SWAMI KESHVANAND INSTITUTE OF TECHNOLOGY, MANAGEMENT AND GRAMOTHAN, JAIPUR

Subject: Engineering. Mathematics-I [Course Code:1FY2-01]

ASSIGNMENT-3
(Vector integration, Beta and Gamma Function)

Section A(Short answer questions)

Q.1 State Gauss Divergence theorem.

Q.2 State Green's Theorem.

Q.3 Evaluate $\int_0^{\pi/2} \sqrt{tan\theta}\, d\theta$

Q.4 Evaluate $\int_0^{\infty} 4x^4 e^{-x^4} dx$

Q.5 Evaluate B(1/2,1/2)

Section B (Analytic / Problem solving questions)

Q.6 Evaluate $\iiint\limits_V f\ dV$ where $f = 2x + y$, V is the closed region bounded by the cylinder $z = 4 - x^2$ and the planes $x = 0, y = 0, y = 2$ and $z = 0$.

Q.7 Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy\hat{i} - 5z\,j + 10xk$ along the curve $x = t^2 + 1,\quad y = 2t^2,\quad z = t^3$ from $t = 1$ to $t = 2$.

Q.8 Evaluate $\int_0^{\infty} \frac{x^2\left(1+x^4\right)}{\left(1+x\right)^{10}} dx$

Q.9 Prove $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$

Q.10 Evaluate $\int_0^{\pi/2} \sec^{1/2} x \sin^{8/3} x dx$

Section C(Descriptive / Analytical questions)

Q.11 Prove that $\int_0^{\pi/2} \cos^m \theta \sin^n \theta d\theta = \frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{m+n+2}{2}\right)}$

Q.12 Show that if $c > 1$ $\int_0^{\infty} \frac{x^c}{c^x} dx = \frac{\Gamma(c+1)}{(\log c)^{c+1}}$

Q.13 Verify Gauss's Divergence theorem and show that $\int_s \vec{F}.\hat{n}\,ds = \frac{1}{3}a^5$, where $\vec{F} = \left(x^3 - yz\right)\hat{i} - 2x^2y\,\hat{j} + 2\hat{k}$, S is the surface of the cube bounded by the coordinate planes: $x = y = z = 0; x = y = z = a$

Q.14 Verify Stokes's theorem for $\vec{F} = (x^2 + y - 4)\hat{i} + 3xy\,\hat{j} + (2xy + z^2)\hat{k}$ over the surface of hemisphere $x^2 + y^2 + z^2 = 16$ above the xy-plane.

Q.15 Verify Green's theorem in a plane for $\vec{F} = \left(3x^2 - 8y^2\right)\hat{i} + \left(4y - 6xy\right)\hat{j}$ and C is the region bounded by the parabolas $y = x^2$ and $x = y^2$.