Kristoffer Skjelanger

*Department of Physics and Technology*
*University of Bergen*

June 2022

# QCD Parton Fragmentation in Vacuum and Medium with Leading Jet Energy-Loss

# Introduction

## Motivation:

Investigate the leading parton distribution in jets.

**Why:**

- Cleaner probe of quark-gluon plasma.
- Improve studies of Jet-quenching.

**What we need:**

- Knowledge of jets and parton showers.
- Analytical and numerical methods.
- Start by considering the inclusive parton distribution.

# Overview

1. Fundamentals
   - QCD
   - Parton branching
   - Jets
2. Analytical
   - The evolution equations in vacuum
   - The evolution equations in medium
3. Numerical
   - Monte-Carlo generated parton showers
4. Leading Parton and Energy-Loss
   - Energy-loss models in medium
   - Leading branches
   - Leading parton evolution equations

# Fundamentals

QCD is a field theory described by the Yang-Mills Lagrangian,

$$\mathcal{L}_{QCD} = \bar{\Psi}^f(x)\left[i\not{D} - m_f\delta_{ij}\right]\Psi^f(x) - \frac{1}{4}G_{i\mu\nu}(x)\,G_i^{\mu\nu}(x). \tag{1}$$

With the tree-level Feynman diagrams:

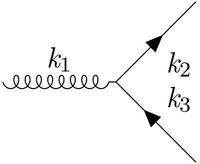



Figure: Gluon splitting into a $q\bar{q}$-pair.

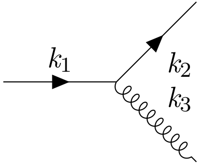



Figure: Quark radiating a gluon.

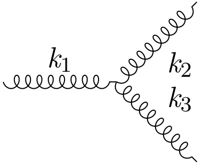



Figure: Gluon radiating a gluon.

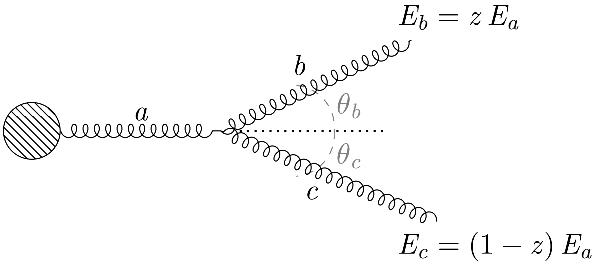



Figure: Gluon $a$ branching into two gluons $b$, $c$. The opening angle is given as $\theta = \theta_b + \theta_c$.

$$P_{gg}(z) = C_A \left[ \frac{1-z}{z} + \frac{z}{1-z} + z(1-z) \right] \tag{2}$$

The probability of $g \to gg$ branching in vacuum is

$$d\mathcal{P}_{1\to 2} \sim \frac{\alpha_s C_A}{\pi} \frac{d\theta}{\theta} \frac{dz}{z}. \tag{3}$$

This highlights the *collinar* and *soft* limits in the branching process.

# Jets

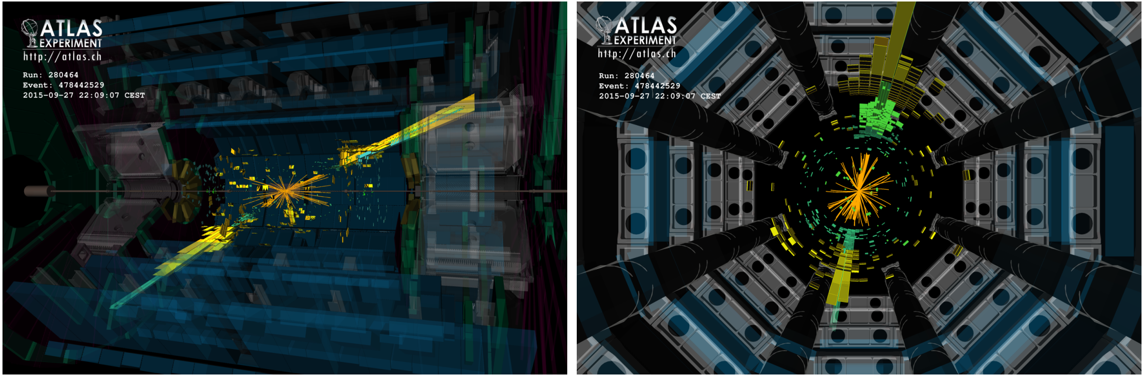



Figure: Dijet event at ATLAS 2015 [1].

[1] ATLAS Collaboration (Dec. 2015). "ATLAS event at 13 TeV - Highest mass dijets angular event in 2015 data". General Photo. URL: https://cds.cern.ch/record/2113240

**Jets in vacuum**

- Created in collisions such as $e^+e^-$ and heavy-ion.
- Parton branchings are angular ordered.
- Well understood.

**Jets in medium**

- Created in relativistic heavy-ion collisions.
- Dominated by soft gluon emissions.
- Broadening and jet quenching due to medium-interactions.

- Inclusive parton distribution:

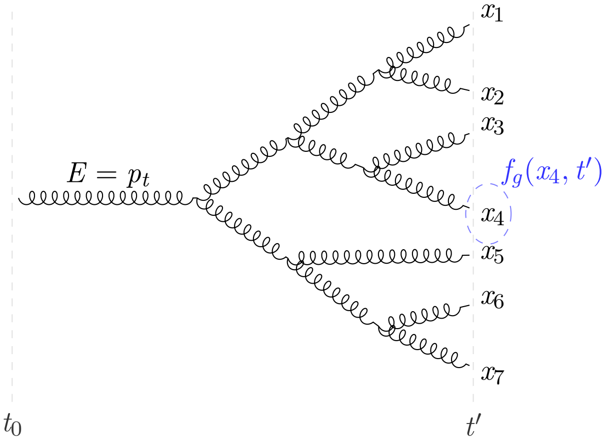
$$f_{q/g}(x,t) = \frac{dN}{dx}$$

such that $\int_0^1 dx\, f_i(x,t) = \langle N_i \rangle$.

- Inclusive energy distribution:

$$D_{q/g}(x,t) = x\frac{dN}{dx}$$

such that $\int_0^1 dx\, D_i(x,t) = 1$.

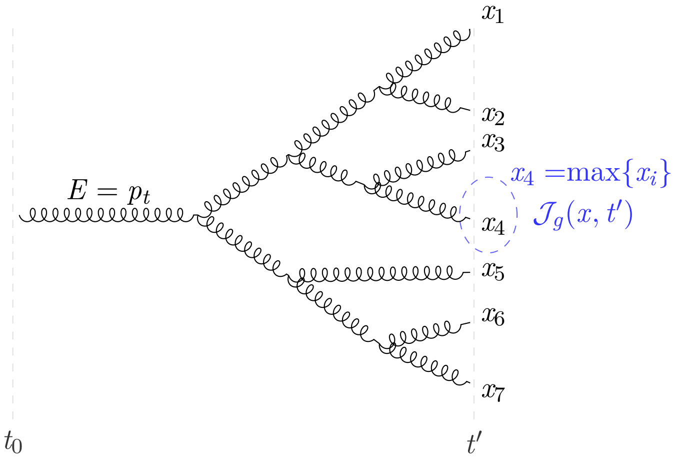


- Leading parton distribution:

$$\mathcal{J}_{q/g}(x,t) = \frac{dN}{dx'}$$

when $x' = \max\{x_i\}$.

- Number of partons is conserved $\int_0^1 dx\, \mathcal{J}_i(x,t) = 1$.
- Energy is not conserved $\int_0^1 dx\, x\, \mathcal{J}_i(x,t) = \langle x \rangle$.



# Analytical

DGLAP equation for gluons only:

$$\frac{\partial}{\partial t} f_g(x,t) = \int_x^1 \frac{dz}{z} 2\, P_{gg}(z) f_g\left(\frac{x}{z}, t\right) - \int_0^1 dz\, P_{gg}(z) f_g(x,t) \qquad (4)$$

Evolution variable (dimensionless)

$$t = \frac{\alpha_s}{\pi} \int_{\theta_{\min}}^{\theta} \frac{d\theta'}{\theta'}, \qquad R \geq \theta \geq Q_0/p_t \qquad (5)$$

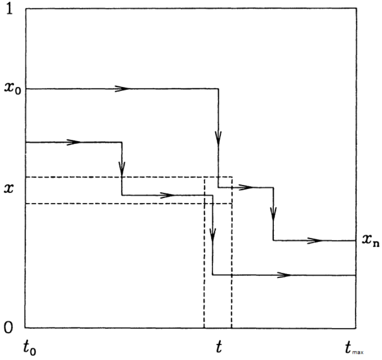



Figure: Partons entering and leaving a volume element $\delta x \delta t$.[2]

[2] Ellis, Stirling, and Webber 2011.

- In the soft limit (small $x$ and large $t$).
- Using a simplified splitting function

$$P_{gg}^{\text{simple}}(z) = \frac{C_A}{z(1-z)}. \tag{6}$$

- Using the Saddle-Point Approximation.

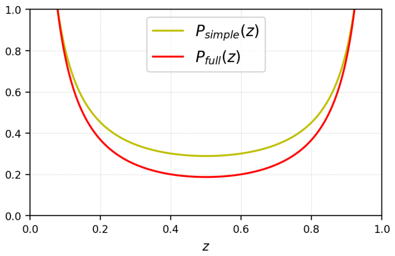



Figure: Vacuum $gg$ splitting functions.

The solution is then

$$D(x,t) = \exp\left(2\sqrt{2C_A}\sqrt{t\ln\frac{1}{x}} - 2C_A\gamma_E\, t\right)\frac{1}{2}\left(\frac{2C_A\, t}{\pi^2\ln^3(1/x)}\right)^{1/4}. \tag{7}$$

Evolution equation for gluons only:

$$\frac{\partial}{\partial \tau} D(x,\tau) = \int_x^1 dz\, \mathcal{K}_{gg}(z) \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \tau\right) - \int_0^1 dz\, \mathcal{K}_{gg}(z) \frac{z}{\sqrt{x}} D(x,t) \tag{8}$$

- Evolution variable ($t$ is now time),

$$\tau = \frac{t}{t_*} = \bar{\alpha} \sqrt{\frac{\hat{q}}{E}}\, t \tag{9}$$

- Splitting function,

$$\mathcal{K}_{gg}(z) = \frac{\left[1 - z(1-z)\right]^{5/2}}{\left[z(1-z)\right]^{3/2}}. \tag{10}$$

- Using a simplified splitting function

$$\mathcal{K}_{gg}^{\text{simple}}(z) = \frac{1}{[z(1-z)]^{3/2}}. \tag{11}$$

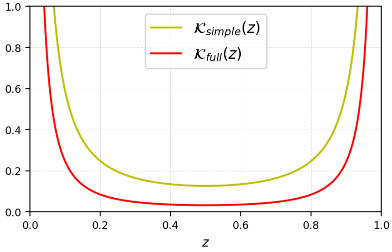



Figure: Medium $gg$ splitting functions.

The solution is then

$$D(x,\tau) = \frac{\tau}{\sqrt{x}(1-x)^{3/2}} \exp\left(-\pi \frac{\tau^2}{1-x}\right). \tag{12}$$

**Sudakov in Vacuum**

$$\Delta(t) = \exp\left(-t\int_{\epsilon}^{1-\epsilon} P_{gg}(z)\,dz\right) \tag{13}$$

**Sudakov in Medium**

$$\Delta(\tau) = \exp\left(-\frac{\tau}{\sqrt{x}}\int_{\epsilon}^{1-\epsilon} dz\, z\mathcal{K}(z)\right) \tag{14}$$

- $\epsilon$ controls the divergences in the splitting functions.
- $\Delta(t)$ can be interpreted as a no-branching probability.
- Will be useful when creating Monte-Carlo parton showers.

# Numerical

1. When can we expect a new parton branching?
2. How do we assign energy-fractions in a splitting?
3. When do we terminate the parton shower?

1. When can we expect a new parton branching?
2. How do we assign energy-fractions in a splitting?
3. When do we terminate the parton shower?

$\Rightarrow$ Can be determined directly from the Sudakov form factor.

1. When can we expect a new parton branching?
2. How do we assign energy-fractions in a splitting?
3. When do we terminate the parton shower?

$\Rightarrow$ Can be determined directly from the Sudakov form factor.

$\Rightarrow$ Can be sampled from the splitting functions.

# Creating a parton shower program

1. When can we expect a new parton branching? $\Rightarrow$ Can be determined directly from the Sudakov form factor.
2. How do we assign energy-fractions in a splitting? $\Rightarrow$ Can be sampled from the splitting functions.
3. When do we terminate the parton shower? $\Rightarrow$ No more partons to branch (too soft) or reached the minimum angle ($t_{min}$).

**Branching interval gluons vacuum**

$$\Delta t = -\frac{\ln(\mathcal{R})}{\int_{\epsilon}^{1-\epsilon} dz\, P_{gg}(z)} \tag{15}$$

**Branching interval gluons medium**

$$\Delta \tau = -\frac{2\sqrt{x}\ln(\mathcal{R})}{\int_{\epsilon}^{1-\epsilon} dz \mathcal{K}(z)} \tag{16}$$

- $\mathcal{R}$ is a random number, $\mathcal{R} \in [0,1]$.
- Equation (16) scales with $\sqrt{x}$ !

## Sampling gluons vacuum

$$\mathcal{R} \int_{\epsilon}^{1-\epsilon} dz \, P_{gg}(z) = \int_{\epsilon}^{y} dz \, P_{gg}(z) \tag{17}$$

- Solving for splitting value $y$ can be difficult.
- Introduce **Metropolis-Hastings algorithm**, we then need:
  1. Proposal distribution $f(x)$.
  2. Target distribution $P(x)$.

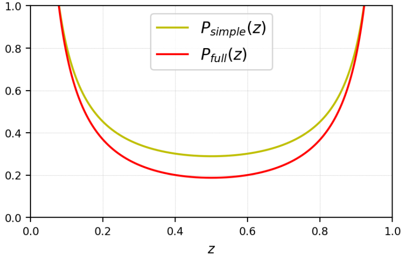



Figure: Vacuum $gg$ splitting functions.

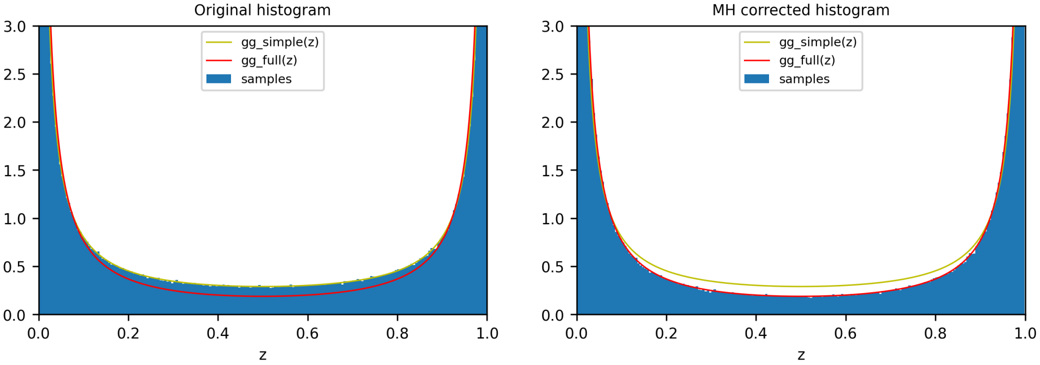
**Algorithm 1** Metropolis-Hastings

1: sample a random value $x'$ from $f(x)$.
2: calculate the acceptance probability,

$$A(x') = \min\left(1,\ \frac{P(x')}{f(x')}\right)$$

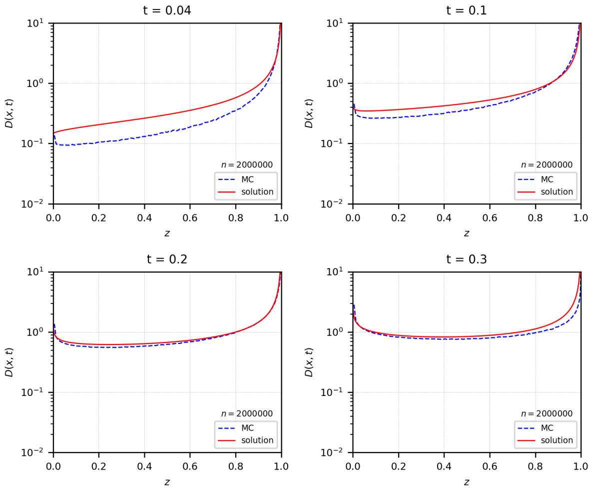
3: generate a random number $\mathcal{R} \in [0, 1]$.
4: **if** $\mathcal{R} \leq A(x')$ **:**
accept the value $x = x'$
5: **else if** $\mathcal{R} > A(x')$ **:**
reject the value $x'$

# Sampling from $P_{gg}(z)$



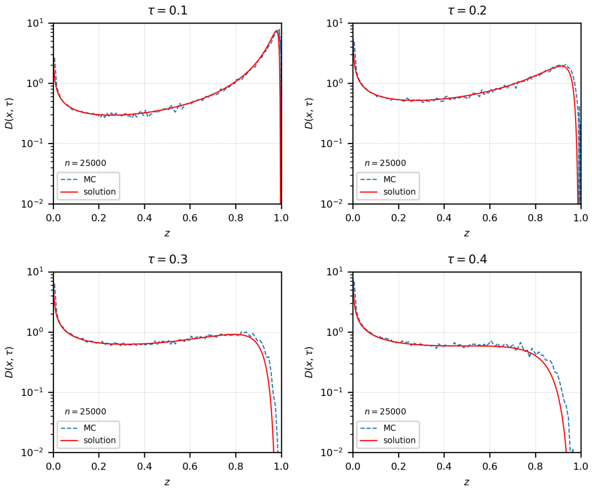
# Results for gluon showers in vacuum

- Inclusive energy distribution $D(x, t)$ for gluons-only in vacuum.
- Solution valid in soft limit (small $x$ large $t$).
- Showers generated using the simple splitting function.



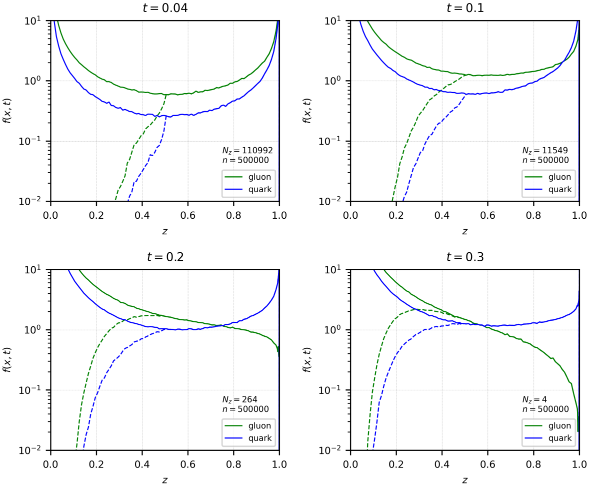
# Results for gluon showers in medium

- Inclusive energy distribution $D(x,t)$ for gluons-only in medium.
- Showers generated using the simple splitting function.



# Results for Quark and Gluon showers in vacuum

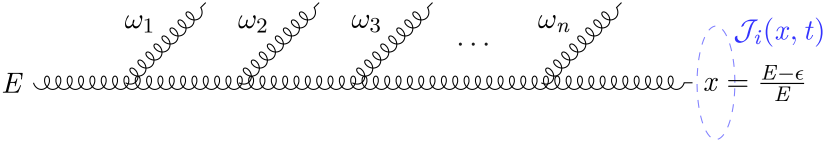
- Inclusive parton distribution $f(x,t)$ for quarks and gluons in vacuum.
- Blue line is initial quark.
- Green line is initial gluon.
- Dotted line is hardest parton of each shower.
- Showers generated using full splitting functions.



# Leading Parton and Energy-Loss

# Simplest model for the leading parton

- Follow only the hardest in each splitting.
- All soft-radiation is treated as energy-loss.



# Simple model for energy-loss in medium

The current model for the energy loss gives a probability of emitting a total energy $\epsilon$ over an arbitrary number $n$ of emissions as as[3]

$$D(\epsilon) = \sum_{n=0}^{\infty} \frac{1}{n!} \left[ \Pi_{i=1}^{n} \int d\omega_i \frac{dI(\omega_i)}{d\omega} \right] \delta \left( \epsilon - \sum_{i=1}^{n} \omega_i \right) \exp\left( - \int_0^{\infty} d\omega \frac{dI}{d\omega} \right) \tag{18}$$

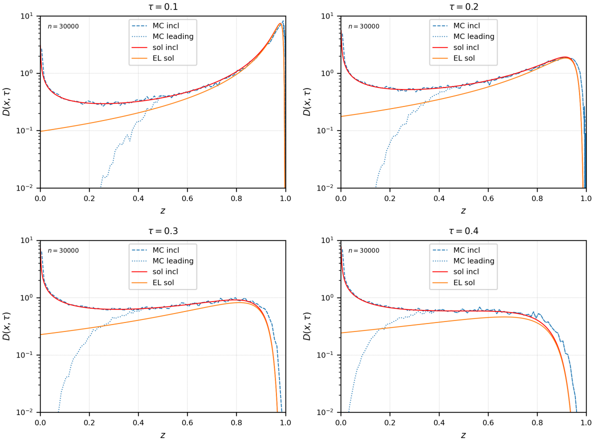
which can be solved by assuming soft emissions,

$$D(x) \approx \frac{\tau}{(1-x)^{3/2}} \exp\left( -\pi \frac{\tau^2}{1-x} \right). \tag{19}$$

[3] Baier et al. 2001.

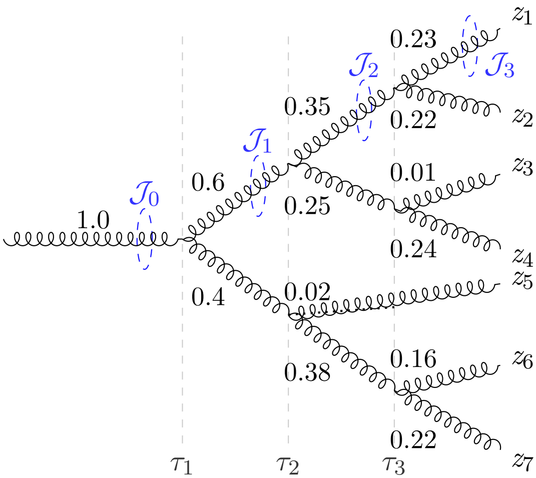
# Results for the simple leading parton in medium

- Blue - inclusive and leading energy distributions for gluons in medium.
- Red line - analytical solution for gluons in medium.
- Yellow line - simple leading parton solution given by the energy-loss formula.

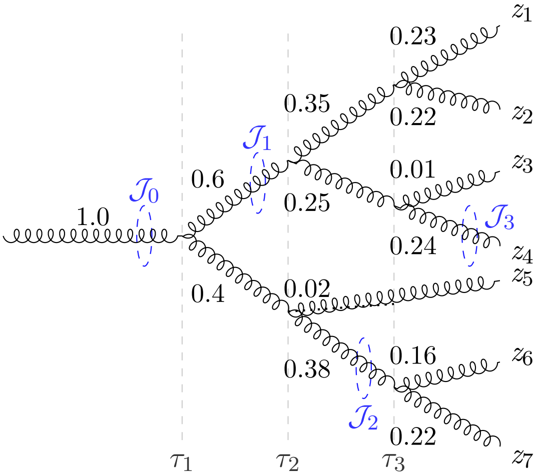


# On-branch and Off-branch

**On-branch**

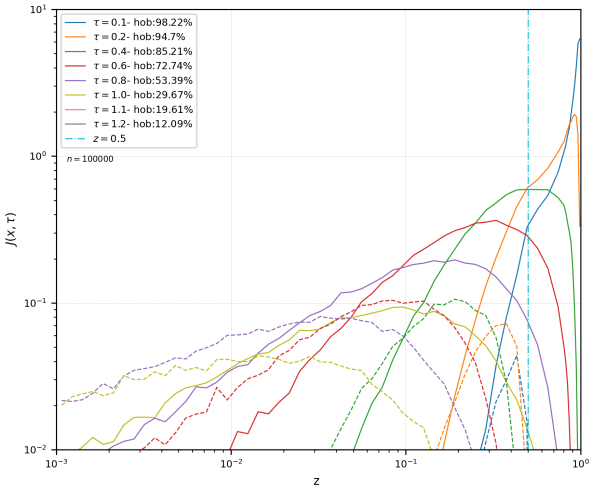



**Off-branch**



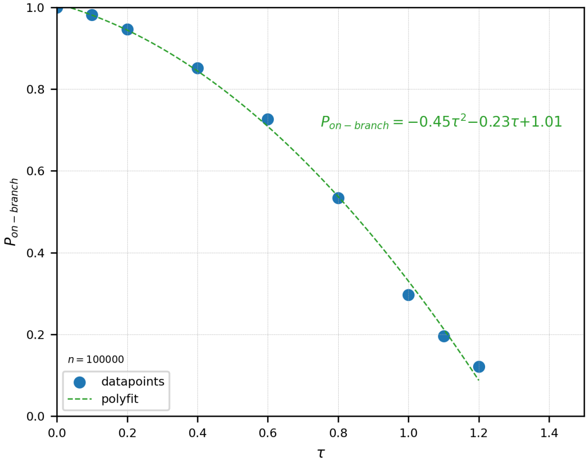
# Leading parton branch distributions for gluons in medium

- Distribution of on-branch and off-branch leading partons for gluons in medium.
- Solid line - leading parton is on-branch.
- Dotted line - leading parton is off-branch.



# Leading parton on-branch percentages in medium

- Percentage of leading partons being on-branch for different values of $\tau$.
- Numpy.Polyfit used to fit data.



# Leading parton evolution equations in vacuum

We propose a new set of leading parton evolution

- In vacuum.
- Gluons only.
- On-branch leading partons.

$$
\begin{aligned}
\frac{\partial}{\partial t} D(x,t) &= \Theta\left(x < \frac{1}{2}\right) x\, 2P(x) \\
&\quad + \int_0^{\min(\frac{1}{2},1-x)} dz\, 2P(z) D\left(\frac{x}{1-z}, t\right) - \int_0^{1/2} dz\, 2P(z) D(x,t).
\end{aligned} \tag{20}
$$

The proposed evolution equation for the leading parton can be solved in Mellin space

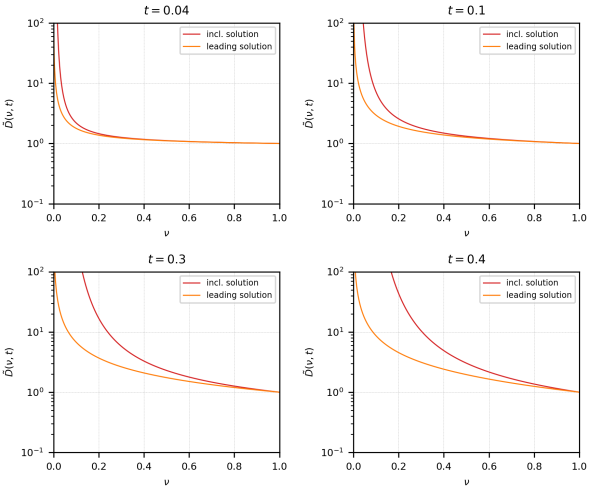
$$\tilde{D}(\nu,t) = \frac{\mathrm{B}_{\frac{1}{2}}(\nu,0) + \int_{1/2}^{1} dz\,\left(\frac{z^{\nu}-1}{z(1-z)}\right)}{\int_{1/2}^{1} dz\,\left(\frac{z^{\nu}-1}{z(1-z)}\right)} \exp\left(2\int_{1/2}^{1} dz\,\left(\frac{z^{\nu}-1}{z(1-z)}\right)\,t\right) - \frac{\mathrm{B}_{\frac{1}{2}}(\nu,0)}{\int_{1/2}^{1} dz\,\left(\frac{z^{\nu}-1}{z(1-z)}\right)} \tag{21}$$

Where the Mellin transform is defined as,

$$\tilde{D}(\nu,t) = \int_0^1 dx\,x^{\nu-1}\,D(x,t) \quad , \quad D(x,t) = \int_{c-i\infty}^{c+i\infty} \frac{d\nu}{2\pi i}\,x^{-\nu}\,\tilde{D}(\nu,t). \tag{22}$$

# Leading parton model in Mellin space

- Red - solution of the DGLAP equation in Mellin space.
- Orange - solution of the leading parton evolution equations in Mellin space.
- Difficult to interpret the values of $\nu$.



# Summary

- Used analytical methods to find approximate solutions to the evolution equations in vacuum and medium.
- Created Monte-Carlo programs where the inclusive and leading distributions were easily obtained.
- Compared a simple model of energy-loss to the leading parton distribution generated by the Monte-Carlo.
- Motivated the differences between on-branch and off-branch leading partons.
- Formulated evolution equations for the leading parton in vacuum, and attempted to solve them.

# Future work

- Use Monte-Carlo methods to validate the Leading parton evolution equations in vacuum.
- Escape from Mellin space.
- Formulate leading parton evolution equations for medium and solve them.
- Implement off-branch leading partons into the evolution equations.

# Acknowledgements


- Konrad Tywoniuk.
- Adam Takacs.
- The heavy-ion group.

# Acknowledgements


- Konrad Tywoniuk.
- Adam Takacs.
- The heavy-ion group.
- All of you.

# The End

# References


Baier, R. et al. (2001). "Quenching of hadron spectra in media". In: *JHEP* 09, p. 033. DOI: `10.1088/1126-6708/2001/09/033`. arXiv: `hep-ph/0106347`.

Collaboration, ATLAS (Dec. 2015). "ATLAS event at 13 TeV - Highest mass dijets angular event in 2015 data". General Photo. URL: `https://cds.cern.ch/record/2113240`.

Ellis, R. Keith, W. James Stirling, and B. R. Webber (Feb. 2011). *QCD and collider physics*. Vol. 8. Cambridge University Press. ISBN: 978-0-521-54589-1. DOI: `10.1017/CBO9780511628788`.