Fuzzy Multiple Moderation and Moderated-Mediation Analysis

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# Abstract

Mechanisms that become more complicated nowadays are explained better by models with multiple moderators than a simple model. Although expressing vague data by crisp numbers can increase the possibility of missing information, most of the analysis of the models above have been conducted with crisp numbers instead of fuzzy numbers so far. In this paper, we newly define fuzzy multiple moderators analysis (FMMA) that contains analysis of multiple moderation model and moderated-mediation model. In FMMA, values with ambiguity were transformed to triangular fuzzy numbers and we estimate regression coefficients with fuzzy least squares estimation. In addition, fuzzy T-test is proposed to judge whether the coefficient is significant, and fuzzy F-test and fuzzy are suggested as model evaluation indicators. They are compared with original version of classical multiple moderator analysis (CMMA).

**Keywords:** Fuzzy multiple moderation, Fuzzy moderated moderated-mediation, Fuzzy least squares estimation, Fuzzy T-test

# 1 Introduction

The causality that occurs in social phenomena related to human has been explained by the relationship between the independent variable () and independent variable (). Mediation and moderation analysis are common statistical methods that add a third variable respectively, a mediator and a moderator [1], to account for the causal relationship in various fields that is more complicated. Those analyses have been studied as an integrated model, moderated-mediation, where a moderator moderates the indirect effect of on through a mediator .

It is obvious that using models with multiple moderators is usually proper to explain mechanisms because the real world is complicated [4]. For example, human mind that affects behavior is influenced by many other factors, not just one factor. Like the example, if multiple moderators control the effect of on , the model is called a multiple moderation [2], and two representative model of the multiple moderation model is multiple additive moderation model and moderated moderation model. Each moderator controls the effect of on respectively and independently in the first model. In the second model, the moderation of the effect of on by a moderator depends on the other moderator. Multiple moderators can also control the indirect effect of on via in mediation analysis. If two moderators influence the indirect effect independently by moderating same path as or , the model is called partial moderated-mediation [3]. On the other hand, if the secondary moderator controls the moderation of indirect effect of on by the primary moderator, the model is called moderated moderated-mediation [3]. Analysis of these models have been done by many researchers so far, but it should be done more actively since we can express more complicated mechanisms with these models.

Analysis of above models have been conducted on crisp numbers although many values have ambiguity that we cannot express them precisely with real numbers. Ignoring their fuzziness can cause the loss of information, so expressing ambiguous data with fuzzy numbers, introduced by Zadeh [10], is better than crisp numbers. Research of fuzzy analysis of simple models and moderated-mediation model were studied previously [5, 6]. However, fuzzy analysis of more complicated models that include multiple moderators have not been conducted yet. In this paper, we suggest several statistical methods of fuzzy analysis of models with multiple moderators to stress the necessity of fuzzy numbers. Crisp numbers that are ambiguous are changed to triangular fuzzy numbers and we apply -estimation and define fuzzy T-test to estimate regression coefficients [5-8] and judge if each product has a linearity to the dependent variable. Fuzzy F-test and fuzzy are proposed as model evaluation criterions.

# 2 Multiple Moderation Analysis & Moderated-Mediation Analysis with Multiple Moderators

# 2.1 Moderated Moderation Model

This model is shown in Fig. 1. In this model, the moderation of the effect of on by a moderator () is adjusted by the other moderator (). The regression equation of this model can be expressed as follows:

The conditional effect of on can be presented as follows:

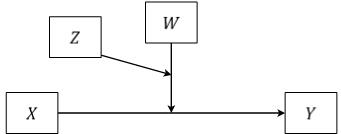
The coefficient of quantifies ’s moderation effect of the moderation by , namely three-way interaction. The significance of this moderated moderation effect can be determined by the significance test of .

# 2.2 Moderated Moderated-Mediation Model

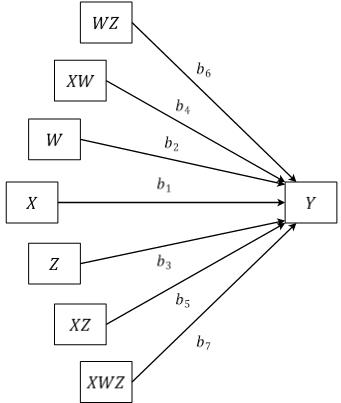
This model is illustrated in Fig. 2. Both moderators controls the indirect effect of on in path. The regression equations are expressed as follows:

The conditional indirect effect is expressed as follows:

quantifies the amount of change of the indirect effect moderated by when is constant. In particular, is called the index of moderated moderated mediation, and its significance determines whether the moderated moderated-mediation effect exists.

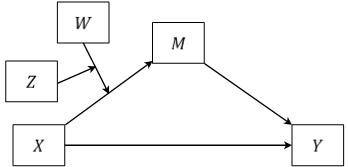


(a) Conceptual diagram

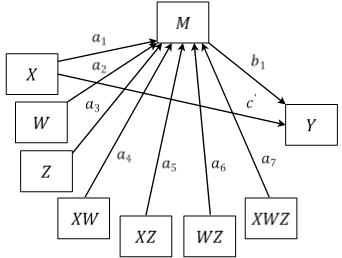


(b) Statistical diagram

Figure 1:A moderated moderation model



1. Conceptual diagram



1. Statistical diagram

Figure 2: A first stage moderated moderated-mediation model

# 3 Proposed Fuzzy Multiple Moderation Analysis & Fuzzy Mediation Analysis

**3.1 Fuzzy Numbers**

Some basic concepts of fuzzy numbers [10] are shown in this section. A fuzzy subset can be expressed with membership function that matches to . This fuzzy set is called a fuzzy number if the fuzzy set is convex and continuous in . -fuzzy numbers are usually used in fuzzy analysis, and its specific form, namely a triangular fuzzy number, is used in this paper. A fuzzy number can be expressed by , where represents the mode of and mean the left and right spread respectively. The spread and express the fuzziness of the number. We used following operations [10] for fuzzy triangular numbers.

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**3.2 Fuzzy Moderated Moderation Model**

Fuzzy moderation analysis was first introduced previously [6]. The moderation effect of on by is controlled by , whose regression equation can be represented as follows:

Fuzzy conditional effect (FCE) of on () can be expressed as . represents ’s moderation of the moderation effect by .

**3.3 Fuzzy Moderated Moderated-Mediation Model**

The regression equation of this model is expressed as follows:

The conditional indirect effect can be expressed as follows:

The meanings of some coefficients are same with those of CMMA.

**3.4 Fuzzy Model analysis based on Distance Approach**

Methods of subtraction on fuzzy numbers vary. In order to analyze the fuzzy Model consisting of regression analysis with the same result, one operation must be defined. Therefore, studies on the fuzzy model were conducted by defining fuzzy L2-distance instead of the square of the subtraction. [7, 9]

**3.4.1 Regression Coefficient Estimation for Fuzzy Model**

For the Fuzzy Least Squared Estimation Method, the following operations were defined in previous study [7], and the regression coefficients of the fuzzy model are estimated using them.

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Through algebraic properties and these operations, the fuzzy regression coefficient is defined as

**3.4.2 and F-tests for Fuzzy Model**

Using the distance approach described above, fuzzy FTSS, FSSE, and FSSR are defined, and the model is evaluated using fuzzy and F-test [9].

*Fuzzy*

**3.4.3 Proposed the Fuzzy T-tests for Fuzzy Model**

We extend studies related to the fuzzy model, and redefine the fuzzy T-statistic. This roughly follows the t-distribution.

The T-test is conducted with this statistic to determine the significance of each coefficient. Through this, the coefficient significance of crisp and fuzzy analysis can be compared.

**4. Data Analysis**

This data set contains information on the weather in Antwerp, Belgium, and the amount of electricity produced by solar panels every day [11, 12]. The variables "X" (Temp), "Z" (Wind), and "M" (Humidity') represent the daily average value, and "W" (Sky Cover) shows the sky cover at 3 p.m. by scoring the linguistic expression on 1-8 scales. "Y" (Day Power) is the amount of electricity produced in one day. Variables for weather, such as temperature and humidity, are not really fixed values but are constantly changing. In addition, "W" is preferably expressed as a fuzzy number because it converts linguistic expressions into integers. Therefore, the value of dividing the difference in the next day's data in half was used as the spread of variables for weather, and the spread of "W" was set to 1. After converting them into triangular fuzzy numbers, fuzzy analysis was conducted using model in Fig.3. The estimated regression coefficients and respective p-values of crisp analysis (CMMA) and fuzzy analysis (FMMA) are shown in Table 1. As a result of FMMA, the absolute value of most coefficients gradually decreased and the p-value slightly increased, but the two variables

도표이(가) 표시된 사진

자동 생성된 설명

(a) Conceptual diagram

도표이(가) 표시된 사진

자동 생성된 설명

(b) Statistical diagram

**Fig. 3.** Fuzzy moderated moderated-mediation model for solar data

showed different results. The coefficient of increased by 0.5348 from 0.4950. In other words, the direct effect from X to Y was actually greater, but it was underestimated in the analysis using crisp data. In addition, the sign of , the influence from W to M, changed from negative to positive and became more significant. In fact, the larger the value of "M" (humidity), the larger the value of "W" (sky cover) due to the more clouds and fog, but it was misinterpreted in CMMA. CMMA and FMMA can be compared with the fuzzy R2 and fuzzy F-test proposed above. In this model, R2 (0.662) is larger than fuzzy R2 (0.645). This does not mean that CMMA is better than FMMA. This result means that it was interpreted exaggeratedly when analyzed with CRISP data, excluding information ambiguity. When looking at the change in F-statistic, it decreased slightly from 1299.55 (CMMA) to 1206.88 (FMMA), but both models are judged to be significant models by F-test.

Table 1**:** Estimated coefficients and p-values from moderated moderated-mediation model using SOLAR data

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Method | Estimated coefficients | | | | | | | | |
|  |  |  |  |  |  |  |  |  |  |
| CMMA |  |  |  |  |  |  |  |  |  |
| FMMA |  |  |  |  |  |  |  |  |  |
|  | p-values |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| CMMA |  |  |  |  |  |  |  |  |  |
| FMMA |  |  |  |  |  |  |  |  |  |

**5. Conclusion**

Analysis of models that includes multiple moderators have been conducted by crisp numbers so far. Even though ambiguous values cannot be exactly expressed by real numbers, most of analysis were done with them. In this paper, fuzzy analysis of models with multiple moderators, namely FMMA, was proposed with several data. Seeing the outcomes of analysis, first of all, some regression coefficients were increased or their sign changed unlike the others. Moreover, the conditional effects controlled by moderators were slightly exaggerated in CMMA than FMMA. It is also noteworthy that F-statistic and was slightly different from those of fuzzy version. Although fuzzy decreased than , it is not the evidence for the fact that fuzzy model is poor, but the result is distorted because of the loss of information. Based on these results, it is obvious that reflecting fuzziness by using fuzzy numbers in analysis shows more exact results.

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