Study of lattice vibrations using electronic circuits

1 Objectives of the experiment

1. Using inductors and capacitors build an analogy of mono-atomic lattice and study of dispersion relation.
2. Using inductors and two different value capacitors build an analogy of diatomic lattice and study of dispersion relation and band gap energy

2 Theory

Mass and spring model for a one direction periodic mono-atomic lattice is given in Fig.1. The atoms having mass ‘m’ connected by force constant (spring constant) ‘f’

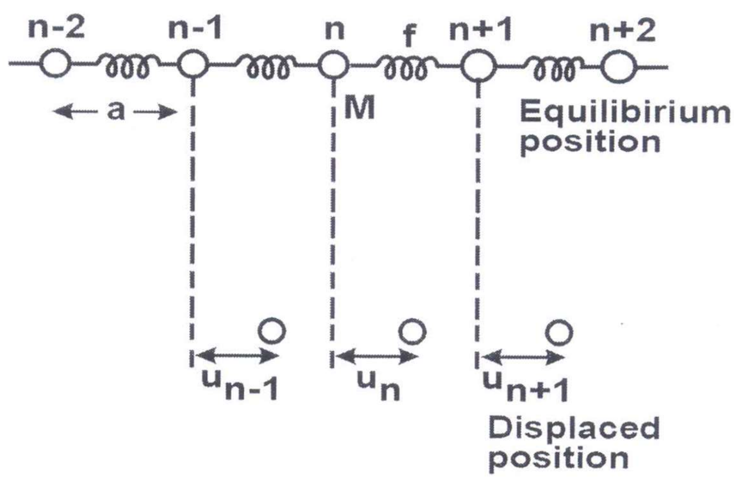

Fig. 1. One-dimensional linear mono-atomic lattice
a → lattice constant
f → Force constant
M → Mass of the atom

The equilibrium distance between the atoms is $'\boldsymbol{a}'$ and the array is assumed to be infinitely long. Assuming only the nearest neighbor interaction, the equation of the motion of the *n*th atom is given by

$$m\ddot{x}_n = f\left(U_{n+1} + U_{n-1}\right) - 2U_n \qquad \ldots(1)$$

After solving equation (1) it gives

$$\omega^2 = \frac{4f}{m}\sin^2\left(\frac{ka}{2}\right) \qquad \ldots(2)$$

$$= \frac{2f}{m}\left(1-\cos\theta\right) \qquad \ldots(3)$$

Where k is the wave vector [$\frac{2\pi}{\lambda}$ $\boldsymbol{or}$ $\frac{\omega}{c}$] and c is the velocity of propagation and $\boldsymbol{\theta} = \boldsymbol{ka}$ is the phase change per unit cell. The relation shows that the velocity of propagation is independent of frequency. It also shows that there is a maximum frequency

$$\nu_{\max} = \frac{\omega_{\max}}{2\pi} = \frac{1}{\pi}\sqrt{\frac{f}{m}} \qquad \ldots(4)$$

Beyond which no transmission occurs. The array may thus considered as a low-pass filter which transmits only in the range 0-ν_{max}.

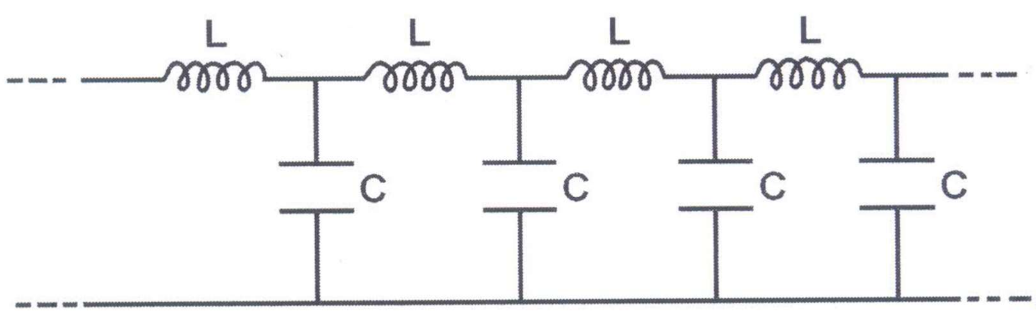

Fig. 2. Electrical analogue of linear mono-atomic lattice

The electrical analogue of the mono-atomic lattice is shown in Fig.2. The dispersion relation for this circuit is

$$\omega^2 = \frac{2}{LC}(1-\cos\theta) \qquad \ldots(5)$$

Where θ is the phase change introduced by ach section (unit cell) of the filter. Thus, one has a precise analogy with the one dimensional mono-atomic lattice with C ↔ m and (1/L) ↔ f. By measuring the phase difference between the input and output voltages of the circuit shown in Fig.2 as the function of frequency, the dispersion relation may be verified.

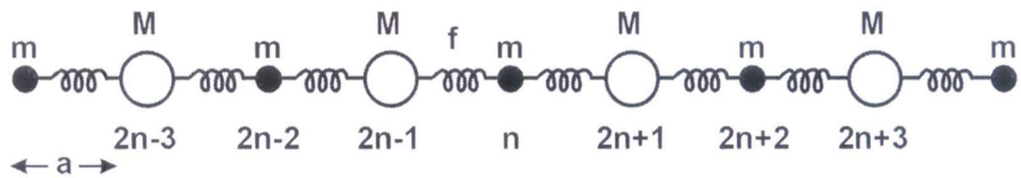

Fig. 3. Linear diatomic lattice of lattice parameter 'a' mass 'm' and 'M' and force constant 'f'

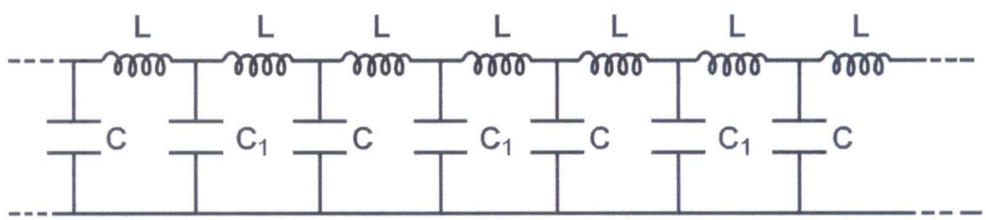

Fig. 4. Linear diatomic lattice -- electrical analogue

The di-atomic lattice with alternative masses 'm' and 'M' shown in Fig.3 can be simulated by the transmission line with alternative capacitors 'C' and 'C_1' shown in Fig.4. The dispersion relations for the mechanical and electrical analogues are given below.

$$\omega^2 = f\left(\frac{1}{m}+\frac{1}{M}\right) + f\left[\left(\frac{1}{m}+\frac{1}{M}\right)^2 - \frac{4\sin^2\theta}{mM}\right]^{1/2} \qquad \ldots(6)$$

$$\omega^2 = \frac{1}{L}\left(\frac{1}{C}+\frac{1}{C_1}\right) + \frac{1}{L}\left[\left(\frac{1}{C}+\frac{1}{C_1}\right)^2 - \frac{4\sin^2\theta}{CC_1}\right]^{1/2} \qquad \ldots(7)$$

In contrast to the mono-atomic lattice, there are now two frequencies ω_+ and ω_- corresponding to a particular value of the wave vector *'k'*. In a plot of ν versus θ (Fig.5) this leads to two branches; the one corresponding to ν_- is called the acoustical branch and the one corresponding to ν_+ is called the optical branch. The frequency gap between the two branches depends on (M/m) ↔ (C/C_1)

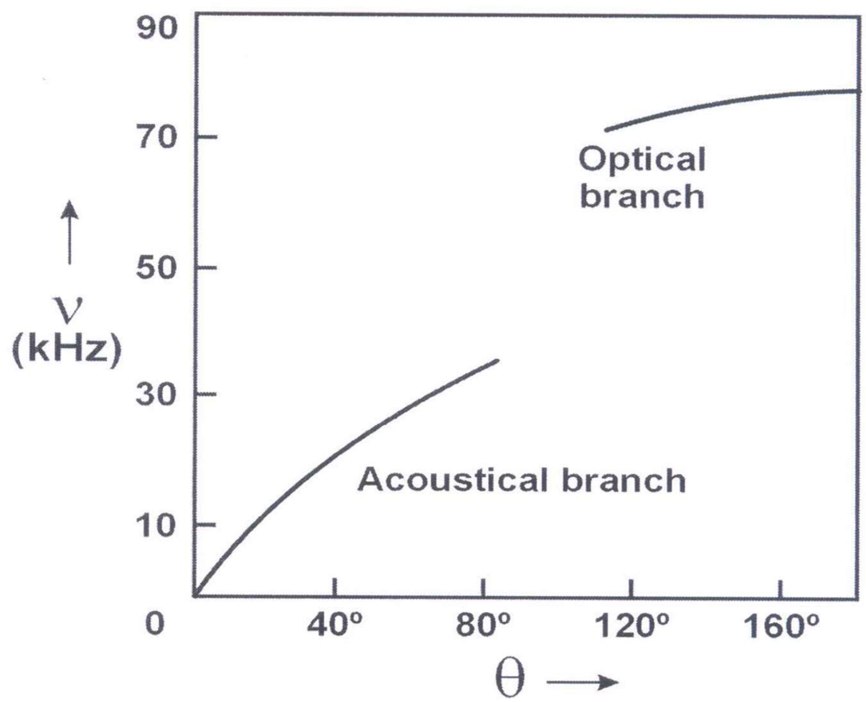

Fig. 5. Dispersion relation for the diatomic lattice

3 Procedure

1. Construct monoatomic lattice using given inductors and capacitors. Initially use 10 unit cells.

2. Input sine wave is given to the circuit through a function generator. Both input and output are given to oscilloscope which is kept in XY mode. Record the frequencies at which phase difference becomes 90 by observing Lissajous figure as circle. To maintain the equal amplitudes for both the channels of oscilloscopes, use a potentiometer.

3. Plot the phase difference per unit cell versus frequency versus frequency and determine the maximum angular frequency and compare with theoretical value (use $\frac{1}{\pi}\sqrt{\frac{L}{C}}$). How this formula is obtained?

4. Construct a diatomic lattice using two different types of capacitors and one type inductors. Use 5 unit cells initially.

5. Record the data of phase difference versus frequency and plot

6. Estimate the band gap using the graph and compare with the theoretical values. Frequency gap for diatomic comes in between $\frac{1}{2\pi}\sqrt{\frac{2}{LC_1}}$ *and* $\frac{1}{2\pi}\sqrt{\frac{2}{LC_2}}$

7. How the above formula for frequency gap is obtained? You may increase the number of lattices and observe the effect and interpret results

4 Tables for Observation

Monoatomic lattice

Table for frequency vs phase per unit cell

Frequency(kHz)	Phase(θ)in degree

Diatomic Lattice

Table for frequency vs phase per unit cell

Frequency(kHz)	Phase(θ)in degree

5 References

1.https://unlcms.unl.edu/cas/physics/tsymbal/teaching/SSP-927/Section%2005_Lattice_Vibrations.pdf

2.Mittal enterprises lattice dynamics kit manual