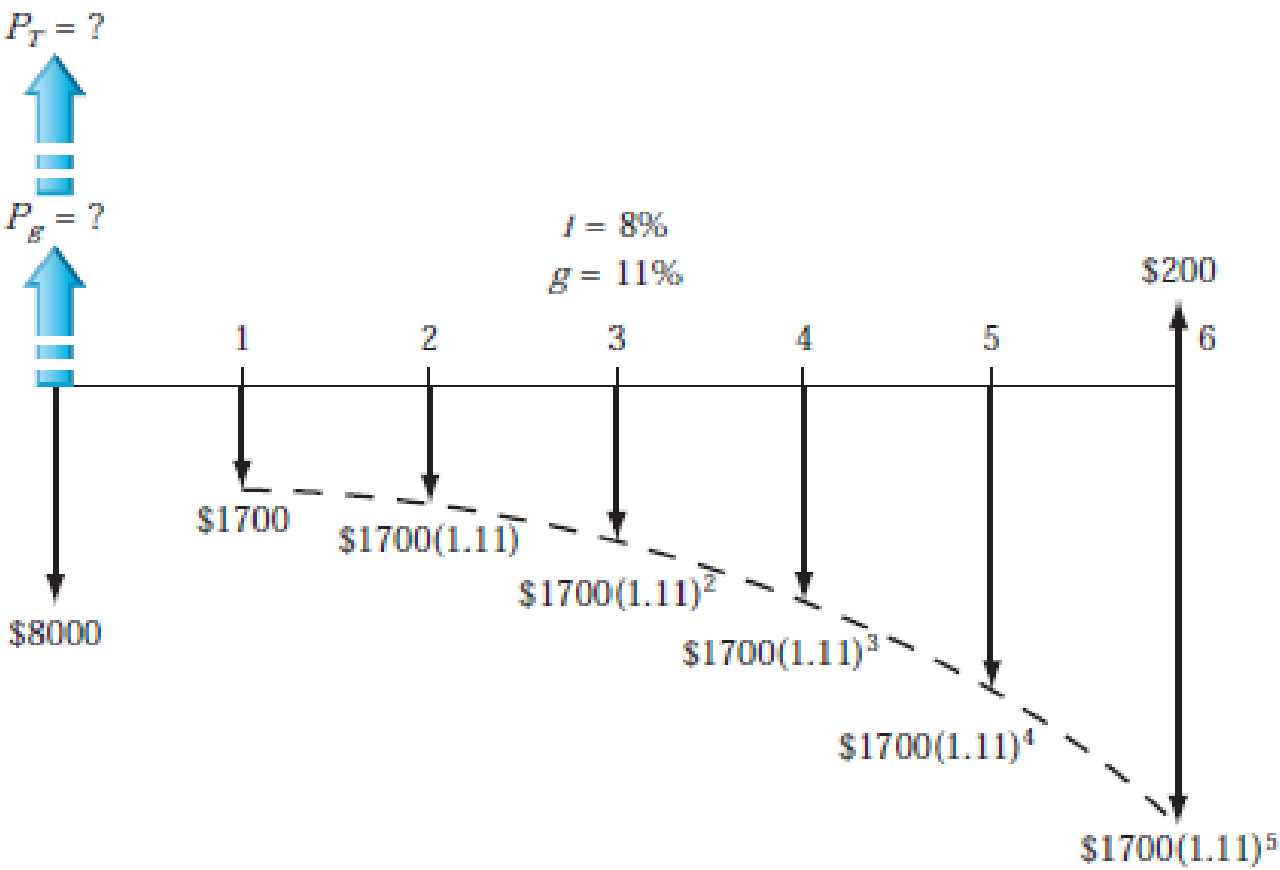
- The $(P/A,g,i,n)$ factor determines the P_g value in period $t = 0$ for a geometric gradient series **starting in period 1** in the amount A_1 and increasing or decreasing by a constant rate of g each period.

- *Note that the present worth of the geometric gradient series is not calculated separately in this case but calculated with the base amount as will be shown in the foregoing example.*

Example 2.7: A coal-fired power plant has upgraded an emission control valve. The modification costs only \$8000 and is expected to last 6 years with a \$200 salvage value. The maintenance cost is expected to be high at \$1700 the first year, increasing by 11% per year thereafter. Determine the equivalent present worth of the modification and maintenance cost at 8% per year (**Leland Blank and Anthony Tarquin, 2012).**

Solution:

The cash flow diagram is shown below:



$$P_T = -8000 - P_g + 200(P/F, 8\%, 6)$$

$$P_T = -800 - 1700\left(\frac{1 - (\frac{1.11}{1.08})^6}{0.08 - 0.11}\right) + 126$$

$$P_T = -800 - 1700(5.9559) + 126$$
$$= -\$17{,}999$$

INTEREST PROBLEMS WITH COMPOUNDING MORE-OFTEN-THAN-ONCE PER YEAR

- This is a situation whereby nominal interest rate is compounded non – annually i.e. quarterly, monthly, weekly etc. The amount of interest earned if it is compounded more frequently is given as:

$$i_{effective} = (1 + {}^{rate}/_{m\ payment\ periods})^{m} - 1$$

- If the interest rate is 20% compounded quarterly, the effective annual interest rate is:

$$i_{\text{effective}} = (1 + 0.20/4)^4 - 1$$

$$= (1 + 0.05)^4 - 1$$

$$= (1.05)^4 - 1 = 0.2155$$

Exercises

1. Look up the numerical value for the following factors from the interest tables.

a. (P/F,6%,8)

b. (A/P,10%,10)

c. (A/G,15%,20)

d. (A/F,2%,30)

e. (P/G,35%,15)

2. Find the numerical value of the following factors using (a) interpolation and (b) the formula.

i. (A/P,13%,15)

ii.(P/G,27%,10)

3. How much can College of Engineering and Environmental Studies, OOU, Ibogun Campus afford to spend now on an energy management system if the software will save the College $21,300 per year for the next 5 years? Use an interest rate of 10% per year.

4. Yomakie Systems makes a panel milling machine with a 2.7-m-diameter milling head that emits low vibration and processes stress-relieved aluminum panels measuring up to 6000 mm long. The company wants to borrow money for a new production/warehouse facility. If the company offers to repay the loan with \$60,000 in year 1 and amounts increasing by \$10,000 each year through year 5, how much can the company borrow at an interest rate of 10% per year?

5. *Tesla Motors manufactures high-performance battery for electric vehicles. An engineer is on a Tesla committee to evaluate bids for new-generation coordinate-measuring machinery to be directly linked to the automated manufacturing of high-precision vehicle components. Three bids include the interest rates that vendors will charge on unpaid balances. To get a clear understanding of finance costs, Tesla management asked the engineer to determine the effective semi-annual and annual interest rates for each bid. The bids are as follows:*

Bid 1*: 9% per year, compounded quarterly*

Bid 2*: 3% per quarter, compounded quarterly*

Bid 3*: 8.8% per year, compounded monthly*

- *Which bid has the lowest effective semi-annual interest rate and effective annual interest rate?*

Questions?

Thank you for Listening