Swami Keshvanand institute of technology Management and Gramothan- Jaipur

Department of Mathematics

Question Bank

Topic: Vector Integration **Faculty: Dr. Vijay Singhal**

1. Evaluate $\iint_S \vec{F}.n\,ds$; where $\vec{F} = x^2\hat{i} + y^2\,j + z^2k$ and S is the part of plane $x+y+z=1$; which is located in first octant.

2. Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy\hat{i} - 5z\,j + 10xk$ along the curve $x = t^2+1, \quad y = 2t^2, \quad z = t^3$ from $t=1$ to $t=2$.

3. Suppose the force field $\vec{F} = \nabla f$ is the gradient of the function $f(x,y,z) = -\dfrac{1}{(x^2+y^2+z^2)}$. Find the work done by F in moving an object along a smooth curve C joining (1, 0, 0) to (0, 0, 2) that do not passes through the origin.

4. If $\vec{F} = 2z\hat{i} - x\,j + yk$ evaluate $\iiint_V \vec{F}.dV$ where V is the region bounded by the surfaces $x=0,\ y=0,\ x=2,\ y=4,\ z=x^2,\ z=2$.

5. Prove that $\vec{F} = \left(2xy + z^3\right)\hat{i} + x^2\,j + 3xz^2k$ is a conservative force field. Also find the scalar potential.

6. If $\vec{F} = y\,\hat{i} - x\hat{j},$ evaluate $\int_C \vec{F}.d\vec{r}$ from (0, 0) to (1, 1) along the following paths.

(i) The parabola $y = x^2$. Ans: -1/3
(ii) The straight lines from (0, 0) to (1, 0) and then to (1, 1) Ans: -1
(iii) The straight line joining (0, 0) to (1, 1) Ans: 0

7. Evaluate $\iint_s \vec{F}.\hat{n}\,ds$ where $\vec{F} = \dfrac{\vec{r}}{r^3}$ and S is the sphere $x^2+y^2+z^2=1$. Ans: 4π

8. Find the constant a so that $\vec{V}$ is a conservative vector field, where $\vec{V} = \left(axy - z^3\right)\hat{i} + \left(a-2\right)x^2\,\hat{j} + \left(1-a\right)xz^2\,\hat{k}$. Calculate its potential and work done in moving a particle from $(1,2,-3)$ to $(1,-4,2)$ in this field.

9. If $\vec{F} = x\,\hat{i} + y\,\hat{j}$ then find the value of $\iint_s \vec{F}.\hat{n}\,ds,$ where S is the unit sphere i.e. $x^2+y^2+z^2=1$ above the XY plane. Ans: $4\pi/3$

10. Evaluate $\iint_s \vec{F}.\hat{n}\,ds,$ where $\vec{F} = z\,\hat{i} + x\,\hat{j} - 3y^2z\,\hat{k}$ and S is the surface of the cylinder $x^2+y^2=16$ included in the first octant between $z=0$ and $z=5$. Ans: 90

11. If $\vec{F} = 4xz\,\hat{i} - y^2\,\hat{j} + yz\hat{k}$ evaluate $\iint_S \vec{F}\,\hat{n}\,ds,$ where S is the surface of the cube bounded by

$x = 0, x = b; y = 0, y = b; z = 0, z = b.$ Ans: $3b^4/2$

12. Find the flux of the vector field $\vec{F} = (x-2z)\hat{i} + (x+3y+z)\hat{j} + (5x+y)\hat{k}$ through the upper side of the triangle ABC with vertices at the points A(1,0,0), B(0,1,0), C(0,0,1)

[Hint: Flux across the triangle ABC = $\iint_S \vec{F}.\hat{n}\, ds$] Ans: 5/3

13. Evaluate $\iiint_V f\ dV$ where $f = 2x + y$, V is the closed region bounded by the cylinder $z = 4 - x^2$ and the planes $x = 0, y = 0, y = 2$ and $z = 0$. Ans: 80/3