SWAMI KESHVANAND INSTITUTE OF TECHNOLOGY, MANAGEMENT & GRAMOTHAN, JAIPUR

Subject: Engineering. Mathematics-I [Course Code:1FY2-01]

ASSIGNMENT-2
(Multiple Integrals and Vector Differentiation)

PART-A (Compulsory)

Q.1 Evaluate the following integrals.

(i) $\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2+y^2)dx\,dy$ (ii) $\int_1^{\ln 8} \int_0^{\ln y} e^{x+y} dx\,dy$ (iii) $\int_0^2 \int_{3y^2-6y}^{2y-y^2} 4xy\,dx\,dy$ (iv) $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy\,dx$

Q.2 Check whether $f(r)\vec{r}$ is irrotational or not?

Q.3 Find the value of $div(curl\,\vec{F})$ where $\vec{F} = 2xy\hat{i} + (xyz^2 - \sin yz)\hat{j} + ze^{x+y}\hat{k}$.

Q.4 Find the value of λ for which the vector $\vec{u} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+\lambda z)\hat{k}$ is a solenoidal vector.

Q.5 If $\vec{A} = x^2yz\,\hat{i} + xyz^2\,\hat{j} + y^2z\,\hat{k}$ Find curl curl $\vec{A}$.

Q.6 Find a unit vector normal to the surface $x^2y + 4xz = 4$ at the point (2, -2, 3).

Q.7 By changing the order of integration evaluate $\int_0^1 \int_x^1 \sin(x^2)\,dx\,dy$

Q.8 Changing the order of integration in the double integral $I = \int_0^8 \int_{\frac{x}{4}}^2 f(x,y)dydx$ leads to $I = \int_r^s \int_p^q f(x,y)dxdy$. What is the value of s and q.

PART-B
(Analytical / Problem solving questions)

Q.9 Find the area of the region R in the xy-plane enclosed by the circle $x^2 + y^2 = 4$ above the line $y = 1$ and below the line $y = \sqrt{3}x$.

Q.10 If $\vec{A} = (x+2y+az)\hat{i} + (bx-3y-z)\hat{j} + (4x+cy+2z)\hat{k}$, find a,b,c so that $\vec{A}$ is irrotational. Also find the scalar potential of $\vec{A}$.

Q.11 Find the mass, coordinates of centre of gravity and moments of inertia relative to x-axis y-axis and origin of a rectangle $0 \le x \le 4, 0 \le y \le 2$ having mass density xy.

Q.12 Determine the volume of the region below the paraboloid $x^2+y^2+z=4$ and above the square in the xy-plane with vertices at $(0,0),(0,1),(1,0),(1,1)$.

Q.13 Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} (a^2-x^2-y^2)\,dx\,dy.$

Q.14 Find the values of the constants a,b,c so that the directional derivative of $\phi = axy^2+byz+cz^2x^3$ at $(1,2,-1)$ has a maximum of magnitude 64 in the direction parallel to the z-axis.

Q.15 If $\vec{a}$ is a constant vector point function, show that $\nabla \times \left(\frac{\vec{a}\times\vec{r}}{r^3}\right) = -\frac{\vec{a}}{r^3} + \frac{3(\vec{a}.\vec{r})}{r^5}\vec{r}.$

Q.16 Evaluate the double integral $\iint_R \frac{y}{1+x^2}\,dA$ where $R = \{(x,y)\,|\,0 \le x \le 4 \; and \; 0 \le y \le \sqrt{x}\}$

PART-C
(Descriptive/ Analytical/ Problem solving/ Design Questions)

Q.17 Find the volume under the surface $z = 2x + y^2$ and above the region bounded by $x = y^2$ and $x = y^3$.

Q.18 Evaluate $I = \iint_R \left(6x+2y^2\right) dA$

where R is the region enclosed by the parabola $x = y^2$ and the line $x + y = 2$

Q.19 Evaluate the following integral by change the order of Integration.

$$\int_0^a \int_{\sqrt{ax}}^a \frac{y^2}{\sqrt{y^4-y^2x^2}}\,dydx.$$

Q.20 A triangular prism is formed by planes whose equations are $ay = bx,\; y = 0$ and $x = a$.

Find the volume of the prism between the planes $z = 0$ and the surface $z = c + xy$.

Q.21 Find the constants a and b so that the surface $ax^2 - byz = (a+2)x$ will be orthogonal to

the surface $4x^2y + z^3 = 4$ at the point (1,-1,2).