School of Mathematics and Statistics University of New South Wales

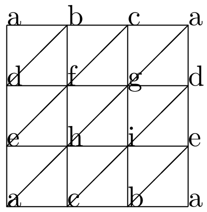
MATH5665: Algebraic Topology (2015,S1)
Assignment 1 [1]

This assignment is due during the week 5 wednesday lecture.

1. Let $A = \{a, b, c, d\}$ be ordered in alphabetical order.
   (a) Which of the following defines a simplicial complex:
   $$K = \{a, b, c, d, ab, bc, bd\} \quad \text{or,}$$
   $$K' = \{a, b, c, d, ab, bc, abc\}$$
   (b) For each of the simplicial complexes in part (a), draw a figure which is homeomorphic to the polytope of that simplicial complex.

2. Show that the following sequence of abelian groups is a chain complex and compute all of its homology groups.
   $$0 \longrightarrow \mathbb{Z} \xrightarrow{\phi} \mathbb{Z} \oplus \mathbb{Z} \xrightarrow{\psi} \mathbb{Z} \longrightarrow 0$$
   where $\phi(n) = (n, -2n), \psi(m, n) = 2m + n$.

3. Consider the triangulation $\theta : |K| \longrightarrow X$ of the Klein bottle given by the labelled surface diagram below.



   Compute the homology groups $H_p(K)$. Hint: Follow the example of the torus in lecture 5.

4. Let $s_\bullet$ be a chain homotopy between chain maps $f_{0\bullet}, f_{1\bullet} : C_\bullet \longrightarrow C'_\bullet$. Let $g_\bullet : C'_\bullet \longrightarrow C''_\bullet$ be another chain map. Prove that $g_\bullet f_{0\bullet}, g_\bullet f_{1\bullet}$ are also chain homotopic, i.e. there is a chain homotopy between them.

[1] by Daniel Chan

5. Let $\theta : |K| \longrightarrow X$ be a triangulation of a topological space. Consider the quotient space $Y = (X \times I)/\sim$ where $\sim$ is the equivalence relation generated by $(x, 1) \sim (x', 1)$ for all $x, x' \in X$. Find a triangulation for Y.