

Database Management Systems

Lecture 10

Evaluating Relational Operators

Query Optimization (IV)

- running example - schema
 - Students (SID: integer, SName: string, Age: integer, RoundedGPA: integer)
 - Courses (CID: integer, CName: string, Description: string)
 - Exams (SID: integer, CID: integer, EDate: date, Grade: integer, FacultyMember: string)
- Students
 - every record has 50 bytes
 - there are 80 records / page
 - 500 pages
- Courses
 - every record has 40 bytes
 - there are 100 records / page
 - 1 page
- Exams
 - every record has 40 bytes
 - there are 100 records / page
 - 1000 pages

IBM's System R Optimizer

- tremendous influence on subsequent relational optimizers
- design choices
 - use statistics to estimate the costs of query evaluation plans
 - consider only plans with binary joins in which the inner relation is a base relation
 - focus optimization on SQL queries without nesting
 - don't eliminate duplicates when performing projections (unless DISTINCT is used)

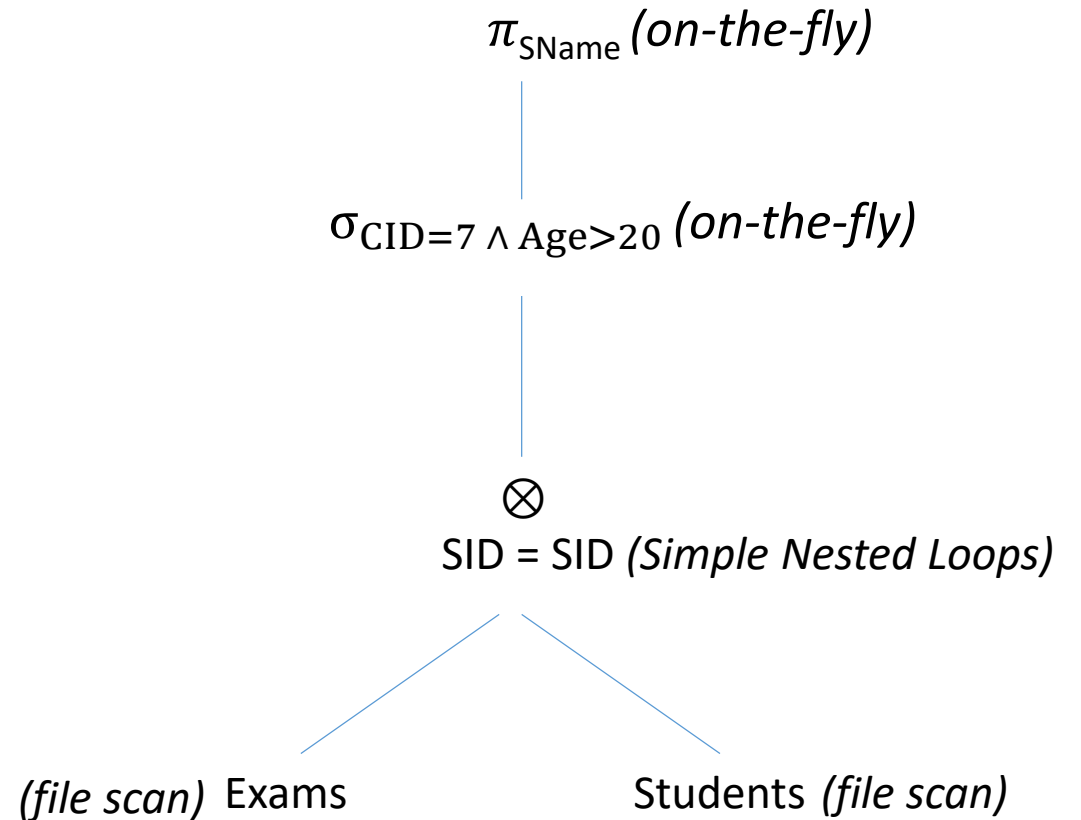
Motivating Example

* E - M pages * * 1000 pages *

* S - N pages * * 500 pages *

```
SELECT S.SName
FROM Exams E, Students S
WHERE E.SID = S.SID AND E.CID = 7
      AND S.Age > 20
```

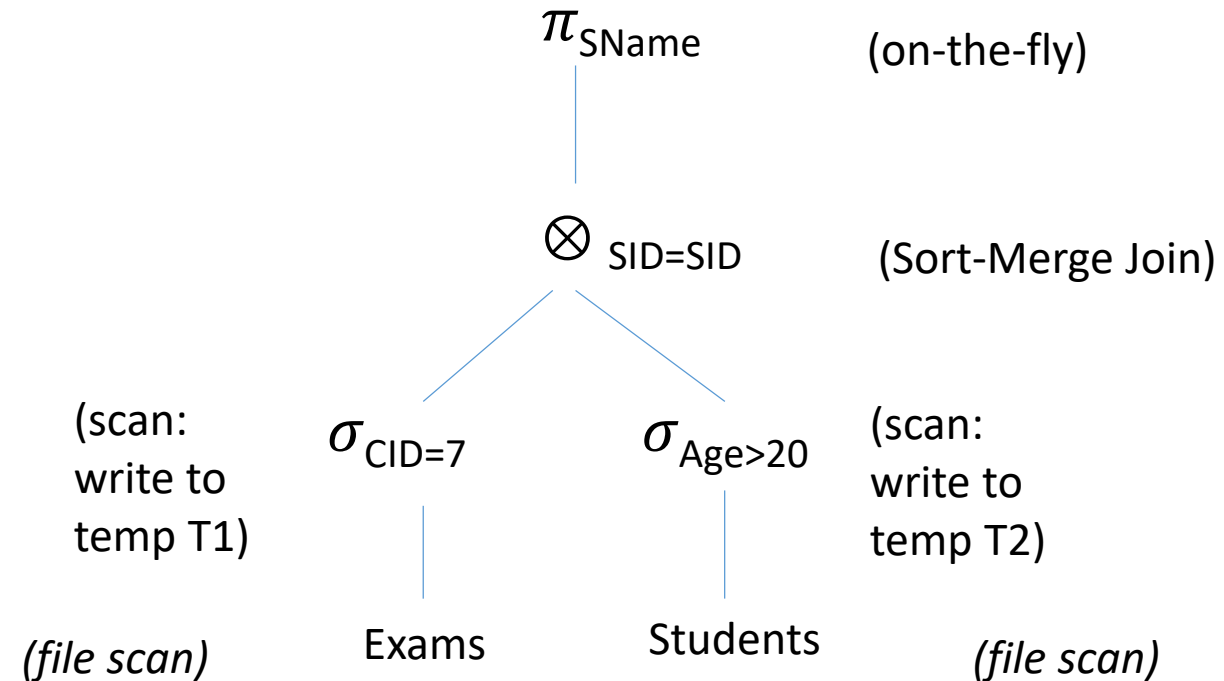
- σ, π – on-the-fly
- cost of plan – very high:
 - $1000 + 1000 * 500 = 501,000$ I/Os
- less efficient plan
 - join: cross-product followed by a selection



Motivating Example

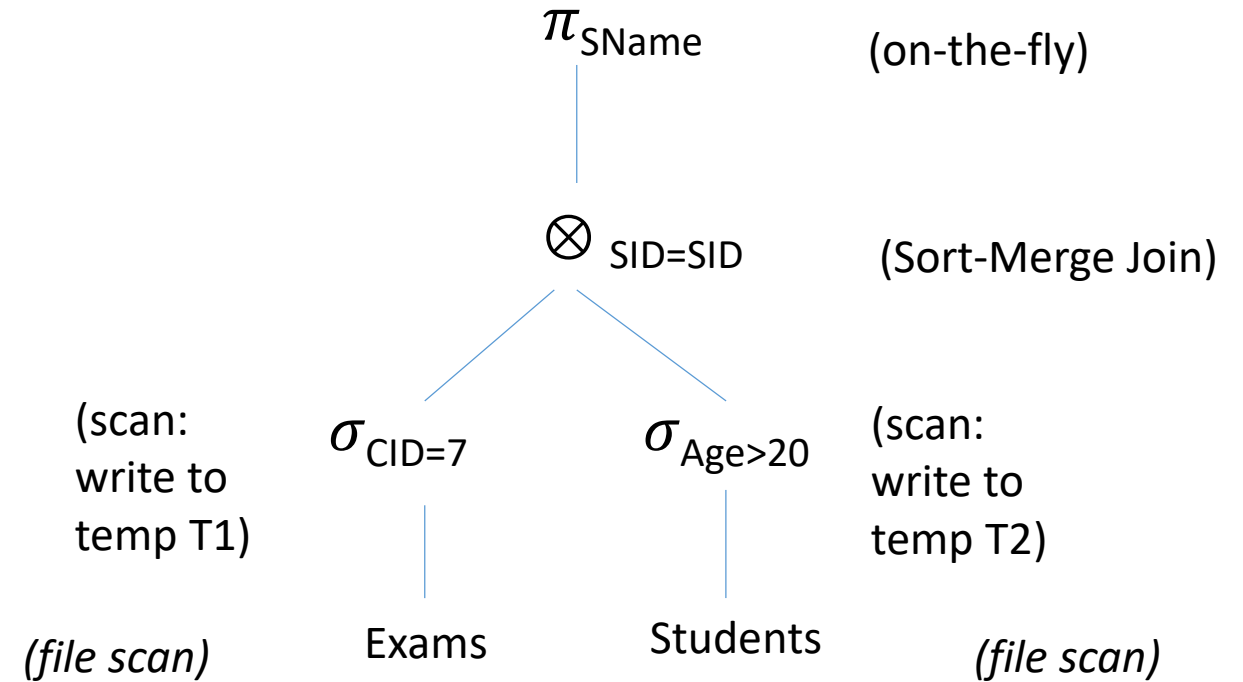
* optimizations

- reduce sizes of the relations to be joined
 - push selections, projections ahead of the join
- alternative plans
 - push selections ahead of joins
- selection
 - file scan
 - write the result to a temporary relation on disk
- join the temporary relations using Sort-Merge Join



Motivating Example

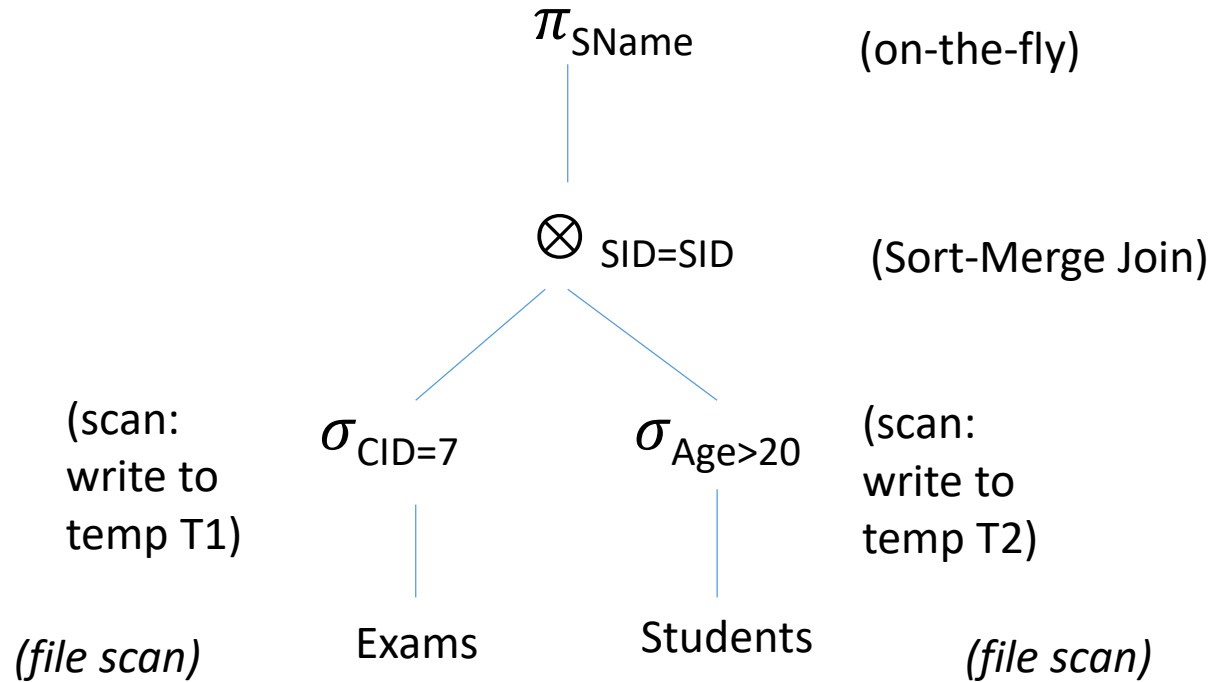
- 5 available buffer pages
- cost
 - $\sigma_{CID=7}$
 - scan Exams: 1000 I/Os
 - write T1
 - assume exams are uniformly distributed across all courses, i.e., T1 has 10 pages
 - $\sigma_{Age>20}$
 - scan Students: 500 I/Os
 - write T2
 - assume ages are uniformly distributed over the range 19 to 22, i.e., T2 has 250 pages



Motivating Example

- 5 available buffer pages
- cost
 - Sort-Merge Join
 - T1 - 10 pages
 - sort T1: $2 * 2 * 10 = 40$ I/Os
 - T2 - 250 pages
 - sort T2: $2 * 4 * 250 = 2000$ I/Os
 - merge sorted T1 and T2
 - $10 + 250 = 260$ I/Os
 - π - on the fly

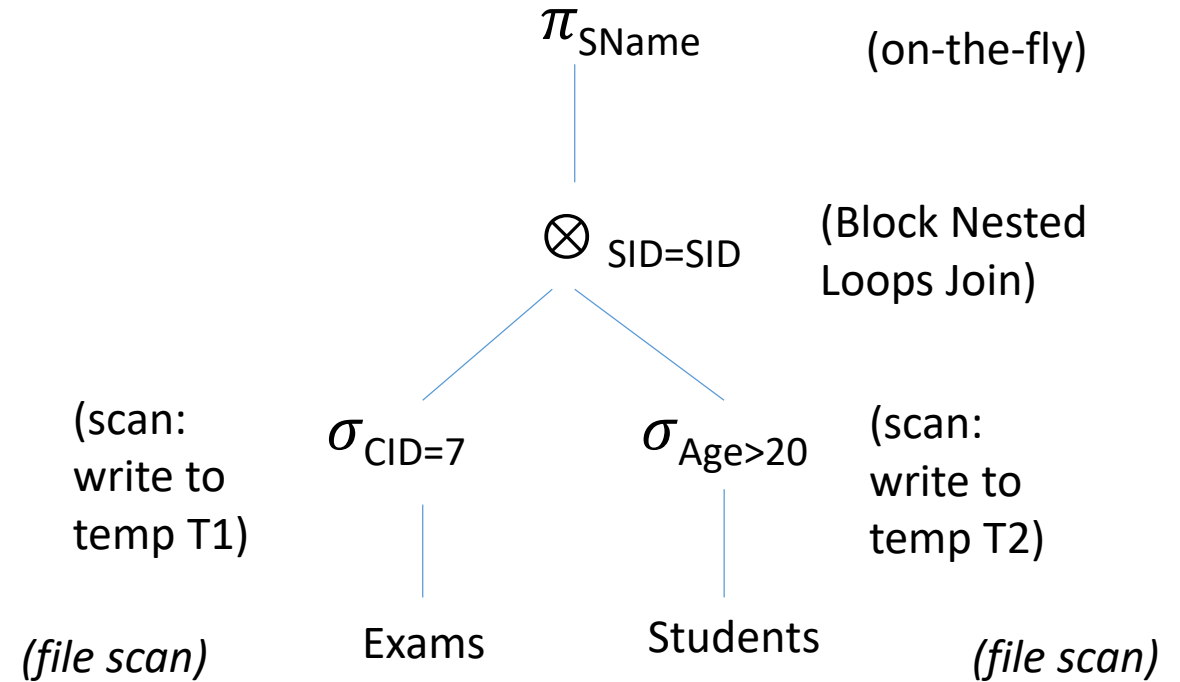
=> **total cost:** $\underbrace{1000 + 10 + 500 + 250}_{\text{selection}} + \underbrace{40 + 2000 + 260}_{\text{join}} = 4060 \text{ I/Os}$



Motivating Example

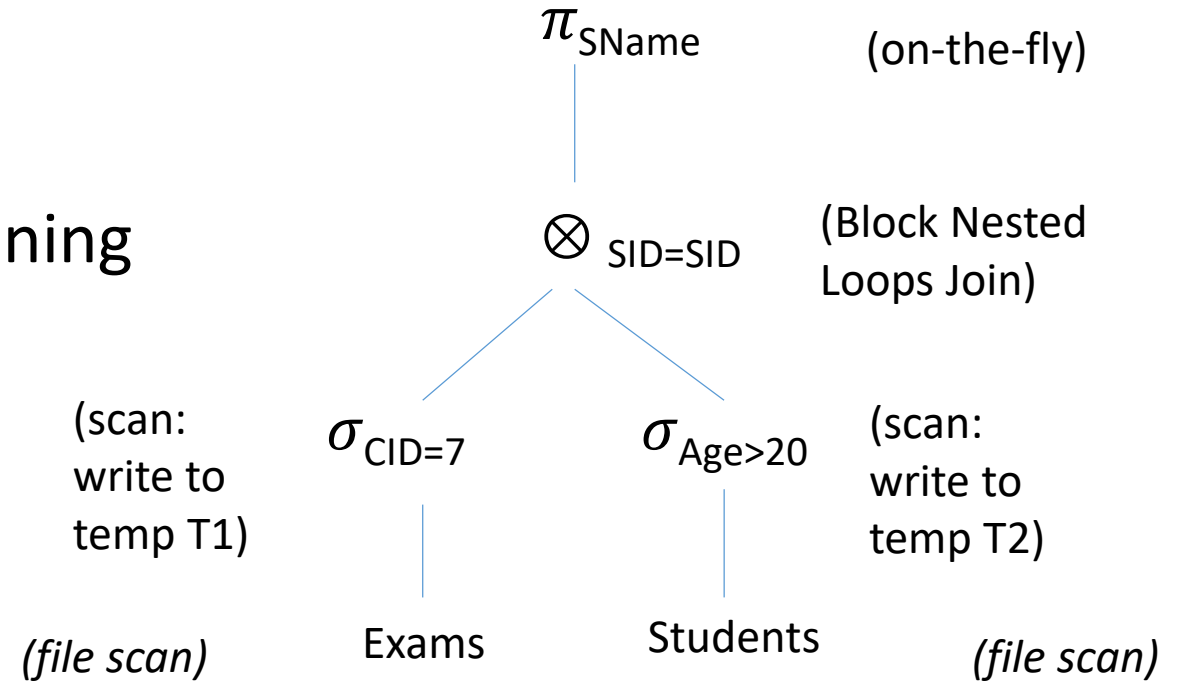
- 5 available buffer pages
- cost
 - Block Nested Loops Join
 - T1 - 10 pages, T2 - 250 pages
 - T1 - outer relation
 - => scan T1: 10 I/Os
 - $\lceil 10/3 \rceil = 4$ T1 blocks
 - => T2 scanned 4 times: $4 * 250 = 1000$ I/Os
 - BNLJ cost: $10 + 1000 = 1010$ I/Os
 - π - on the fly

=> **total cost:** $\underbrace{1000 + 10 + 500 + 250}_{\text{selection}} + \underbrace{10 + 1000}_{\text{join}} = \mathbf{2770 \text{ I/Os}}$



Motivating Example

- push projections ahead of joins
 - drop unwanted columns while scanning Exams and Students to perform the selections
=> T1[SID], T2[SID, SName]
- T1 fits within 3 buffer pages
=> T2 scanned only once
=> cost of BNLJ drops to under 250 I/Os
=> **total cost: \approx 2000 I/Os**



Motivating Example

* optimizations

- investigate the use of indexes

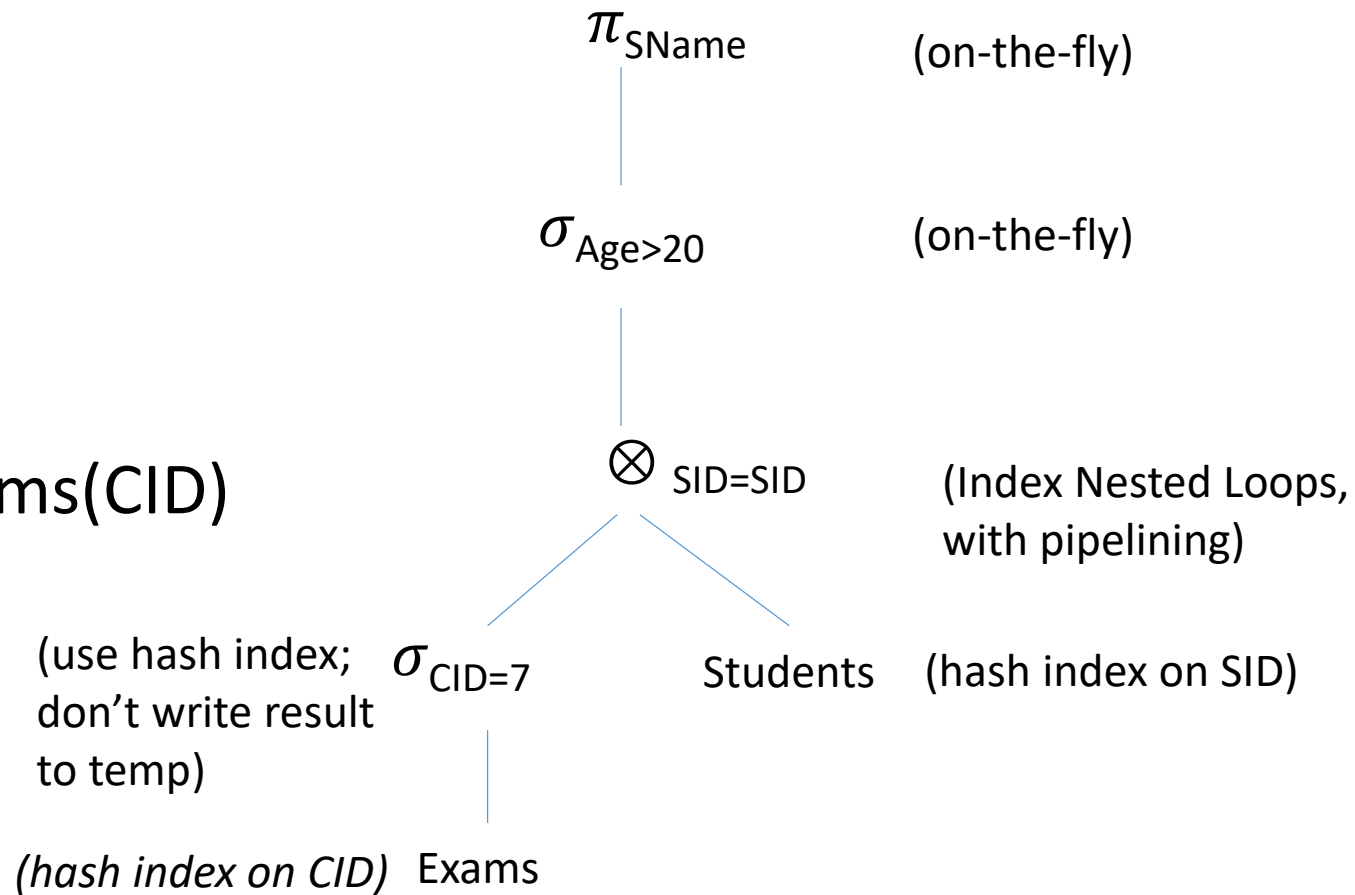
- clustered static hash index on Exams(CID)

- hash index on Students(SID)

• cost

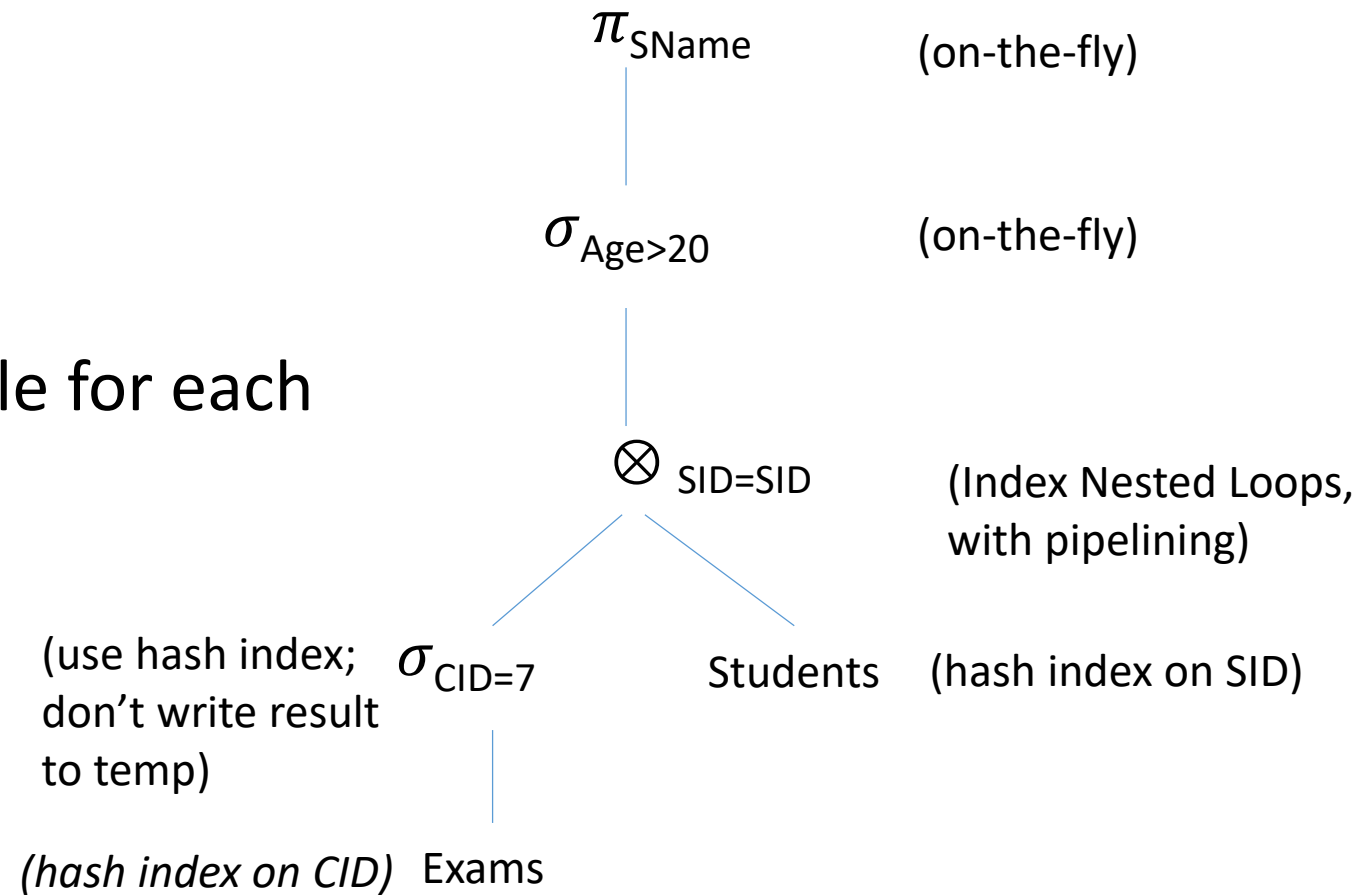
- $\sigma_{CID=7}$

- assume exams are uniformly distributed across all courses => 100,000 exams / 100 courses => 1,000 exams / course
- clustered index on CID => 1,000 tuples for course with CID=7 appear consecutively within the same bucket => cost: 10 I/Os
- the result of the selection is not materialized, the join is pipelined



Motivating Example

- cost
 - Index Nested Loops
 - find matching Students tuple for each selected exam
 - use hash index on SID
 - assume the index uses a1 => cost of 1.2 I/Os (on avg.)
 - σ, π – performed on-the-fly on each tuple in the result of the join

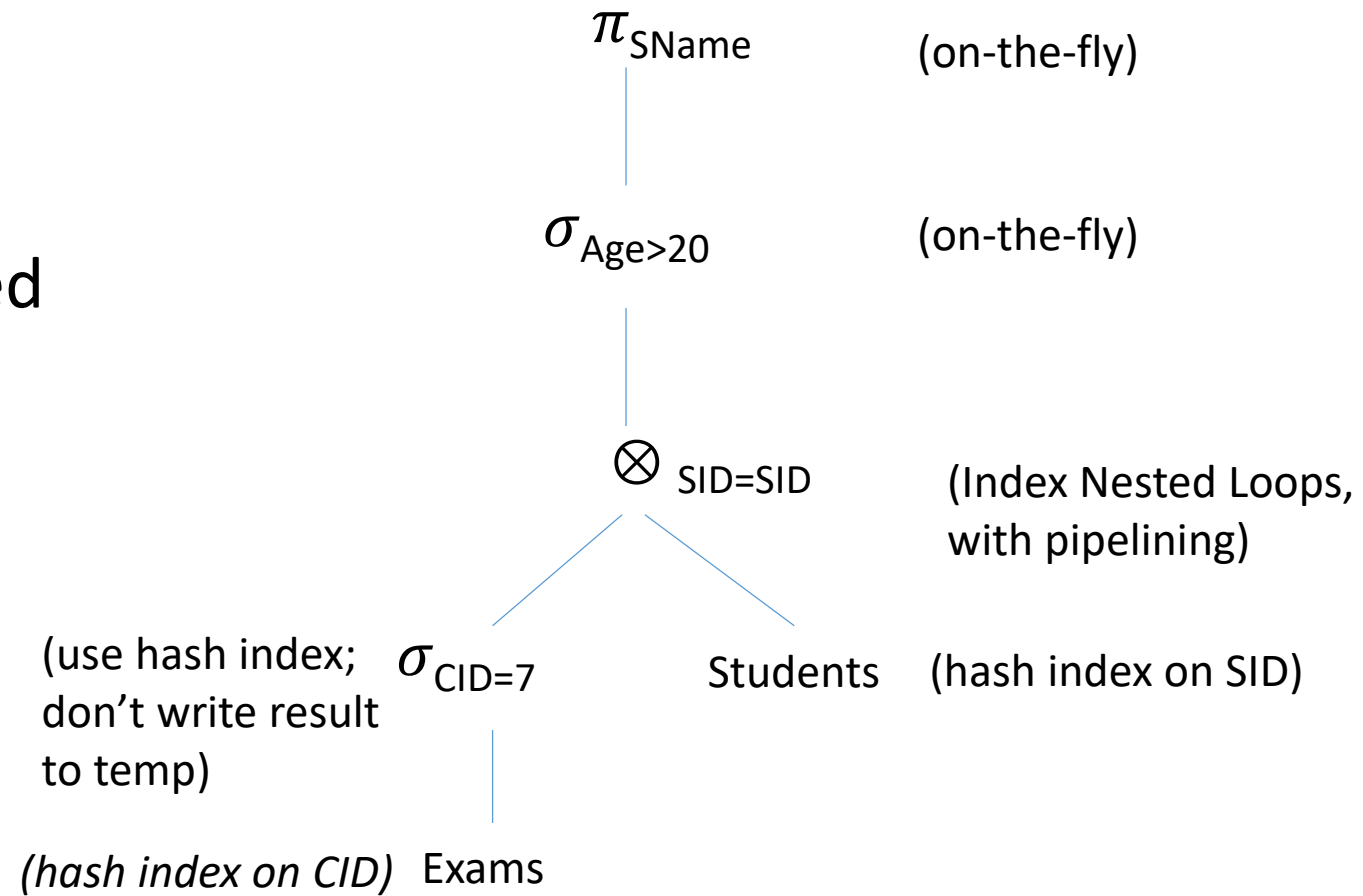


$$\Rightarrow \text{total cost} = \underbrace{10}_{\sigma \text{ on Exams}} + \underbrace{1000}_{\text{num. of Exams tuples}} * \underbrace{1.2}_{\text{find matching Students tuple (on avg.)}} = \mathbf{1210 \text{ I/Os}}$$

Motivating Example

* obs.

- the selection $Age > 20$ is not pushed ahead of the join in this case
- there is a hash index on SID in Students
- but there aren't any indexes on the result of the selection $Age > 20$



Estimating the Cost of a Plan

- * estimating the cost of an evaluation plan for a query block
 - for each node N in the tree:
 - estimate the cost of the corresponding operation
 - pipelining versus temporary relations
 - estimate the size of N's result and whether it is sorted
 - N's result is the input of N's parent node
 - these estimates affect the estimation of cost, size, and sort order for N's parent

Estimating the Cost of a Plan

- estimating costs
 - use data about the input relations (such statistics are stored in the DBMS's system catalogs)
 - number of pages, existing indexes, etc.
 - obtained estimates are at best approximations to actual sizes and costs
- => one shouldn't expect the optimizer to find the best possible plan
- optimizer - goals:
 - avoid the worst plans
 - find a good plan

Statistics Maintained by the DBMS

- updated periodically, not every time the data is changed
 - relation R
 - cardinality - $NTuples(R)$
 - the number of tuples in R
 - size - $NPages(R)$
 - the number of pages in R
 - index I
 - cardinality - $NKeys(I)$
 - the number of distinct key values for I
 - size - $INPages(I)$
 - the number of pages for I
 - B+ tree index
 - number of leaf pages

Statistics Maintained by the DBMS

- index I
 - height - $IHeight(I)$
 - maintained for tree indexes
 - the number of nonleaf levels in I
 - range - $ILow(I), IHigh(I)$
 - the minimum / maximum key value in I

Estimating Result Sizes

- query Q
SELECT attribute list
FROM relation list
WHERE $term_1 \wedge \dots \wedge term_k$
- the maximum number of tuples in Q's result:
 - $\prod |R_i|$, $R_i \in \text{relation list}$
- every $term_j$ in the WHERE clause eliminates some candidate tuples
 - associate a reduction factor RF_j with each term $term_j$
 - RF_j models the impact $term_j$ has on the result size
- estimate the actual size of the result:
 - $\prod |R_i| * \prod RF_j$
 - the maximum result size times the product of the reduction factors for the terms in the WHERE clause
- assumption: the conditions tested by the terms in the WHERE clause are statistically independent

Estimating Result Sizes

- compute reduction factors for terms in the WHERE clause
- assumptions:
 - uniform distribution of values
 - independent distribution of values in different columns

SELECT attribute list

FROM relation list

WHERE term₁ AND ... AND term_k

- *column = value*
 - index I on *column*
=> RF approximated by $1/NKeys(I)$
 - no index on *column*
=> RF: 1/10
 - maintain statistics on *column* (e.g., number of distinct values in *column*) to obtain a better value

Estimating Result Sizes

- *column1 = column2*
 - indexes *I1* on *column1*, *I2* on *column2*
=> RF: $1/\text{MAX}(\text{NKeys}(I1), \text{NKeys}(I2))$
 - only one index *I* (on one of the 2 columns)
=> RF: $1/\text{NKeys}(I)$
 - no indexes
=> RF: $1/10$
- *column > value*
 - index *I* on *column*
=> RF: $(\text{IHigh}(I) - \text{value}) / (\text{IHigh}(I) - \text{ILow}(I))$
 - no index on *column* or *column* not of an arithmetic type
=> a value less than 0.5 is arbitrarily chosen
 - similar formulas can be obtained for other range selections

Estimating Result Sizes

- *column IN (list of values)*
=> RF: (RF for *column = value*) * number of items in list (but at most 0.5)
- *NOT condition*
=> RF: 1 - RF for *condition*
- obtain better estimates
 - use more detailed statistics (e.g., histograms of the values in a column)

Relational Algebra Equivalences

- central role in generating alternative plans
- different join orders can be considered
- selections, projections can be pushed ahead of joins
- cross-products can be converted to joins
- selections
 - *cascading selections*
 - $\sigma_{c_1 \wedge \dots \wedge c_n}(R) \equiv \sigma_{c_1}(\dots(\sigma_{c_n}(R)))$
 - *commutativity*
 - $\sigma_{c_1}(\sigma_{c_2}(R)) \equiv \sigma_{c_2}(\sigma_{c_1}(R))$

Relational Algebra Equivalences

- projections
 - *cascading projections*
 - $\pi_{a_1}(R) \equiv \pi_{a_1}(\dots(\pi_{a_n}(R)))$
 - a_i – set of attributes in R
 - $a_i \subseteq a_{i+1}$, for $i = 1..n-1$

Relational Algebra Equivalences

- joins and cross-products
 - assumption
 - fields are identified by their name, not by their position
 - *associativity*
 - $R \times (S \times T) \equiv (R \times S) \times T$
 - $R * (S * T) \equiv (R * S) * T$
 - *commutativity*
 - $R \times S \equiv S \times R$
 - $R * S \equiv S * R$
 - can choose the inner / outer relation in a join

Relational Algebra Equivalences

- joins and cross-products
- e.g., check that $R * (S * T) \equiv (T * R) * S$
 - commutativity
 - $R * (S * T) \equiv R * (T * S)$
 - associativity
 - $R * (T * S) \equiv (R * T) * S$
 - commutativity
 - $(R * T) * S \equiv (T * R) * S$

Relational Algebra Equivalences

- can commute σ with π if σ uses only attributes retained by π
 - $\pi_a(\sigma_c(R)) \equiv \sigma_c(\pi_a(R))$
- can combine σ with \times to form a join
 - $R \otimes_c S \equiv \sigma_c(R \times S)$
- can commute σ with \times or a join when the selection condition includes only fields of one of the arguments (to the cross-product or join)
 - for instance:
 - $\sigma_c(R * S) \equiv \sigma_c(R) * S$
 - $\sigma_c(R \times S) \equiv \sigma_c(R) \times S$
 - condition c must include only fields from R
- in general: $\sigma_c(R \times S) \equiv \sigma_{c_1}(\sigma_{c_2}(R) \times \sigma_{c_3}(S))$
 - c_1 – attributes of both R and S
 - c_2 – only attributes of R
 - c_3 – only attributes of S

Relational Algebra Equivalences

- can commute π with \times
 - $\pi_a(R \times S) \equiv \pi_{a_1}(R) \times \pi_{a_2}(S)$
 - a_1 – attributes in a that appear in R
 - a_2 – attributes in a that appear in S
- can commute π with join
 - $\pi_a(R \bowtie_c S) \equiv \pi_{a_1}(R) \bowtie_c \pi_{a_2}(S)$
 - every attribute in c must appear in a
 - a_1 – attributes in a that appear in R
 - a_2 – attributes in a that appear in S
 - a doesn't contain all the attributes in c – generalization
 - eliminate unwanted fields, compute join, eliminate fields not in a
 - $\pi_a(R \bowtie_c S) \equiv \pi_a(\pi_{a_1}(R) \bowtie_c \pi_{a_2}(S))$
 - a_1 – attributes of R that appear in either a or c
 - a_2 – attributes of S that appear in either a or c

Enumeration of Alternative Plans

- query Q
 - consider a certain set of plans
 - choose the plan with the least estimated cost
 - algebraic equivalences
 - implementations techniques for Q's operators
- not all algebraically equivalent plans are enumerated (optimization costs would be too high)
- two main cases
 - queries with one relation in the FROM clause
 - queries with two or more relations in the FROM clause

Enumeration of Alternative Plans

- queries with one relation in the FROM clause
 - no joins; only σ , π , grouping, aggregate operations
 - one σ or π or aggregate operation \Rightarrow consider implementation techniques and cost estimates discussed in previous lectures
 - combination of several σ , π , aggregate operations
 - plans with / without indexes
- example query:

```
SELECT S.RoundedGPA, COUNT(*)  
FROM Students S  
WHERE S.RoundedGPA > 5 AND S.Age = 20  
GROUP BY S.RoundedGPA  
HAVING COUNT(DISTINCT S.SName) > 5
```

$\pi_{S.RoundedGPA, COUNT(*)}(\text{HAVING}_{COUNT\ DISTINCT(S.SName) > 5}(\text{GROUP BY}_{S.RoundedGPA}(\pi_{S.RoundedGPA, S.SName}(\sigma_{S.RoundedGPA > 5 \wedge S.Age = 20}(Students))))))$

Enumeration of Alternative Plans

* plans without indexes

- apply σ, π while scanning Students

- file scan

=> NPages(Students): 500 I/Os

- write out tuples to a temporary relation T

- $\text{NPages(Students)} * \text{RF(RoundedGPA} > 5) * \text{RF(Age} = 20) *$
(size of a *pair* $\langle \text{RoundedGPA}, \text{SName} \rangle$ / size of a *Students tuple*)

- RF for *RoundedGPA* > 5

- 0.5

- RF for *Age* = 20

- 0.1

- size of $\langle \text{RoundedGPA}, \text{SName} \rangle$

- $\approx 0.8 * \text{size of a Students tuple}$

=> $500 * 0.5 * 0.1 * 0.8 = 20$ I/Os (temporary relation T)

Enumeration of Alternative Plans

* plans without indexes

- GROUP BY
 - sort T in 2 passes
 - $4 * 20 = 80$ I/Os
- HAVING, aggregations
 - no additional I/O
- **total cost:** $500 + 20 + 80 = 600$ I/Os

Enumeration of Alternative Plans

* plans that use an index

- available indexes on Students - a2
 - hash index on <Age>
 - B+ tree index on <RoundedGPA>
 - B+ tree index on <RoundedGPA, SName, Age>
- single-index access path
 - choose the index that provides the most selective access path
 - apply π , nonprimary selection terms
 - compute grouping and aggregation operations
- example
 - use the hash index on Age to retrieve Students with Age = 20
 - cost: retrieve index entries and corresponding Students tuples

Enumeration of Alternative Plans

* plans that use an index

- single-index access path
 - apply condition *RoundedGPA* > 5 to each retrieved tuple
 - retain RoundedGPA and SName
 - write out tuples to a temporary relation
 - sort the temporary relation by RoundedGPA to identify groups
 - apply the HAVING condition (to eliminate some groups)

Enumeration of Alternative Plans

* plans that use an index

- multiple-index access path
 - several indexes using a_2 / a_3 match the selection condition, e.g., I_1 , I_2
 - retrieve $Rids_{I_1}$, $Rids_{I_2}$ using I_1 , I_2
 - get tuples with rids in $Rids_{I_1} \cap Rids_{I_2}$ (tuples satisfying the primary selection terms of I_1 and I_2)
 - apply π , nonprimary selection terms
 - compute grouping and aggregation operations
- example
 - use the index on Age \Rightarrow rids of tuples with Age = 20 (R_1)
 - index on RoundedGPA \Rightarrow rids of tuples with RoundedGPA > 5 (R_2)
 - retrieve tuples with rids in $R_1 \cap R_2$
 - keep only RoundedGPA and SName

Enumeration of Alternative Plans

* plans that use an index

- multiple-index access path
 - write out tuples to a temporary relation
 - sort the temporary relation by RoundedGPA to identify groups
 - apply the HAVING condition (to eliminate some groups)

Enumeration of Alternative Plans

* plans that use an index

- sorted index access path
 - works well when the index is clustered
 - B+ tree index I with search key K
 - GROUP BY attributes – prefix of K
 - use the index to retrieve tuples in the order required by the GROUP BY clause
 - apply σ , π
 - compute aggregation operations
- example
 - use the B+ tree index on RoundedGPA to retrieve Students tuples with $\text{RoundedGPA} > 5$, ordered by RoundedGPA
 - aggregations in HAVING, SELECT - computed on-the-fly

Enumeration of Alternative Plans

* plans that use an index

- index-only access path

- dense index I with search key K
- all the attributes in the query are included in K

=> index-only scan, don't need to retrieve tuples from the relation

- data entries
 - apply σ
 - perform π
 - sort the result (to identify groups)
 - compute aggregate operations

* obs.

- index I doesn't need to match the selections in the WHERE clause

Enumeration of Alternative Plans

- * plans that use an index
 - index-only access path
- * obs. I – tree index, GROUP BY attributes – prefix of K
 - => can avoid sorting
- example
 - use the B+ tree index on <RoundedGPA, SName, Age> to retrieve entries with RoundedGPA > 5, ordered by RoundedGPA
 - select entries with Age = 20
 - aggregation operations in the HAVING and SELECT clauses - computed on-the-fly

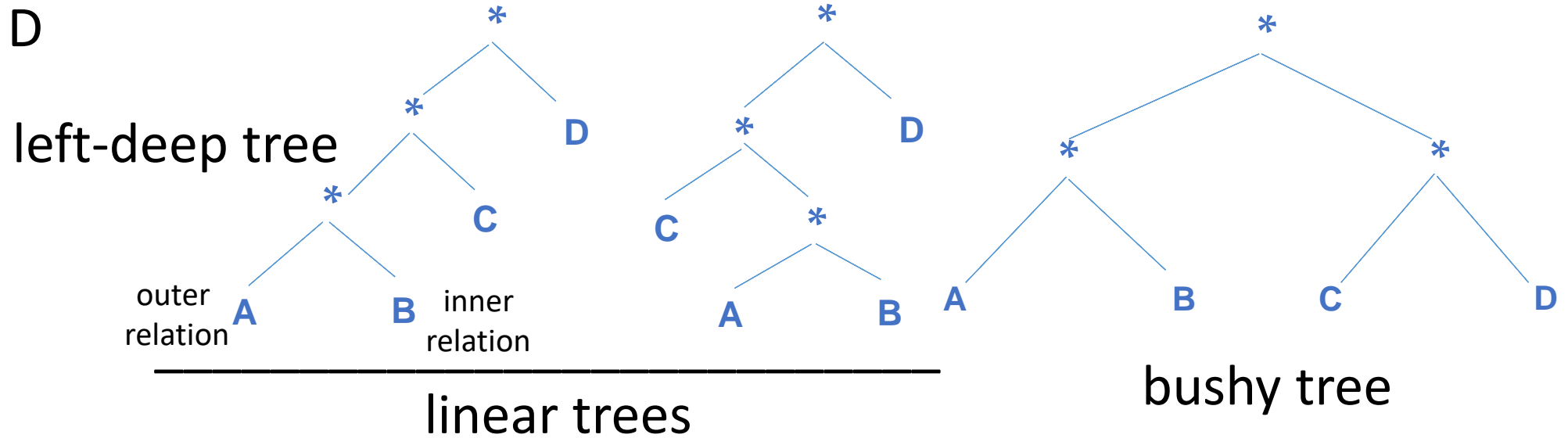
Enumeration of Alternative Plans

- queries with several relations in the FROM clause
 - joins, cross-products => queries can be quite expensive
 - different join orders => intermediate relations of widely varying sizes => plans with very different costs
- class of plans considered by the optimizer
- plan enumeration

Enumeration of Alternative Plans

- queries with several relations in the FROM clause

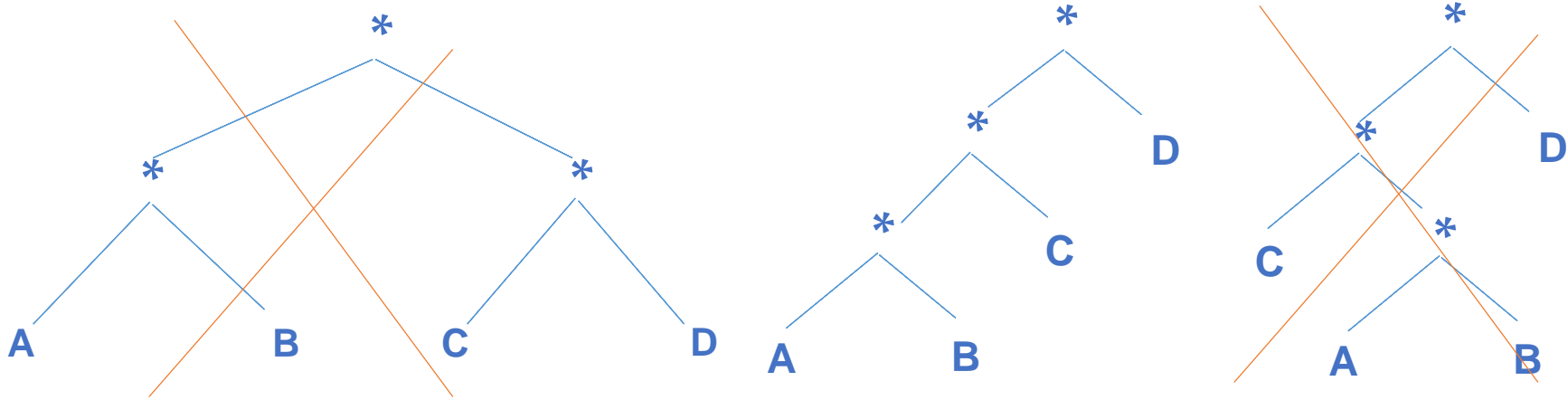
$A * B * C * D$



- linear trees
 - at least one child of a join node is a base relation
- left-deep trees
 - the right child of each join node is a base relation
- bushy tree – not linear

Enumeration of Alternative Plans

- queries with several relations in the FROM clause
- fundamental decision in System R
 - only *left-deep trees* are considered



- motivation
 - number of joins increases => number of alternative plans increases quickly => must prune the search space
 - left-deep trees
 - generate all fully pipelined plans (all joins are evaluated using pipelining)

Enumeration of Alternative Plans

* obs.

- not all plans based on left-deep trees are fully pipelined
- e.g., sort-merge join
 - outer tuples may have to be retrieved in a particular order, so the outer relation should be materialized

References

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