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The output of a quartz transducer oscillating in its fundamental and higher harmonics

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The output from a quartz crystal oscillating in three liquids, at its fundamental, 3rd harmonic and 5th harmonic have been measured by the calorimetric method keeping the crystal amplitude the same. The output is found to be much less than that one can expect from theoretical considerations.

1. Introduction

The phenomenon of longitudinal vibrations of piezoelectric crystals has been extensively discussed by CADY [1]. Exact expression for the amplitude ξ_h of oscillation at the hth harmonic of a quartz plate of thickness l, when the applied stress is X_0 is given by [2]

$$-\xi_{h(l/2)} = \frac{2x_0}{\pi \varrho c \cdot h \cdot \alpha_h} \tag{1}$$

where ϱ is the density and c the velocity of elastic waves in it and α_h the frictional damping coefficient at the hth harmonic.

Since the stress is proportional to the applied voltage it follows that [3] the amplitude at the fundamental ξ_1 and that at the hth harmonic ξ_h are related by

$$\frac{\xi_1}{\xi_h} = \frac{V_1}{V_h} \cdot \frac{h \cdot \alpha_h}{\alpha_1} \tag{2}$$

where V_1 is the voltage applied to the crystal in its fundamental and V_h the voltage at the hth harmonic, α_1 and α_h are the damping coefficients at the fundamental and hth harmonic respectively.

To obtain an expression between voltage and current in the crystal we note that the admittance of the crystal y is $1/R$ at resonance. We make the assumption here that the capacity of the electrodes etc. is compensated for by connecting a suitable inductance across it so as to make the total impedance of this parallel circuit infinite. We then have

$$\frac{y_1}{y_h} = \frac{R_h}{R_1}. \tag{3}$$

Since the damping terms α_h and α_1 are related to the resistances R_h and R_1 of the crystal in the hth harmonic and the fundamental one has

$$\frac{R_h}{R_1} = \frac{\alpha_h}{\alpha_1}. \tag{4}$$

Whence it follows that

$$\frac{Y_1}{Y_h} = \frac{\alpha_h}{\alpha_1}. \tag{5}$$

Since $I = V \cdot Y$ we obtain

$$\frac{I_1}{I_h} = \frac{V_1}{V_h} \cdot \frac{\alpha_h}{\alpha_1}. \tag{6}$$

Combining equation (6) with equation (2)

$$\frac{\xi_1}{\xi_h} = h \cdot \frac{I_1}{I_h}. \tag{7}$$

It is clear that if $\xi_1 = \xi_h$

$$I_h = hI, \tag{8}$$

which means that in order to obtain the same amplitude of oscillation of the crystal at its hth harmonic as its fundamental all that one has to do is to send h times the current through the crystal at its hth harmonic as its fundamental.

2. Experiment

The experiment consisted in tuning off the crystal capacity by a suitable inductance and measuring the current through the crystal at its fundamental by the method which has been fully described elsewhere [*4*].

The crystal used in this experiment had a fundamental frequency of 2·84 Mc./sec. and it was excited in its fundamental, third and fifth harmonics. As a preliminary experiment the crystal was made to oscillate in several liquids and the current passing through the crystal oscillating in its fundamental were measured. In some liquids like carbon tetrachloride, chloroform and benzene which show a high value of sound absorption the current passing through the crystal, even at such a high voltage as 160 volts across the crystal surfaces, was quite small with the result that the third and fifth harmonic currents required a high value of potential across the crystal to maintain the crystal amplitude the same as in its fundamental, a potential which the signal generator was incapable of producing.

Three liquids: methyl ethyl ketone, hexane and butyl alcohol were found to be suitable for this experiment. The amount of electrical energy converted by the quartz crystal into heat was measured by the calorimetric experiment as described in our earlier communication [*4*].

The experiment was repeated in the 3rd harmonic viz. 8·52 Mc./sec. and in the 5th harmonic viz. 14·20 Mc./sec. In the former experiment the current through the crystal was thrice its value in the fundamental and in the latter it was 5 times that value.

3. Results and Discussion

The results are shown in Table 1. It is seen that while the crystal oscillates in its fundamental frequency the total amount of heat generated varies from liquid to liquid and it is highest in methyl ethyl ketone and nearly the same in hexane and butyl alcohol. The applied voltage is the same in these 3 cases. More current flows through the crystal immersed in methyl ethyl ketone than in hexane or butyl alcohol.

Table 1. *Oscillations of a Quartz Crystal in its Fundamental 3rd and 5th Harmonics*

Liquid	Mass of liq. gm	Voltage V	Current m.a.	Elec. power Watts	Temp. rise in 10 min.			Thermal power (Watts)		Efficiency %	
					Total	Dielec. °C	Ultra-sonic °C	Total	Ultra-sonic	Total	Ultra-sonic
1	2	3	4	5	6	7	8	9	10	11	12
I. *Fundamental* 2·84 Mc.											
(a) Methyl ethyl ketone	91·7	81	6·1	0·494	0·950	0·525	0·425	0·417	0·185	84·0	37·5
(b) Hexane	82·4	81	4·3	0·348	0·600	0·100	0·500	0·259	0·216	74·3	68·1
(c) Butyl alcohol(n) .	98·4	81	4·6	0·373	0·550	0·100	0·450	0·262	0·214	70·2	57·6
II. *3rd Harmonic* 8·52 Mc.											
(a) Methyl ethyl ketone	97·7	117	18·4	2·140	1·875	0·875	1·000	0·866	0·462	40·5	21·6
(b) Hexane	83·4	125	12·9	1·625	1·400	0·600	0·800	0·610	0·349	37·5	21·4
(c) Butyl alcohol(n) .	98·0	125	13·9	1·750	1·550	0·625	0·925	0·735	0·439	42·0	25·1
III. *5th Harmonic* 14·20 Mc.											
(a) Methyl ethyl ketone	96·6	98	30·6	2·990	1·025	—	—	0·468	—	15·6	—
(b) Hexane	81·6	90	21·5	1·940	0·725	—	—	0·311	—	16·03	—
(c) Butyl alcohol(n) .	97·9	85	23·0	1·955	0·700	—	—	0·332	—	16·8	—

Water equivalent of calorimeter etc. = 12·31 gm.

In the third harmonic one would expect when the amplitude remains the same, a power output nine times greater than that at the fundamental. This follows from the fact that the intensity is proportional to the square of the amplitude and frequency. An examination of the total thermal output in the 3rd harmonic indicates only a two-fold increase in thermal output whereas the electrical input increases at least four times.

In the fifth harmonic the amplitude is maintained the same as at the fundamental by sending five times the current at the fundamental. The experimentally measured output at the 5th harmonic is between 12 to 20% more than the output at the fundamental while theoretically one would expect at twentyfive fold increase.

The voltages required to maintain the same amplitude at a higher harmonic as compared to the fundamental are about fifty percent more in the 3rd harmonic but only five to twenty-two percent more in the fifth harmonic.

Substracting the amount of heat produced in the 1st and 3rd harmonic due to dielectric heating of the quartz crystal by the method already described [4], one finds that the total ultrasound output is much less and the percentage of electrical energy converted into heat decreases as the crystal oscillates in its higher harmonics.

The reason for this enormous difference in measured and theoretically anticipated values may be due to some physical property of the liquid as yet unknown. The theory of the oscillations of the bar or plate from which equation (7) was derived makes no assumption regarding the nature of the medium surrounding the crystal. It is well known that the liquid surrounding the quartz alters the Q of the crystal as shown by experiments of PARTHASARATHY and CHHAPGAR [5]. This may account for a small reduction in output. Even the requirements of matching the impedance of the crystal to that of the medium cannot account for the enormously reduced output.

4. Conclusion

Assuming that the relation between amplitude and current, as given by equation (8) viz. to maintain the same amplitude at the hth harmonic as at the fundamental, the current passing through the crystal should be h times that at the fundamental, is correct, the quantity of sound emanating from the crystal oscillating at its fundamental, 3rd and 5th harmonics with the same amplitude is not according to theory; the electrical power input increases whereas the output decreases. Similar results have been obtained by PARTHASARATHY [6].

References

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