UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF MATHEMATICS AND STATISTICS

Assignment 2

Algebraic Topology

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Question 1

Proposition 1. *Let $\mathcal{C}$, $\mathcal{D}$ and $\mathcal{E}$ be categories, and $\mathcal{F} : \mathcal{C} \to \mathcal{D}$ and $\mathcal{G} : \mathcal{D} \to \mathcal{E}$ are functors. Then the composite $\mathcal{GF} : \mathcal{C} \to \mathcal{E}$, defined by $(\mathcal{GF})(x) = \mathcal{G}(\mathcal{F}(x))$ for $x \in \mathrm{Obj}(\mathcal{C})$ and $(\mathcal{GF})(f) = \mathcal{G}(\mathcal{F}(f))$ for a morphism f, is a functor.*

Proof. It is necessary to prove,

1. If $x \in \mathrm{Obj}(\mathcal{C})$, then $(\mathcal{GF})(\mathrm{id}_x) = \mathrm{id}_{\mathcal{GF}(x)}$.
2. If f and g are morphisms in $\mathcal{C}$ such that gf is defined, then $(\mathcal{GF})(gf) = (\mathcal{GF})(g)(\mathcal{GF})(f)$.

To prove 1, we simply compute,

$$\begin{aligned}(\mathcal{GF})(\mathrm{id}_x) &= \mathcal{G}(\mathrm{id}_{\mathcal{F}(x)}) \\ &= \mathrm{id}_{\mathcal{GF}(x)}.\end{aligned}$$

Similarly, we prove 2,

$$\begin{aligned}(\mathcal{GF})(gf) &= \mathcal{G}(\mathcal{F}(g)\mathcal{F}(f)) \\ &= (\mathcal{GF})(g)(\mathcal{GF})(f).\end{aligned}$$

□