NAME: ....
STUDENT NO.: ....

The University of New South Wales

School of Mathematics and Statistics

MATH5535 Lie Groups S2 2014

ASSIGNMENT 1

This assignment is due at 4pm on Friday 5^{th} September.

Your answers can be typed or written in neat hand writing and may be handed to the lecturer in person or submitted via Moodle.

If you submit your answers to the lecturer on paper, this question paper must be **attached to the front** of your answers and you **must** give your name (PRINT BLOCK LETTERS) and student number above and you **must** complete the declaration below.

Your assignment MUST BE YOUR OWN WORK.

Declaration
I declare that this assessment item is my own work, except where acknowledged, and has not been submitted for academic credit elsewhere, and acknowledge that the assessor of this item may, for the purpose of assessing this item:

- Reproduce this assessment item and provide a copy to another member of the University; and/or
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I certify that I have read and understood the University Rules in respect of Student Academic Misconduct.

Signed: .... Date: ....

Please turn over for the questions.

1. Using only linearity and the Leibniz rule for derivations, show that if X and Y are derivations then $[X, Y] = XY - YX$ is a derivation.

2. For smooth manifolds M and N with a smooth map $f : M \to M'$, the vector field $X \in \mathfrak{X}(M)$ is said to be f-related to $X' \in \mathfrak{X}(M')$ if $df(X_p) = X'_{f(p)}$ for all $p \in M$. Show that if $X, Y \in \mathfrak{X}(M)$ are f-related to $X', Y' \in \mathfrak{X}(M')$ then $[X, Y]$ is f-related to $[X', Y']$.

3. Let G and H be Lie groups with identity elements e_G and e_H. If $f : G \to H$ is a Lie group homomorphism show that $df_{e_G} : T_{e_G}G \to T_{e_H}H$ is a Lie algebra homomorphism.

4. The Heisenberg group $\mathbb{H}$ can be considered to be the set of matrices

$$\mathbb{H} = \left\{ \begin{pmatrix} 1 & a & c \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} \ : \ x, b, c \in \mathbb{R} \right\}$$

with matrix multiplication as the group operation.

   (a) Show that $\mathbb{H}$ is a Lie subgroup of $GL(n, \mathbb{R})$.

   (b) Find a basis for the left invariant vector fields of $\mathbb{H}$ and determine the Lie bracket on this basis.

   (c) If $\mathfrak{h}$ is the Lie algebra of $\mathbb{H}$, find $[\mathfrak{h}, \mathfrak{h}]$, the Lie subalgebra of $\mathfrak{h}$ consisting of all Lie brackets of pairs of elements in $\mathfrak{h}$. What is $[\mathfrak{h}, [\mathfrak{h}, \mathfrak{h}]]$?