How do I prove that gcd (a, b, c) = gcd (a, gcd (b, c))? Note that if you want to use any facts about gcd (a, b, c) beyond the definition, you will need to prove them.

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It is actually pretty easy.

Let $g = gcd(a, b, c)$ and let $h = gcd(a, gcd(b, c))$.

Note that both are positive integers.

Clearly $h \mid a, \underline{h \mid gcd(b, c)}$ so we indeed we have 要点1

$h \mid a, h \mid b, h \mid c$ $\quad h \mid gcd(a,b) \Rightarrow h \mid a,\ h \mid b$

so, by definition of gcd, also

$h \mid g$

On the other hand, since

$g \mid a, g \mid b, g \mid c$ $\quad$ 要点2

we also have (again by definition of gcd)

$\underline{g \mid a, g \mid gcd(b, c)}$ $\Leftarrow x \mid a,\ x \mid b \Rightarrow$ 证明.

$\Rightarrow \underline{x \mid gcd(a,b)}$

∵ $a = k_1 \, gcd(a,b)$

$b = k_2 \, gcd(a,b)$

若 $x \mid a,\ x \mid b$

则 $x \mid k_1 gcd(a,b)$

$x \mid k_2 gcd(a,b)$

若 $x \nmid gcd(a,b)$ 不能整

则 $x \mid k_1$ 且 $x \mid k_2$

设 $k_1 = k_3 x,\ k_2 = k_4 x$

$a = k_3 x \, gcd$

$b = k_4 x \, gcd$

与gcd为最大公因数定义不符

and therefore

$g \mid h$

And since two positive integers that are factors of each other must be equal, the conclusion follows.

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