School of Mathematics and Statistics University of New South Wales

MATH5665: Algebraic Topology (2015,S1)
Problem Set 1 [1]

This problem set covers material from lectures 1-3 and also has some revision material concerning abelian groups.

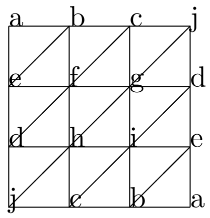
1. Recall that *free abelian group with generators* $\sigma_i, i \in I$ denoted
$$\bigoplus_i \mathbb{Z}\,\sigma_i =: F$$
consists of all formal finite linear combinations of the σ_i. If $I = \{1, \ldots, r\}$ we write this as $\mathbb{Z}\,\sigma_1 \oplus \ldots \oplus \mathbb{Z}\,\sigma_r$. Show that in this case $F \simeq \mathbb{Z}^r$.

2. Given an abelian group A and $a_1, \ldots, a_s \in A$. We denote the *subgroup generated by* $a_1, \ldots, a_s$ by $\mathbb{Z}\,a_1 + \ldots + \mathbb{Z}\,a_s$. Let $A = \mathbb{Z}\,\sigma_1 \oplus \ldots \oplus \mathbb{Z}\,\sigma_r$ and $B = \mathbb{Z}(\sigma_1 - \sigma_2) + \ldots + \mathbb{Z}(\sigma_{r-1} - \sigma_r)$. Show that $B \cap \mathbb{Z}\,\sigma_1 = 0$ and that $B = \sum_{i,j=1}^{r} \mathbb{Z}(\sigma_i - \sigma_j)$.

3. Show that the span of $a_0, \ldots, a_n$ is their convex hull.

4. Find a labelled surface diagram for the Klein bottle.

5. Is the following a labelled surface diagram for the real projective plane?



[1] by Daniel Chan