

First task consists of 5 questions. Try doing them as rigorously as possible.

Q1: Let $\zeta(z) = \sum_{n=1}^{\infty} n^{-z}$. At which domain ζ is complex differentiable? Extend ζ to the domain $\{\Re z > 0, z \neq 1\}$ as complex differentiable function.

Q2: Let $f_1, f_2 : \mathbb{C} \rightarrow \mathbb{C}$ be complex differentiable. Can $f = \frac{f_1}{f_2}$ have infinitely many poles within unit ball?

Q3: Find a conformal mapping from (the interior of) ellipse to a unit ball.

Q4: Let D be a unit ball. Let H be the set of all complex differentiable function on D which are square integrable. Does the norm $f \rightarrow (\int_D |f|^2)^{1/2}$ make H a Banach space?

Q5: Find a function f complex differentiable at a unit ball such that $\lim_{z \rightarrow z_0} f(z)$ does not exist at every boundary point z_0 of a unit ball.