Reference point on the image: $\begin{bmatrix} u \\ v \\ w \end{bmatrix} \equiv \begin{bmatrix} u/w \\ v/w \end{bmatrix}$

Reference point in the world coordinate: $\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$

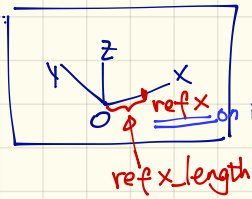
Now, the projection matrix is $[V_x \; V_y \; V_z \; 0]$ up to a scale

$\equiv [aV_x \; bV_y \; cV_z \; 0]$

↑ the point correspond to world orign, on image

Select a reference point on X coordinate, and give its reference length:

image:



on image coordinate

ref point on real-world X coordinate
the refx_length is also the value of X

$$\Rightarrow \begin{bmatrix} aV_x(1) & bV_y(1) & cV_z(1) & O(1) \\ aV_x(2) & bV_y(2) & cV_z(2) & O(2) \\ a & b & c & 1 \end{bmatrix} \begin{bmatrix} X \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} aV_x(1)X + O(1) \\ aV_x(2)X + O(2) \\ aX + 1 \end{bmatrix} = \begin{bmatrix} refx(1) \cdot w \\ refx(2) \cdot w \\ w \end{bmatrix}$$

$w = aX + 1$

*(1), (2) are 1st/2nd elements of that vector

$$\therefore \begin{cases} aV_x(1)X + O(1) = refx(1)(1 + aX) \\ aV_x(2)X + O(2) = refx(2)(1 + aX) \\ aV_x(3)X + O(3) = refx(3)(1 + aX) \end{cases}$$

do subtraction to get a

Vectorize ⇒ $a(X)(V_x - refx) = refx - O$

X = refx_length

The same for b and c

∴ matlab code:

$$a = \left((V_x - refx) \backslash (refx - O)\right) / refx\_length$$

$$b = -\left((V_y - refy) \backslash (refy - O)\right) / refy\_length$$

$$c = -\left((V_z - refz) \backslash (refz - O)\right) / refz\_length$$

in matlab, vertical image coordinate is the opposite direction of default, ∴ need negative sign for b & c