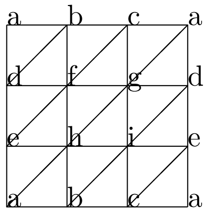
MATH5665: Algebraic Topology (2015,S1)
Problem Set 2 [1]

This problem set covers material from lectures 4-8.

1. Consider the triangulation of $|K| \longrightarrow \mathbb{T}^2$ given by the labelled surface diagram below (as given in e.g. 5.4).



   Show that $\gamma = [ah] + [hg] + [ga]$ is a 1-cycle in K. Express the corresponding homology class $\gamma + B_1(K)$ as a $\mathbb{Z}$-linear combination of the canonical generators $\alpha = [ab] + [bc] + [ca], \beta = [ad] + [de] + [ea]$.

2. Fix a simplicial complex K giving a triangulation of the Klein bottle. Compute the homology of K.

3. Fix a simplicial complex K giving a triangulation of the real projective plane. Compute the homology of K.

4. After lecture 9, you may wish to repeat the last two questions, but with co-efficients over the field $\mathbb{F}_p$ where p is prime.

5. Prove proposition 5.2, that $H_0(K) \simeq \mathbb{Z}^n$ where n is the number of connected components of $|K|$.

6. Consider the sequence of abelian groups (and group homomorphisms) below
$$0 \longrightarrow \mathbb{Z}/4\mathbb{Z} \xrightarrow{2} \mathbb{Z}/4\mathbb{Z} \xrightarrow{2} \mathbb{Z}/4\mathbb{Z} \longrightarrow 0$$
   where the 2 indicates the multiplication by 2 map $a+4\mathbb{Z} \mapsto 2(a+4\mathbb{Z}) = 2a + 4\mathbb{Z}$. Show it is a complex and compute its homology.

[1] by Daniel Chan

7. Let $f_1, f_2 : (X, A) \longrightarrow (Y, B)$ be continuous maps of pairs and $F : f_1 \approx f_2$ be a homotopy. Given $X' \subset X$, we say F is a homotopy *relative to* X' if furthermore, $F(x', t) = F(x', 0)$ for all $x' \in X'$, in otherwords, the functions $F(x', -) : I \longrightarrow Y$ are constant for all $x' \in X'$. Show that if $f_1, f_2 : I \longrightarrow I$ are any two maps with $f_i(0) = 0, f_i(1) = 1$, then f_1, f_2 are homotopic relative to $\{0, 1\}$.

8. Compute the homology of the suspension $S(K^{(1)}_{abc})$.