MATH 5645

TOPICS IN NUMBER THEORY. 2014

Assignment 4. Due Week 9.

Marks will be deducted for poor and illogical setting out or for solutions that are unnecessarily complicated or obscure.

1a. Use the character table given in lectures for $\mathbb{Z}_5$, extended to a Dirichlet character, to evaluate

$\sum_{i=1}^{4} \chi_i(n)\overline{\chi_i(b)}$, for each $b \in \mathbb{U}_5$.

b. Use the results of (a) to prove, in detail, that there are infinitely many primes congruent to 1 mod 5, 2 mod 5, and 3 mod 5 and 4 mod 5.

You should derive the following (or similar) bounds on the L functions: $\frac{1}{6} < L(s, \chi_2) < 1$ if $s > 1$ and $\frac{3}{4} < \Re(L(s, \chi_3), \Re(L(s, \chi_4) < 1$ as part of your proof.

2. Let χ be any Dirichlet character. Then, for $s > 1$, prove that

$$\frac{1}{L(s, \chi)} = \sum_{n=1}^{\infty} \frac{\chi(n)\mu(n)}{n^s}.$$

3. Suppose χ_4 and χ_6 are the (unique) non-principal characters modulo 4 and 6 respectively.

Show that $L(1, \chi_4) = \dfrac{\pi}{4}$ and $L(1, \chi_6) = \dfrac{\pi}{2\sqrt{3}}$.