Multi-Level, Multi-Resource Lot Sizing Problems

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Abstract

Presentation of models and basic heuristics for Multi-Level, Multi-Resource Lot Sizing Problems.

1 Modelling

1.1 Sets

- I is the set of items
- $S(i)$ is the set of items used to produce item i, i.e., one level below in the bill of material (BOM)
- K is the set of resources
- T is the set of time periods

1.2 Variables

- s_t^t is the number of units of item i on stock at time t
- x_t^{ik} is the number of units of production of item i on resource k at time t. No resource (k) index for big-bucket model.
- y_t^{ik} indicates if resource k is setup for production of item i at time t. No resource (k) index for big-bucket model. time t
- y_t^{ik} indicates if a setup of item i has happened on resource k at time t

1.3 Constants

- h_t^i is the unit inventory cost of item i at time t
- p_t^{ik} is the unit production cost of item i on resource k at time t
- f_t^{ik} is the fixed cost of resource k for having it set up for item i at time t
- d_t^i is the demand of item i at time t

- r^{ij} is the number of units of item j used for the production of item i
- α_t^{ik} is the unit capacity for producing item i at resource k at time t
- β_t^{ik} is the capacity used for setting up the production of item i at resource k at time t
- C_t^k is the capacity of reource k at time t
- M is a large constant

1.4 Small-bucket

Multi-Level Multi-Resource Capacitated Lot Sizing and Scheduling Problem with Setup Times (MLMR-CLSSP):

$$\min \sum_{i\in I}\left[h_0^i s_0^i + \sum_{t\in T}\left(h_t^i s_t^i + \sum_{k\in K}(p_t^{ik}x_t^{ik} + f_t^{ik}y_t^{ik})\right)\right] \tag{1}$$

$$\text{s.t. } s_{t-1}^i + \sum_{k\in K} x_{t-\gamma^{ik}}^{ik} = d_t^i + \sum_{j\in S(i)}\sum_{k\in K} r^{ij}x_t^{jk} + s_t^i \qquad t\in T,\ i\in I \tag{2}$$

$$\alpha_t^{ik}x_t^{ik} + \beta_t^{ik}z_t^{ik} \le C_t^k y_t^{ik} \qquad t\in T,\ i\in I,\ k\in K \tag{3}$$

$$\sum_{i\in I} y_t^{ik} \le 1 \qquad t\in T\ ,k\in K \tag{4}$$

$$y_t^{ik} \ge z_t^{ik} \ge y_t^{ik} - y_{t-1}^{ik} \qquad t\in T,\ i\in I,\ k\in K \tag{5}$$

$$s_t^i, s_0^i \ge 0,\ x_t^{ik}\in\mathbb{Z}^+,\ y_t^{ik}, z_t^{ik}\in\mathbb{B} \qquad t\in T,\ i\in I,\ k\in K \tag{6}$$

A big issue when solving this model is the possibility for an item to be produced in parallel on different resources.

If $r^{ij} = 0$ for all items i, j then (2) can be rewritten to:

$$\text{s.t. } s_{t-1}^i + \sum_{k\in K} x_{t-\gamma^{ik}}^{ik} = d_t^i + s_t^i \qquad t\in T,\ i\in I \tag{7}$$

That is, by substituting (2) with (7) one obtains the Multi-Item Multi-Resource Capacitated Lot Sizing and Scheduling Problem with Setup Times (MIMR-CLSSP).

1.5 Big-Bucket

By aggregating the time slots in the small-bucket model one obtains a big-bucket-model. Now items can share resources in the same time slot and an item can consume capacity of several resources in the same time slot (this is not equivalent with parallel production as described for the small-bucket model, but is an estimate of the production of a big-bucket item consisting of part of a product structure for a small-bucket item). Multi-Level Multi-Resource Capacitated Lot Sizing Problem with Setup Times

(MLMR-CLSP):

$$\min \sum_{i \in I} \left(h_0^i s_0^i + \sum_{t \in T} (h_t^i s_t^i + p_t^i x_t^i + f_t^i y_t^i) \right) \tag{8}$$

$$\text{s.t. } s_{t-1}^i + x_{t-\gamma^i}^i = d_t^i + \sum_{j \in S(i)} r^{ij} x_t^j + s_t^i \qquad t \in T,\ i \in I \tag{9}$$

$$x_t^i \geq M y_t^i \qquad t \in T,\ i \in I \tag{10}$$

$$\sum_{i \in I} \left(\alpha_t^{ik} x_t^i + \beta_t^{ik} y_t^i \right) \leq C_t^k \qquad t \in T,\ k \in K \tag{11}$$

$$s_t^i, s_t^0, x_t^i \geq 0,\ y_t^i \in \mathbb{B} \qquad t \in T,\ i \in I \tag{12}$$

This can be aggregated further into a single resource.

$$\sum_{i \in I} \left(\alpha_t^i x_t^i + \beta_t^i y_t^i \right) \leq C_t \qquad t \in T \tag{13}$$

Substitute (11) with (13) to obtain the Multi-Level Capacitated Lot Sizing Problem with Setup Times (MI-CLSP).

By aggregating time further one can aim for a single-level product structure.

$$\text{s.t. } s_{t-1}^i + x_{t-\gamma^i}^i = d_t^i + s_t^i \qquad t \in T,\ i \in I \tag{14}$$

Substitute (9) with (14) to obtain the Multi-Item Multi-Resource Capacitated Lot Sizing Problem with Setup Times (MIMR-CLSP).

Obviously both can be applied simultaneously to obtain the Multi-Item Capacitated Lot Sizing Problem with Setup Times (MI-CLSP).

2 Construction Heuristics

Two basic approaches to construct solutions for the small- and big-bucket models.

For simplicity we assume that the problem to solve is the MLMR-CLSP. The heuristics are adaptable to the MLMR-CLSSP

Both heuristics solve a series of sub-problems of the original formulation. The sub-problems are also $\mathcal{NP}$-hard but due to their size they are expected to be manageable with the use of general MIP heuristics available in CPLEX, e.g., diving and Relaxation Induced Neighborhood Search (RINS).

2.1 Per Product Structure Level

Given the product structure of the demanded items partition the structures into sets given by their level in a given product structure. In this case there are two approaches to divide the items: (i) an item that occur on different levels in different production structures can either be put in several sets according to the levels or (ii) the item is put in the set given by the lowest level of the structures.

Next the a MIMR-CLSP for each set is solved starting from the top down. The solution of on iteration is fixed before proceeding to the next iteration. Due to initial stock settings a feasible solution can always be obtained.

The algorithm is sketched as follows:

1. Divide items into sets each corresponding to a level in the product structures.

2. For each set of items sorted with top level items first

3. Solve a MIMR-CLSP for a given set of items.

4. Fix the solution and goto step 2.

An obvious drawback of such a heuristic is the potential of producing a lot of initial stock for the lower level items of the product structure since these items are placed last. Hence, the production plan for first time periods are not expected to be very good compared to later time periods.

2.2 Per Item Demand

An alternative simple procedure is to plan the production of a product structure one at a time. Two basic approaches are suggested: (i) plan the product structure for one unit of end item demanded at a given time, or (ii) plan the product structure for the entire demand of end item at a given time. If (i) is used the end items should be planned with some kind of randomization to diversify the production. If (ii) is used the end items can be planned in decreasing order of time for the demand.

Planning a the product structure of an end item is done by solving a MLMR-CLSP. The problem is easier than the original problem since only the demand of one time period is considered. However, if the product structure is very deep, we suggest to split product structure into smaller subtrees and solve them top down.

The algorithm is sketched as follows:

1. Produce a list of end items for planning

2. For each end item in list

3. Solve a MLMR-CLSP for the demand of the end item

4. Fix the solution and goto step 2.

The drawback with this heuristic is the possibility of high initial stock for the end items being planned late in the procedure. This is, due to capacity restrictions and consumed capacity by the production of the first planned product structures.

3 Uncertainty

In the classical stochastic approach the problem is decomposed using Benders Decomposition and the numerous scenarios (samples) are solved as Benders subproblems. Examples of uncertain events are:

- Demand
- Capacity
- Lead-time

In principle, any constant in the above models can be considered under uncertainty.