**605.632 Graph Analytics Midterm**

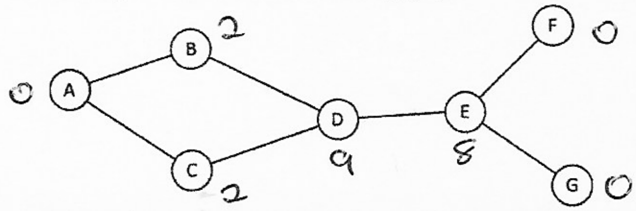
**Time:** 150 minutes.

1. (6) Consider an Erdos Renyi graph with average degree λ = 2. What is the probability that a node will have more than 2 neighbors ?
2. (4) What is the neighbor's degree distribution in an Erdos Renyi graph with average degree λ = 2 ?
3. (6) Consider an Erdos Renyi graph with a giant component whose size is 80% of the graph. What is the average degree in this graph ?
4. (4) Provide a comparison of clustering coefficients and degree distributions of social networks with Erdos Renyi, Barabasi Albert, and Watts Strogatz graph models

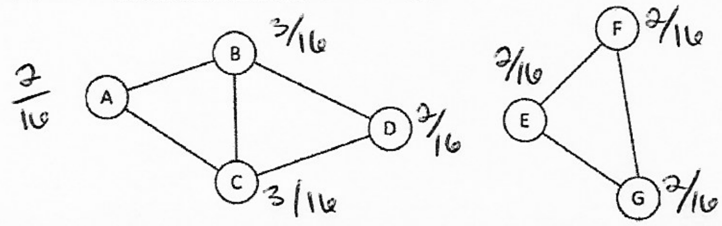
see below

| | Social Networks | ER | BA | WS |
|---|---|---|---|---|
| High Clustering (Y/N) | Y | N | N | Y |
| Degree distribution (Long/Short Tail) | Long | short | Long | short |

5. (4) Given an adjacency matrix A for an undirected graph, express the number of triangles in terms of A. $trace(A^3)/6$
6. (6) Calculate betweenness centrality for vertices in the following graph:

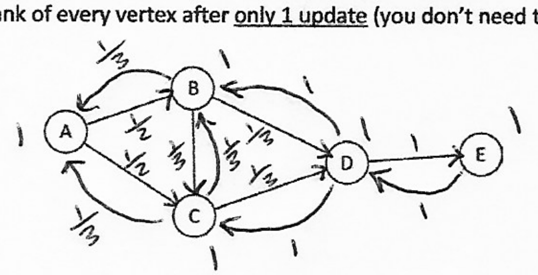



7. (6) Consider the preferential attachment network growth model (Barabasi-Albert). If a newcomer to the community wants to be friends with a person in the network shown below, what is the probability that he will join the larger group (assume <u>strictly preferential</u> attachment) ?




P(large group) = $\frac{10}{16}$

8. (6) Calculate the page rank centrality for vertices in the following graph based on <u>a single iteration</u>. Start with assigning page rank value of 1 to each vertex and assume links are bi-directional. Explicitly draw 2 links between every connected pair and show how much page rank flows in each direction. Assume that there are no random jumps. Calculate the page rank of every vertex after <u>only 1 update</u> (you don't need to normalize the result).

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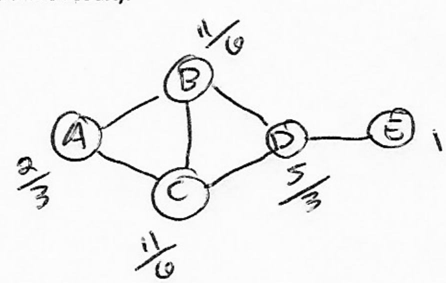


$A = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

$B = \frac{1}{3} + \frac{1}{2} + 1 = \frac{11}{6}$

$C = \frac{1}{3} + \frac{1}{2} + 1 = \frac{11}{6}$

$D = \frac{1}{3} + \frac{1}{3} + 1 = \frac{5}{3}$

$E = 1$

1. $P(x>2) = 1 - P(x \leq 2)$

$$= 1 - \sum_{i=0}^{2} \frac{e^{-2} 2^i}{i!}$$

$$= 1 - (e^{-2} + e^{-2} \cdot 2 + e^{-2} \cdot 2)$$

$P(x>2) = 1 - 0.677$

$P(x>2) = 0.323$

2. $P(k) = \frac{2^k}{k!} e^{-2}$

3. $0.8 = 1 - e^{-\lambda \cdot 0.8}$

$e^{-\lambda \cdot 0.8} = 0.2$

$\lambda = \frac{\ln(0.2)}{-0.8}$

$\lambda = 2.011$