Today's Lecture

- Introductions
- Syllabus and notes

  Discuss grade breakdown, office hours, instructions for accessing Webwork, and labs.

- Lecture format and expectations

  Remind students to:

  - Read syllabus
  - Buy textbook
  - Register for Webwork
  - Get an approved calculator
  - Write down your contact info

- Sections 1.1-1.3, & 2.1

Contact Information

Name:
E-mail:
Office:
Office Hours:

Lecture Expectations

I expect that you don't do any of the following.

- Come to class late or leave early (if this is an issue out of your control you should switch sections)
- Have side conversations
- Have your phone out
- Have your laptop out
- Pack up early
- Listen to music, sleep, work on materials from another class, etc...

I expect you do your best to do the following.

- Raise your hand or ask whenever you have a question.
- Challenge yourself to ask at least one question in lecture this semester.
- Sit in the front of the class
- Set up times to meet with myself or your TA outside of class
- Share contact information with others in the class. Set up weekly study sessions.

Introductory Sections: 1.1-1.3, 2.1-2.3

Sections 1.1-1.3 and 2.1-2.3 in the text cover the definition of a function, interval notation, and definitions of specific types of functions and their use. This material is pre-requisite for this course, thus we will only spend the first few lectures going over the tools from those sections you will need throughout the course.

Have students help you fill in these sections by asking, "What do you know about linear functions?" They should already know most of this material.

We will regularly use polynomials(constant, linear, quadratic, cubic, etc..), exponential, logarithmic, and power functions throughout the course. In order to solve the problems given in the homework, labs, and exams you will need to have a thorough understanding of these functions and their properties. In particular, given each type of function you should know:

1. *The general form or an example of the function.*
2. *What the graph of the function tends to look like.*
3. *How the values of any constants effect the behavior of the function. For example given the general exponential function Ae^{kx}, where A and k are constants, how do the values of A and k affect the behavior of the function?*

Linear Functions

- Function & Related Equations

A linear function has the form $y = mx + b$ where m is slope and b is y-intercept. Slope can be calculated via

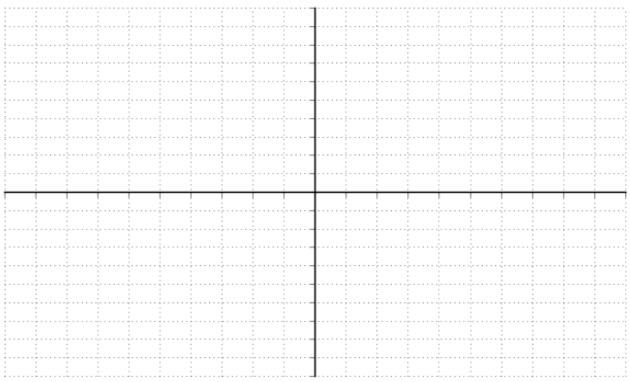
$$m = \frac{\text{change in y}}{\text{change in x}} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Point-slope form is $y - y_0 = m(x - x_0)$. The graph is a line.

Table

x	$f(x)$

Graph



- Related Keywords

x-intercept, y-intercept

Quadratic Functions

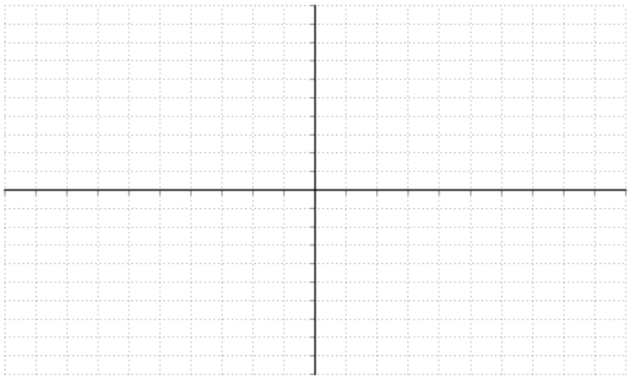
- Function & Related Equations

A <u>quadratic function</u> has the form $f(x) = ax^2 + bx + c$ where a, b, c are numbers (with $a \neq 0$. The graph is a *parabola*.

Table

x	$f(x)$

Graph



- Related Keywords

quadratic formula (for solving for zeros)

Exponential Functions

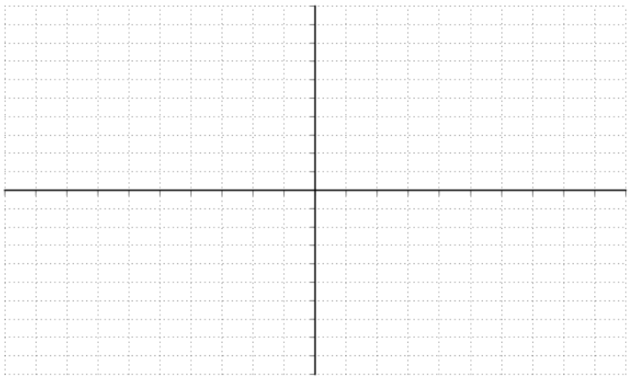
- Function & Related Equations

  Exponential functions have the form $f(x) = ka^x$ where a is positive and k is any constant. a is called the base.

Table

x	$f(x)$

Graph



- Related Keywords

  growth, decay, compounded interest, compounded continuously, doubling time, half-life

Logarithmic Functions

- Function & Related Equations

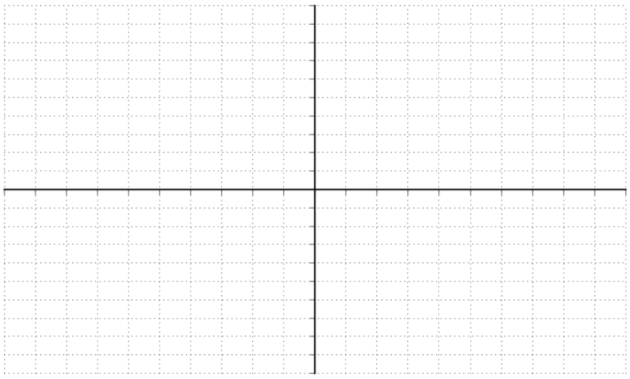
The base b logarithm of x, $log_b x$ is the power to which we need to raise b to get x, i.e.

$$log_b x = y \iff b^y = x$$

Table

x	$f(x)$

Graph



- Related Keywords

Base 10: $logx$, base e: lnx, change-of-base formula: $log_b a = \frac{loga}{logb} = \frac{lna}{lnb}$

The skills from this material we will use regularly include, but are not limited to, the following. ***During the first week of class you should go through these sections, look over the problems at the end of each section, and write down any questions you may have to ask during lecture or office hours.***

1. Identify if a plot or expression represents a function using the vertical line test.

   Definition

   A ***function*** is a rule that assigns an output number to an input number. The ***domain*** is the set of input numbers and the ***range*** is the set of output numbers.

2. Use interval notation to describe intervals of interest. Closed intervals, open intervals etc..
   e.g. $(0, 2] \cup (4, \infty)$, $[-2, 5)$, $(40, 60]$

3. Build a linear function/model procedurally or from a context.

   (a) Determine the slope of a line.

   (b) Equation(s) of a line *(point-slope, slope-intercept)*

   (c) Understand what linear growth means.

   (d) Understand the meaning of slope and how to calculate it including any appropriate units.

   (e) Calculate x and y intercepts.

4. Build an exponential function/model procedurally or from a context. Understand what exponential growth/decay means. Understand the different interpretations of exponential growth *(annual, continuous, etc...)*.

5. Recognize and distinguish if a function is linear or exponential given a context.

   Example *The number of cases of flu is doubling every 3 days....build a function modeling the number of people infected...*

   Example *The number of cases of flu is increasing by 10 cases every three days....build a function modeling the number of people infected...*

6. Solve and interpret problems given a table, analytic function, word problem, or plot.

7. Be comfortable interpreting and evaluating piecewise functions.

8. Given a quadratic function be able to determine x and y intercepts and the vertex of the parabola.

9. Know exponent rules, distributive property, and function notation.

Word Problems

When choosing examples to do in class, aim for a combination of *procedural* (so students can see the computation) and *interpretation* (so students can understand the idea in context) problems.

In addition to the ***procedural*** skills used in solving the problems in this course you will need ***interpretation*** skills. These are the skills that allow you to interpret a problem given in a context. As you learn the material you should always be sure to practice both procedural and contextual problems.

> ***Example*** *Determine the equation for the line that intersects the points* $(-1, 5)$ *and* $(-4, -8)$.
>
> *This question requires only procedural knowledge. In other words as long as we know the equation of a line, we can solve this problem using arithmetic and algebra.*

> ***Example*** *In 2009 population of a species of bat in a small cave has an initial population of 500 bats. 5 years later the bat population has increased to 3000 bats. Assuming the increase in the number of bats per year is constant, determine a formula for the number of bats t years after 2009.*
>
> *This problem actually uses the same procedural knowledge. However, we are unable to proceed directly to the procedure of finding the equation of this line. Here we have a* ***context*** *that we must first unpack. This includes recognizing the following facts.*
>
> - *First we have to decide what kind of equation is appropriate here (linear, quadratic, exponential, etc...)*
> - *After convincing ourselves the function should be linear we must interpret what information we can use to solve our equation.*
> - *In particular, what is the meaning of the numbers 5, 500, 2009, and 3000 in the given context?*
>
> *Only after we have answered the questions above can we move on to apply our procedural knowledge.*

Tips for answering questions in this course

1. ***When asked to interpret your answer you should use units to explain all numbers relevant to your answer within the context of the problem.*** For example, suppose you determine the equation $f(t)$, which represents the number of volleyballs produced by a factory t hours after production begins, and have used f to determine that $f(2) = 4200$. Then an acceptable interpretation could be the following.

   After 2 hours the factory has produced 4200 volleyballs.

2. If you are uncertain about a problem on an exam you should try something that shows what you know about the material related to the problem. This doesn't guarantee you any credit but I can't give you any partial credit if you leave a question blank.

Practice Problems from 1.1-1.3, 2.1

Have students work in pairs on a couple of these problems for a few minutes, and then talk about the problems altogether by either having students come up and write their solutions or walking you through the solution. If running short on time, have them start with problems 3, 5, and 6.

This selection of problems is meant to give examples of different types of problems covering material from sections 1.1-1.3 and 2.1. ***In addition to these problems it is important to look at the problems at the end of each of these sections.***

1. List the interval(s) where the following functions are increasing.
   (a) $f(x) = (x-2)^2 + 5$
   (b) $h(t) = 32 - 5t$
   (c) $g(x) = x^3$
2. In 2009 population of a species of bat in a small cave has an initial population of 500 bats. 5 years later the bat population has increased to 3000 bats. Assuming the increase in the number of bats per year is constant, determine a formula for the number of bats t years after 2009.
3. The population of otters in a river watershed is modeled by $P(t) = t^2 - 10t + 200$ where t is the number of years since 2005.
   (a) What time does the otter population reach a minimum?
   (b) At what time(s) is the otter population 180?
   (c) Determine $P(1)$ and interpret the meaning of your answer. Is the otter population increasing or decreasing at this time? How can you tell?

4. The concentration, in $\mu g/ml$, of a blood pressure medication in the bloodstream t hours after it is ingested is given by $C(t) = -2t^2 + 16t$.

   (a) Determine $C(0)$ and interpret the meaning of your answer.

   (b) Determine $C(10)$ and interpret the meaning of your answer.

   (c) At what time does the medication have the highest concentration? What is the concentration at that time?

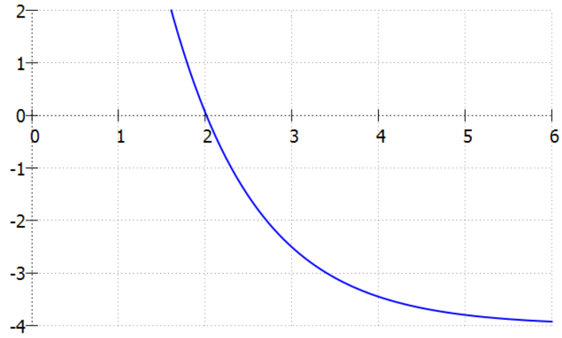
5. The tables below list the different values of three functions given a set of points in their domain. Determine if any of the function values given in the following table are linear. Determine the equation for any functions that are linear.

x	2	4	6	8
$k(x)$	3	15	63	255

t	1	3	7	9
$g(t)$	10	12	14	16

x	15	20	25	30
$P(x)$	319	299	279	259

The graph below represents the temperature in Celsius of an object, $h(t)$, t minutes after the start of an experiment.



t	2		3.5		5
$h(t)$		-2.5		-4	

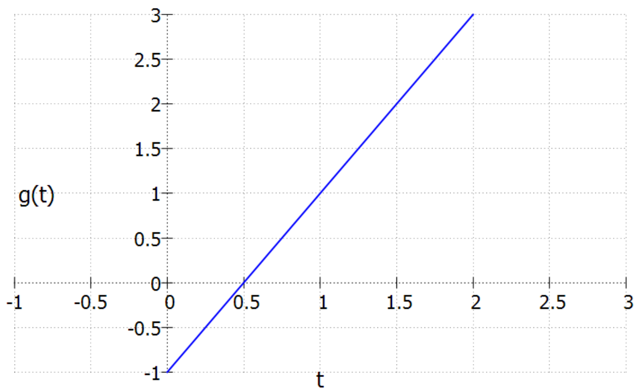
6. (a) Use the figure above to fill in the the missing values in the table.

   (b) In the context of this problem, what significance does $h(2)$ have?

   (c) Is the temperature of the object increasing, decreasing, neither, or both? How can you tell?

   (d) Is the temperature increasing/decreasing linearly? Why or why not?

7. Given the graph below find a function which describes the line.



8. Use the following functions to evaluate the expressions below.

$$n(t) = \begin{cases} 4t & 0 \le t \le 3 \\ 12 + 23(t-3)^{2.7} & 3 < t \le 6 \end{cases}, \qquad S(x) = \begin{cases} x^2 & -2 \le x \le 2 \\ -0.5x + 3.5 & 2 < x \le 4 \\ 5e^{-0.2x} & 4 < x \le 10 \end{cases}$$

   (a) $n(1)$

   (b) $S(2)$

   (c) $n(-1)$

9. Determine the equation for the line that has a slope of 4 and intersects the point $(5, 20)$.

10. Given that $f(x) = x^2 - 2x$ determine the following expressions.

    (a) $f(5)$

    (b) $f(h)$

    (c) $f(x-3)$

    (d) $f(x+h)$

11. The area of a forest is 13,000 acres in 1990 and is decreasing by 200 acres per year. Determine a model which gives the area of the forest t years after 1990.

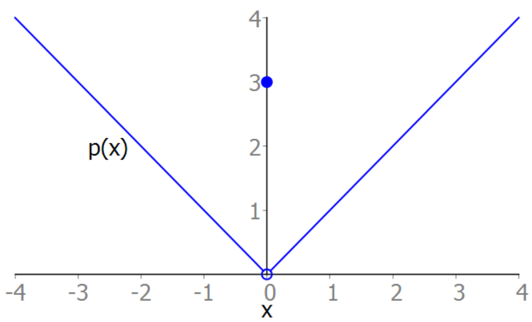
12. The yield $Y(x)$ of my vegetable garden in pounds of produce as a function of x pounds of fertilizer added to my garden beds is modeled by $Y(x) = -x^2 + 30x + 50$.

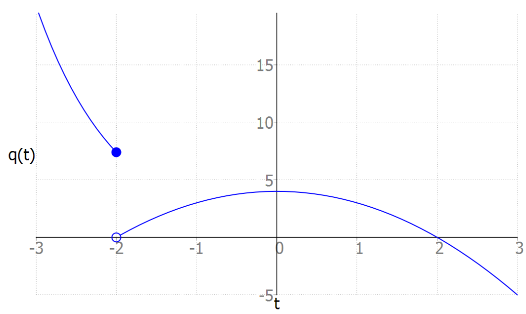
    (a) What amount of fertilizer will maximize the total number of pounds of produce from the garden?

    (b) At the maximized amount of fertilizer how many pounds of vegetables will the garden produce?

    (c) Do you see any limitations of this model? Explain why or why not.

    (d) Which is better, using 31 pounds of fertilizer or none at all? Explain.





13. Use the figures above to evaluate the following expressions.

    (a) $p(0)$

    (b) $q(-2)$