School of Mathematics and Statistics University of New South Wales

MATH5665: Algebraic Topology (2015,S1)
Problem Set 3 [1]

This problem set covers material from lectures 9-13.

1. Compute the Betti numbers of the Klein bottle and the projective plane.

2. Show that the relation of being isomorphic in a category is an equivalence relation. (You probably should restrict to the case where the class of objects form a set. These are called *small categories*.)

3. Show that $\tilde{H}_0 : \underline{\text{Simp}} \longrightarrow \underline{\text{Ab}}$ is a functor.

4. Prove that the figure 8 is not homeomorphic to S^1.

5. Let $f : S^2 \longrightarrow S^2$ be rotation about some axis through angle θ. Compute $\deg f$.

6. (Argument principle in complex analysis). Let $f : \mathbb{C} \longrightarrow \mathbb{C}$ be an entire function. We consider $S^1 = \{z \in \mathbb{C} || z| = 1\}$ and let $r : \mathbb{C} - 0 \longrightarrow S^1 : z \mapsto \frac{z}{|z|}$ be the usual retraction. We assume that f has no zeros on S^1. Let $h = rf : S^1 \longrightarrow S^1$. Show that $\deg h$ is the number of zeros (counting multiplicity) of f in the unit disc, which is also of course

$$\frac{1}{2\pi i} \int_{S^1} \frac{f'(z)}{f(z)} dz.$$

[1] by Daniel Chan