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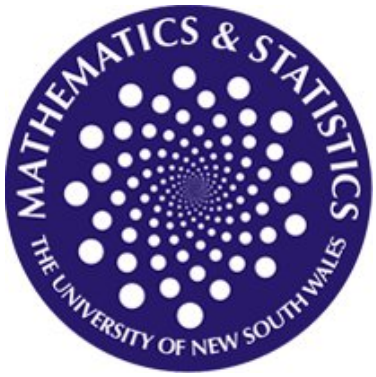

University of New South Wales

School of Mathematics and Statistics

Assignment 1

Complex Analysis

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Question 1

For this question, define the function

$$\zeta(z) = \sum_{n=1}^{\infty} n^{-z}.$$

Lemma 1. *ζ is defined and complex differentiable on the domain $\{z \in \mathbb{C} : \mathrm{Re}(z) > 1\}$*

Now define the function $\zeta_1(z) =$...

Lemma 2. *$\zeta_1(z) = \zeta(z)$ for all $\mathrm{Re}(z) > 1$.*

Theorem 1. *ζ_1 is defined for $z \in \{z : \mathrm{Re}(z) > 0, z \neq 1\}$.*

Question 2

For this question, $f_1, f_2 : \mathbb{C} \to \mathbb{C}$ are complex differentiable with $f_2 \neq 0$.

Theorem 2. *The function $f = f_1/f_2$ cannot have infinitely many poles within the unit ball.*

Proof. If f is undefined precisely when $f_2 = 0$, so the poles of f within the unit ball must be zeros of f_2 within the unit ball.

However, since $f_2 \neq 0$, the zeros of f_2 are a discrete set.

By the Bolzano-Weierstrass theorem, any infinite set in the open unit ball has a limit point in the closed unit ball. Hence if f_2 has infinitely many zeros inside the unit ball, the zeros of f_2 are not discrete and this is a contradiction. □

Question 3