

## Short Communication

# Compression of aircraft aerodynamic database using multivariable Chebyshev polynomials

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An automated procedure for the modeling of aerodynamic data obtained from wind-tunnel measurements has been developed. Heuristics and multivariable Chebyshev orthogonal polynomials are combined to construct function approximations of the tabular aerodynamic database. This method results in a significant reduction in the size of the database and the man-hours required for developing an explicit mathematical model. The software design philosophy and validation results using actual aerodynamic data for the F16 fighter aircraft are presented here. © 1997 Elsevier Science Limited. All rights reserved.

**Key words:** function approximation, data compression, multivariable orthogonal polynomials, aerodynamic database.

### 1 INTRODUCTION

In aircraft design, aerodynamic forces and moments acting on the aircraft may be determined from wind-tunnel measurements. For example, in a static force wind-tunnel test, a scaled model of the actual aircraft is held fixed at different orientation to the air flow and the resulting forces and moments acting on the model are then recorded. The test is repeated with different combinations of control surface deflections and at different air speeds. A similar series of tests using a rotary balance or an oscillatory rig is also performed to determine the aircraft dynamic damping characteristics. Considering that a typical combat aircraft will have three primary control surfaces, namely the ailerons, elevators and rudder, in addition to high-lift devices like leading and trailing edge flaps, together with speed brakes and different external stores combinations, the entire series of measurements results in a fairly large aerodynamic database in tabular form.

For subsequent performance, flight control analysis and flight simulation, the experimental data must be further refined to remove measurement noise and to identify dominant trends. A more compact representation of the aerodynamic data would be extremely

desirable. Performance and flight dynamics analysis at large angles of attack and sideslip, using methods from bifurcation theory, is best performed using explicit function approximations of the nonlinear aerodynamics.<sup>1</sup> Computation of stability derivatives will also be expedited by a one-time symbolic differentiation of an explicit mathematical model instead of using repeated numerical interpolation of the tabular data. For real-time flight simulation, especially with hardware or pilot ‘in-the-loop’, extensive interpolation of a large database would be unacceptably slow and the database would also require significant computer memory storage. Initial attempts to simplify the aerodynamic database by actual physical inspection of the data proved to be a time-consuming and tedious process. These considerations led to the development of an automated procedure for constructing simple function approximations for the aerodynamic database.

In the next section the heuristics of function approximations and the reasons for using Chebyshev polynomials are stated. Properties of Chebyshev polynomials in one variable are reviewed and the extension to the multivariable case is then given. Section 3 is an overview of the development of the aircraft database modeling package (ADMOP). This package is meant to

complement ADCAP, the aircraft dynamics and control analysis package reported earlier.<sup>2</sup> The design philosophy and capabilities of ADMOP are discussed. Special attention is paid to the problem of determining an appropriate trade-off between simplicity and accuracy of the function approximation. Finally, a case study is provided in Section 4 using actual aerodynamic data for the F16 fighter aircraft.

## 2 FUNCTION APPROXIMATION

The function approximation problem in one variable can be stated as follows:

- Given (1) a list of *breakpoints* or values of the independent variable:  $\{x_i\} i = 0, 1, \dots, N - 1$ .  
(2) a list of the dependent variables measured at each breakpoint:  $\{f_i\} i = 0, 1, \dots, N - 1$ .

Find (1) an explicit function  $F(x)$  that maps each breakpoint to the corresponding measured value  $f_i$ . The function approximation is constructed in terms of a set of preselected *basis functions*  $\{\phi_i(x_i)\}$ ,  $i = 0, \dots, N - 1$ , as

$$F(x) = c_0\phi_0(x) + \dots + c_{N-1}\phi_{N-1}(x) \quad (1)$$

where the coefficients  $c_i$  are to be determined according to some criteria. For example, one may use the basis functions  $\{1, x, x^2, \dots, x^{N-1}\}$  with the requirement that the  $c_i$ s are chosen so that the sum of the squared error at the breakpoints is minimized. This is the least-squares approach:

$$\overline{\min}_{c_i} \sum_{i=0}^{N-1} (F(x_i) - f_i)^2 \quad (2)$$

By using a high order of fit, i.e. a large number of basis functions, the least squares error may be reduced but unfortunately this often results in a function approximation that changes sharply in between breakpoints. For the computation of stability derivatives where the function will be differentiated, this oscillation between breakpoints is unacceptable. A better approach is to minimize the maximum deviation, not just at the breakpoints but over the entire region of interest:

$$\overline{\min}_{c_i} (\max |F(x) - f(x)|), \quad x_0 < x < x_{N-1} \quad (3)$$

The resulting function approximation is referred to as Chebyshev or uniform approximation. Chebyshev showed that the error  $F(x) - f(x)$  between the uniform ( $N - 1$ )th order polynomial approximation and the actual function has to change sign at  $N$  points. Since  $F(x)$  is continuous, this implies that the uniform approximation must be equal to the actual function at

$N$  specific points. Thus if a correct set of breakpoints is chosen and an interpolation polynomial can be found, then the interpolation polynomial is the uniform function approximation. By itself, this theorem does not lead to a useful approximation procedure. The key idea is that the special set of  $N$  breakpoints can be approximated by the zeros of the  $N$ th order Chebyshev polynomial and the resulting function approximation using this family of Chebyshev polynomials will converge to the actual uniform approximation as the order increases. The entire procedure may be summarized as follows:

- (1) Normalize the  $N$  breakpoints  $x_i$  to lie on the interval  $[-1, 1]$ :  $z_i = 2(x_i - x_0)/(x_{N-1} - x_0) - 1$ .
- (2) Use the Chebyshev polynomials  $T_i(z)$  as the basis functions:  $T_i(z) = \cos(i \arccos z)$ ,  $0 < i < N - 1$ .
- (3) Interpolate to get functions values  $f_i(z_k)$  at the  $N$  zeros of  $T_N(z)$  given by  $z_k = \cos((k + 0.5)\pi/N)$ ,  $k = 0, 1, \dots, N - 1$ . For example, if  $x$  falls between the breakpoints  $\{x_1, x_2\}$  with corresponding data values  $\{f_1, f_2\}$  then using Taylor's expansion about  $x_1$  and approximating the first derivative by a forward difference yields:

$$f(x) \approx f_1 + (f_2 - f_1)/(x_2 - x_1)(x - x_1) \quad (4)$$

Similarly expanding about  $x_2$ :

$$f(x) \approx f_2 - (f_2 - f_1)/(x_2 - x_1)(x_2 - x) \quad (5)$$

Taking the average of eqns (4) and (5) yields the interpolation formula:

$$f(x) \approx 0.5(f_1 + f_2) + (f_2 - f_1)/(x_2 - x_1) \times (x - 0.5(x_1 + x_2)) \quad (6)$$

Thus the interpolated value  $f(x)$  is simply a weighted average of the data values of the two closest breakpoints:

$$f(x) = wf_1 + (1 - w)f_2$$

where

$$w = 0.5 - 1/(x_2 - x_1)(x - 0.5(x_1 + x_2)) \quad (7)$$

- (4) Now obtain the function approximation using the normalized breakpoints  $\{z_k\}$  and interpolated data points  $\{f(z_k)\}$ ,  $k = 0, 1, \dots, N - 1$

$$F(x) = c_0 T_0(z) + \dots + c_{N-1} T_{N-1}(z) \quad (8)$$

where the  $c_i$ s are then given by:

$$c_0 = \frac{1}{N} \sum_{k=0}^{N-1} f(z_k)$$

$$c_i = \frac{2}{N} \sum_{k=0}^{N-1} f(z_k) \cos\left(\frac{i(k + 0.5)\pi}{N}\right),$$

$$i = 1, \dots, N - 1 \quad (9)$$

This results from the orthogonal property of the Chebyshev polynomials

$$\sum_{k=0}^{N-1} T_i(z_k) T_j(z_k) = \begin{cases} 0, & i \neq j \\ N/2, & i = j \neq 0 \\ N, & i = j = 0 \end{cases} \quad (10)$$

where  $i, j < N - 1$ .

This function approximation will be of limited use if the basis functions and the coefficients cannot be computed easily. Fortunately the basis functions satisfy a recurrence relation

$$\begin{aligned} T_{n+2}(z) &= 2zT_{n+1}(z) - T_n(z) \\ T_0(z) &= 1, \quad T_1(z) = z \end{aligned} \quad (11)$$

Hence eqn (8) can be evaluated in a recursive manner similar to Horner's method for evaluating polynomials without repeated functions calls. In the image processing community, eqn (9) is referred to as the discrete cosine transform (DCT) of the interpolated data points and fast algorithms can be used to perform the computation without doing  $N^2$  multiplications. The use of the discrete cosine transform in image processing to compress data is well established<sup>3</sup> but to the best of our knowledge no attempts to apply these concepts for the function approximation of multivariable aerodynamic data has been reported.

For multivariable aerodynamic data, the ideas mentioned can be extended in a natural manner. For simplicity, we will illustrate the multivariable case mainly with two independent variables. The extension to three or more variables is straightforward. Without loss of generality, we will assume that the breakpoints have been normalized to lie on the square  $\{x, y\} = [-1, 1] \times [-1, 1]$ . The key idea is to choose the multivariable basis functions as the product of Chebyshev polynomials in each variable, i.e.

$$\begin{aligned} \phi_{ij}(x, y) &= T_i(x)T_j(y), \quad 0 < i < N - 1, \\ 0 < j < M - 1 \end{aligned} \quad (12)$$

The function approximation then takes the form:

$$F(x, y) = \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} c_{ij} T_i(x) T_j(y) \quad (13)$$

In this case, using the orthogonal property stated as eqn (10), the coefficients  $c_{ij}$  are given by:

$$\begin{aligned} c_{ij} &= \frac{\epsilon}{NM} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} f(x_k, y_l) \cos\left(\frac{i(k+0.5)\pi}{N}\right) \\ &\quad \times \cos\left(\frac{j(l+0.5)\pi}{M}\right), \quad i = 0, \dots, N - 1, \\ j &= 0, 1, \dots, M - 1 \end{aligned} \quad (14)$$

where  $\epsilon = 4$  for  $i \neq 0$  and  $j \neq 0$

$$\begin{aligned} \epsilon &= 2 \text{ for } i = 0 \text{ and } j \neq 0 \text{ or } j = 0 \text{ and } i \neq 0 \\ \epsilon &= 1 \text{ for } i = 0 \text{ and } j = 0 \end{aligned}$$

Note that the interpolated data values  $f(x_i, y_j)$  at the Chebyshev zeros  $\{x_i, y_j\}$  must be used. The location of these  $N \times M$  zeros are given by:

$$\begin{aligned} x_i &= \cos((i+0.5)\pi/N), \quad i = 0, 1, \dots, N - 1 \\ y_j &= \cos((j+0.5)\pi/M), \quad j = 0, 1, \dots, M - 1 \end{aligned} \quad (15)$$

The interpolated data value is obtained as a weighted average of the data values of the four closest breakpoints. For example, referring to Fig. 1, suppose the Chebyshev zero  $\{x_z, y_z\}$  falls within the rectangle defined by the breakpoints  $\{x_1, x_2\}$  and  $\{y_1, y_2\}$ . Let the measured data values at the four corners be  $f_1, f_2, f_3$  and  $f_4$ . By using a Taylor expansion about  $\{x_1, y_1\}$ :

$$\begin{aligned} f(x_z, y_z) &\approx f_1 + (f_2 - f_1)/(x_2 - x_1)(x_z - x_1) \\ &\quad + (f_4 - f_3)(y_2 - y_1)(y_z - y_1) \end{aligned} \quad (16)$$

Expanding about the other corners and then taking the average yields the interpolation formula:

$$\begin{aligned} f(x_z, y_z) &\approx 0.25(f_1 + f_2 + f_3 + f_4) \\ &\quad + 0.5(f_4 - f_3 + f_2 - f_1)/(x_2 - x_1) \\ &\quad \times (x_z - 0.5(x_1 + x_2)) \\ &\quad + 0.5(f_4 - f_2 + f_3 - f_1)/(y_2 - y_1) \\ &\quad \times (y_z - 0.5(y_1 + y_2)) \end{aligned} \quad (17)$$

Equation (14) is the 2-D discrete cosine transform of the interpolated data points  $f(x_i, y_j)$ . Note that the 2-D DCT is separable and may be computed through a row/column decomposition using only 1-D DCT. This holds even in higher dimensions. While fast multi-dimensional DCT algorithms are useful for real-time signal processing, they are not essential for the functional approximation problem.

For three variables, the basis functions are chosen as  $\phi_{ijk}(x, y, z) = T_i(x)T_j(y)T_k(z)$  and the associated  $c_{ijk}$  is computed from a 3-D DCT. A 3-D interpolation formula similar to eqn (17) may be derived for the Chebyshev zero falling in the cube bounded by

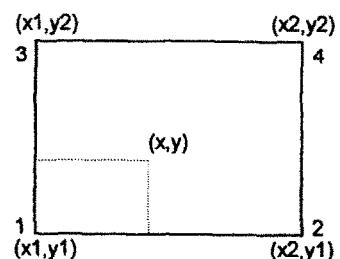


Fig. 1. Interpolation in two variables.

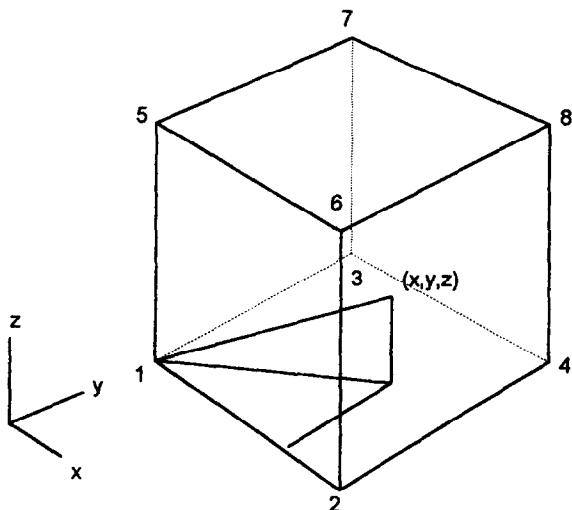


Fig. 2. Interpolation in three variables.

the breakpoints  $\{x_1, x_2\} \times \{y_1, y_2\} \times \{z_1, z_2\}$  (see Fig. 2) as:

$$\begin{aligned}
 f(x_z, y_z, z_z) \approx & 0.125(f_1 + f_2 + f_3 + f_4 + f_5 \\
 & + f_6 + f_7 + f_8) + 0.25(f_4 - f_3 + f_2 - f_1 \\
 & + f_6 - f_5 + f_8 - f_7)/(x_2 - x_1) \\
 & \times (x_z - 0.5(x_1 + x_2)) + 0.25(f_4 - f_2 \\
 & + f_3 - f_1 + f_7 - f_5 + f_8 - f_6)/(y_2 - y_1) \\
 & \times (y_z - 0.5(y_1 + y_2)) + 0.25 \\
 & \times (f_5 - f_1 + f_6 - f_2 + f_8 - f_4 + f_7 - f_3) \\
 & /(z_2 - z_1)(z_z - 0.5(z_1 + z_2)) \quad (18)
 \end{aligned}$$

These ideas can now be integrated into a general purpose modeling package.

### 3 SOFTWARE DESIGN AND VALIDATION

The main design specifications for ADMOP are

- (1) The package must run on a variety of PCs ranging from 386s to Pentium machines using DOS or WINDOWS as well as UNIX workstations.
- (2) Current human expertise, rules-of-thumb in processing aerodynamic data should be incorporated.
- (3) Interactive graphical display for varying the order of fit must be provided for.

Based on these requirements ADMOP is divided into 3 parts:

- (1) The front end user-interface.
- (2) The numerical routines for data manipulation and the discrete cosine transform computation.
- (3) The rule-base for recommending a trade-off between accuracy and data compression.

In order that the package can work on DOS,

Windows PCs and Unix workstations, ANSI C was chosen as the programming language. Non-standard routines specific to a particular operating system or compiler were confined to the user interface. A major difficulty was determining the number of basis functions required. Discussion with aerospace engineers revealed no clear-cut definition on what constitutes a 'good' fit. Experience with different aircraft aerodynamic databases indicated that for simplicity, polynomial basis functions used rarely went beyond eighth order but some local variation in the data will not be captured. This was perceived as an acceptable consequence since raw wind-tunnel measurements are often noisy and it would be unwise to insist on a close match. Hence high accuracy and model simplicity are conflicting demands. Ultimately this conflict was resolved by probing the given data at up to a maximum of 16 Chebyshev zeros for each variable. The interpolated data is then transformed using the DCT and the number of basis functions for each variable was determined by the number of 'significant' DCT coefficients following a user-defined cut-off level. The final function approximation is then constructed using only these significant coefficients. This approach turned out to be a suitable compromise.

A second difficulty was the need to display multi-dimensional data and function approximations on a 2-D screen. Feedback from the trial version indicated a preference to compare the function approximation with the raw data at different breakpoints. Since data involving at most two independent variables can be plotted at any one time on the screen, a color code was used to plot the data points corresponding to different values of the breakpoints for each variable. The code is not unlike that commonly used to display stress levels in finite element software and is limited only by the graphics capabilities of a particular machine. For example, using a PC with a VGA monitor at full resolution, up to a maximum of 16 breakpoints can be plotted as a function of some other variable at any one time. Another alternative was also provided which allowed the user to store any intermediate results in a specified format. These results can then be used by a post-processing package. This procedure was also extremely useful for symbolic manipulation of the function approximation and subsequent automatic generation of computer simulation routines.

The ideas discussed were further refined and a sample of the validation results using the aerodynamic database for the F16 fighter aircraft is presented here. For example, 110 data points for the Y side force coefficient as a function of the angle of attack from  $-20$  to  $20^\circ$  and the sideslip angle from  $-10$  to  $10^\circ$  is reproduced from Ref. 4 as Table 1. The size of the data array is not atypical. Sometimes even smaller step sizes ( $1-2^\circ$ ) in the angle of attack and sideslip are used in wind-tunnel testing. These data are read by ADMOP and in Fig. 3,

**Table 1.** Side-force coefficient

Sideslip angle (deg)	Angle of attack (deg)								
	-20	-15	-10	-5	0	5	10	15	20
-10	-0.1062	0.1332	0.1513	0.1833	0.2014	0.2028	0.2016	0.1837	0.1814
-8	0.0850	0.1039	0.1156	0.1449	0.1553	0.1607	0.1597	0.1473	0.1504
-6	0.0677	0.0753	0.0760	0.1055	0.1138	0.1133	0.1131	0.1069	0.1169
-4	0.0380	0.0422	0.0434	0.0662	0.0726	0.0767	0.0748	0.0652	0.0703
-2	0.0186	0.0175	0.0161	0.0325	0.0371	0.0331	0.0345	0.0298	0.0332
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	-0.0232	-0.0188	-0.0124	-0.0420	-0.0394	-0.0383	-0.0282	-0.0383	-0.0248
4	-0.0467	-0.0402	-0.0430	-0.0763	-0.0764	-0.0819	-0.0786	-0.0770	-0.0558
6	-0.0747	-0.0681	-0.0792	-0.1177	-0.1191	-0.1233	-0.1204	-0.1200	-0.0984
8	-0.1078	-0.1004	-0.1171	-0.1575	-0.1674	-0.1705	-0.1668	-0.1642	-0.1366
10	-0.1421	-0.1317	-0.1542	-0.2072	-0.2134	-0.2173	-0.2171	-0.2056	-0.1729

the magnitude of the Chebyshev coefficients computed from the 2-D discrete cosine transform is displayed in color-coded form. This plot was generated from a symbolic manipulation post-processing package developed using Mathematica.<sup>5</sup> From the plot, significant coefficients are presented up to a third

order in the angle of attack and second order in the sideslip angle. Since the lower-order coefficients are dominant, this suggests that significant data compression can be achieved. The coefficients are scanned and ADMOP recommends a model up to third order in angle of attack and second order in sideslip angle,

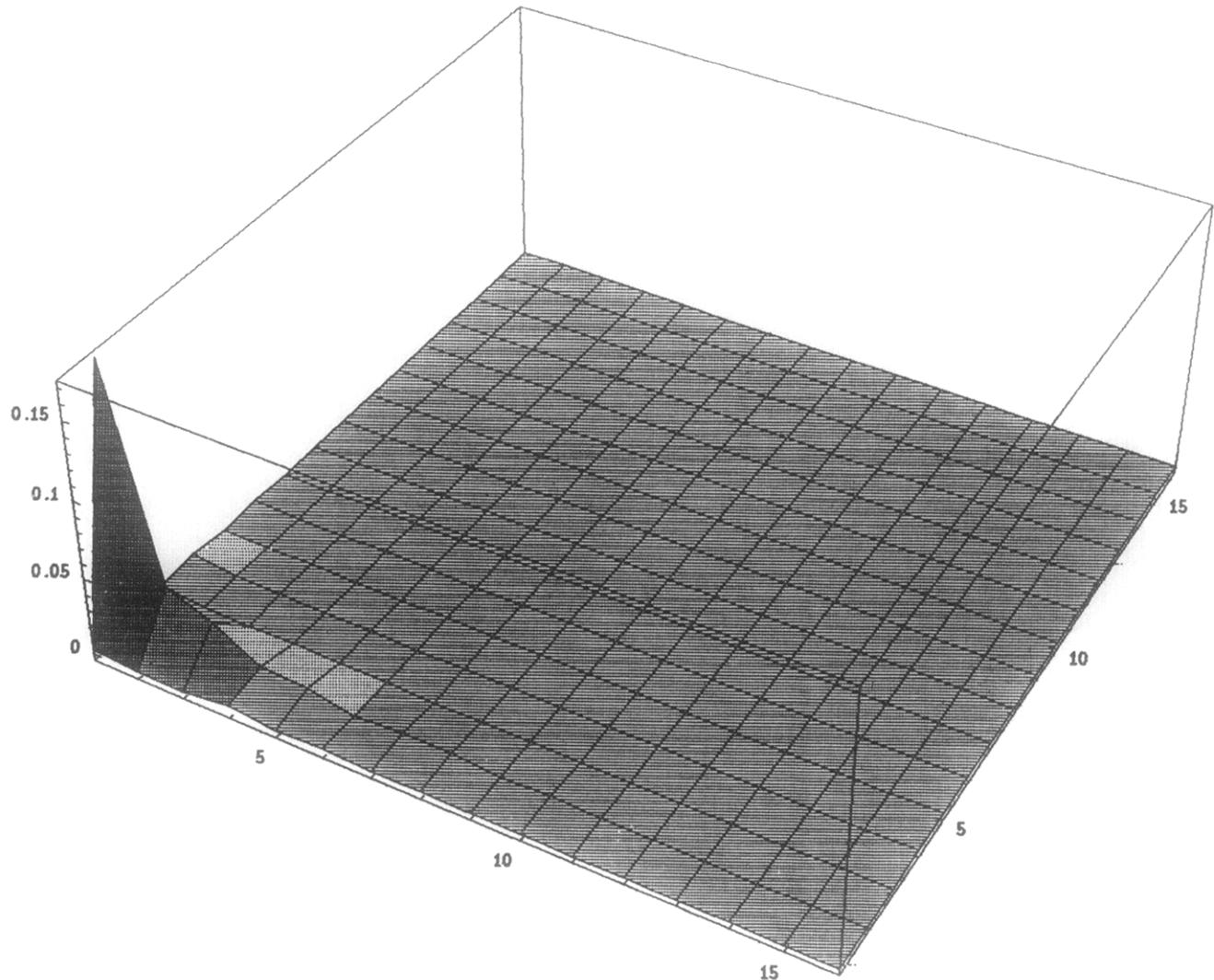
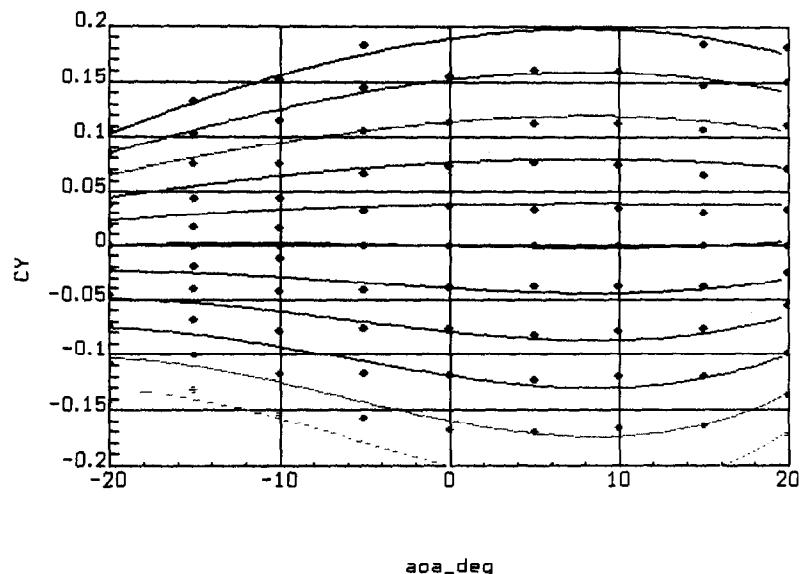
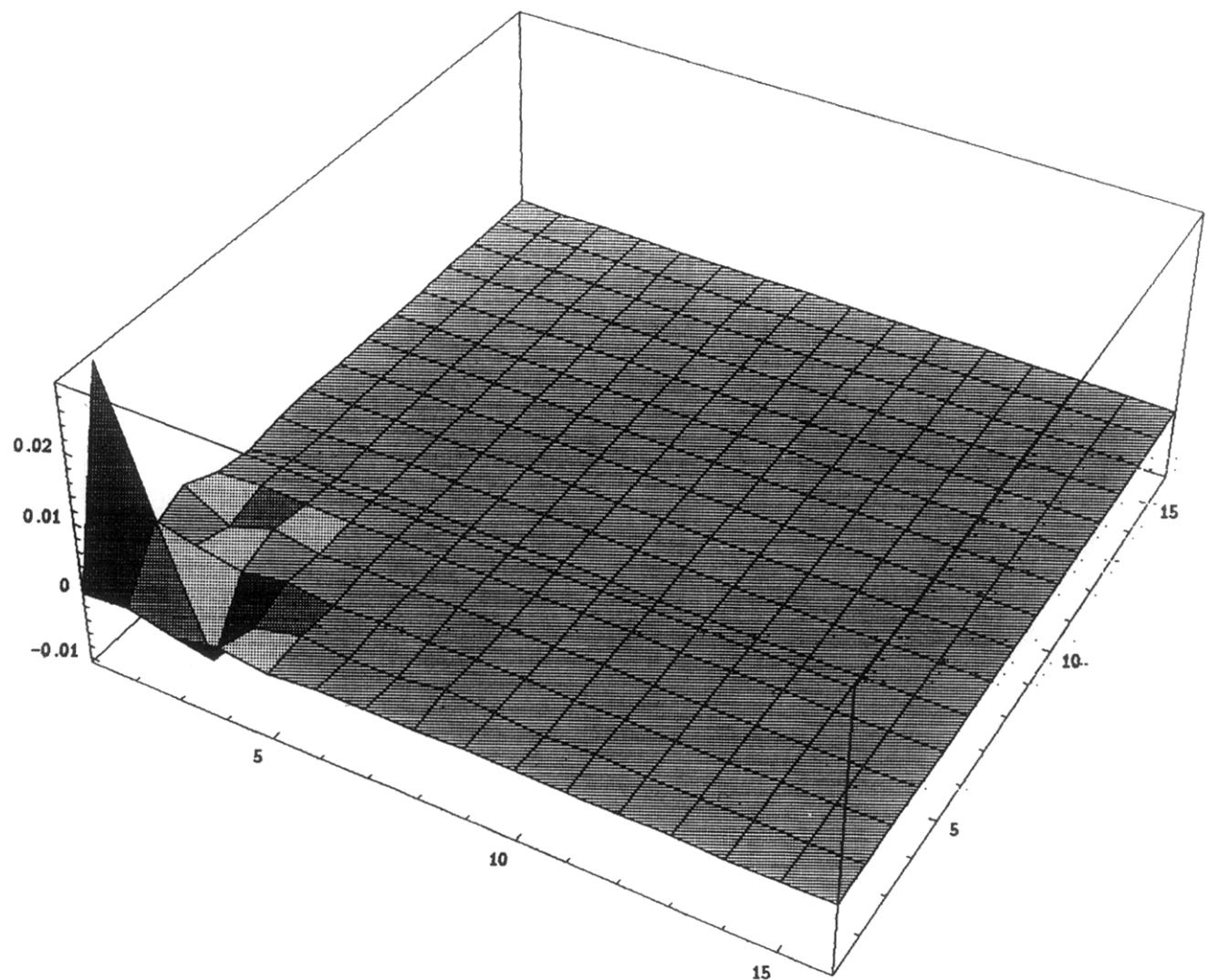


Fig. 3. Chebyshev coefficients for CY.



**Fig. 4.** Side-force coefficient  $C_Y$  as a function of angle of attack for different sideslip angles (circles: data points, lines: function approximation).



**Fig. 5.** First 256 Chebyshev coefficients for  $C_n$ .

consistent with our observations. The simplified function approximation (solid lines) and the original data (circles) for each sideslip angle breakpoint are shown in Fig. 4 as a function of angle of attack. On the computer screen, a different color is used for each sideslip angle breakpoint value. It can be observed that dominant trends in the side force are captured with a fairly close match with the original data. The resulting function approximation in terms of Chebyshev polynomials obtained from ADMOP is:

$$\begin{aligned}
 CY(\alpha, \beta) = & -0.003564 + 0.001106T_1(\alpha) \\
 & + 0.001001T_2(\alpha) + 0.003565T_3(\alpha) \\
 & + (-0.169824 - 0.035534T_1(\alpha)) \\
 & + 0.025447T_2(\alpha) + 0.007882T_3(\alpha))T_1(\beta) \\
 & + (-0.002271 + 0.001593T_1(\alpha)) \\
 & - 0.000435T_2(\alpha) + 0.002864T_3(\alpha))T_2(\beta)
 \end{aligned} \quad (19)$$

where  $\alpha$  and  $\beta$  are the normalized angle of attack and sideslip,  $-1 < \alpha, \beta < 1$ . The function approximation requires only 12 coefficients as opposed to the original 110, i.e. a 90% decrease. This explicit function for the side-force coefficient can now be used in analytical performance and flight control studies or incorporated into a subroutine for simulation studies.

Similar results can be obtained for data depending on three or more variables. For illustration, 297 data points of the yawning moment coefficient as a function of angle of attack, sideslip angle and stabilizer deflection for the F16 are used.<sup>4</sup> The data points are specified at 9 angle of attack settings, 11 sideslip angles and 3 stabilizer deflection angles. In Figs 5 and 6, DCT coefficients for  $T_i(x)T_j(y)T_0(z)$  and  $T_i(x)T_j(y)T_1(z)$  are plotted respectively for  $0 < i, j < 15$ . Based on these significant coefficients, ADMOP recommends using up to third-order Chebyshev polynomials in angle of attack and up to first-order polynomials in sideslip and stabilizer settings. This function approximation is plotted in Figs 7–9 for the three stabilizer settings. Once again,

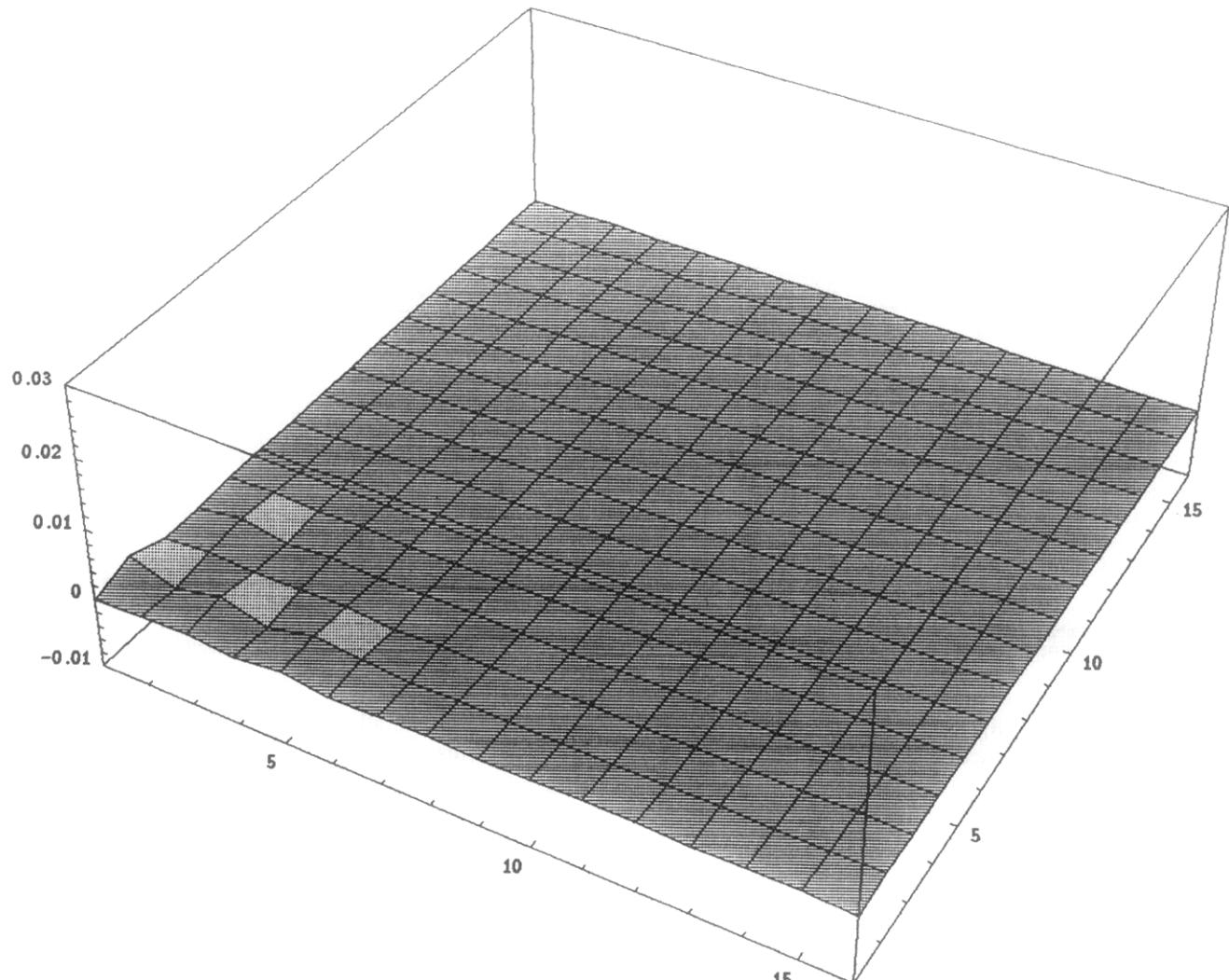
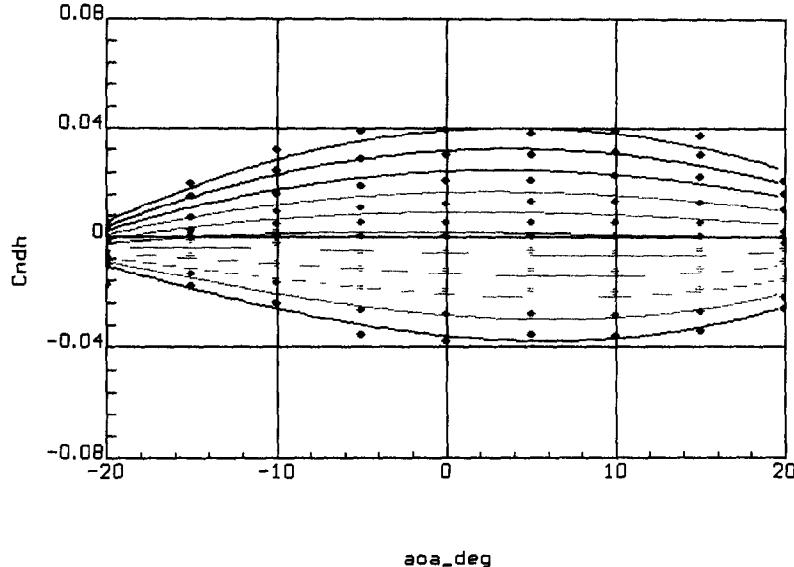
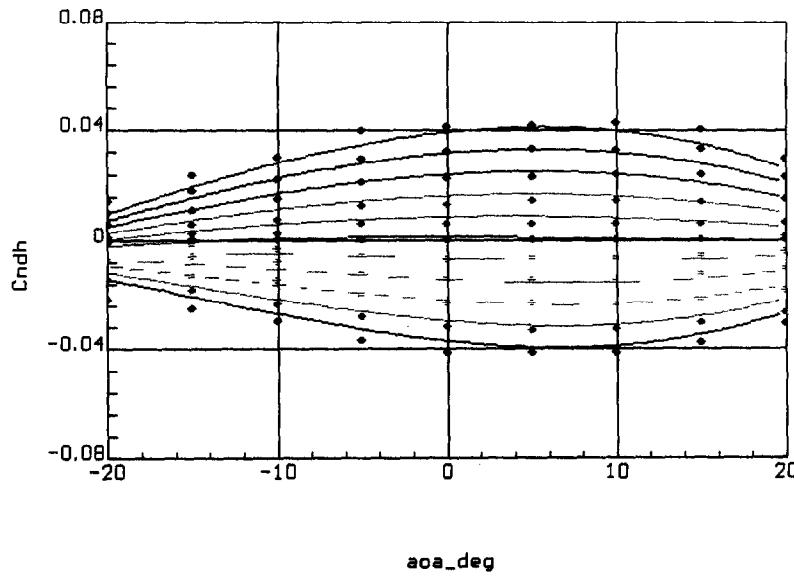


Fig. 6. Second 256 Chebyshev coefficients for  $C_n$ .



**Fig. 7.** Yawing moment coefficient  $C_n$  as a function of angle of attack for different sideslip angles,  $\delta h = -25^\circ$  (circles: data points, lines: function approximation).

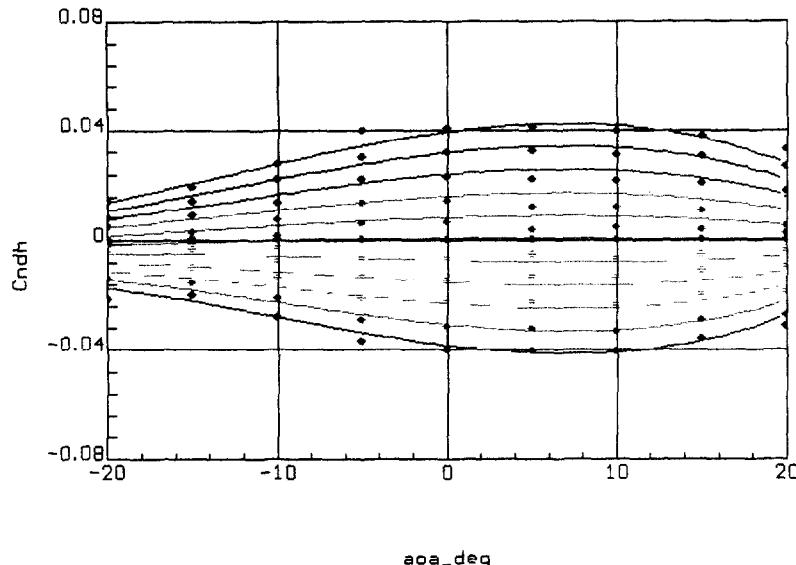


**Fig. 8.** Yawing moment coefficient  $C_n$  as a function of angle of attack for different sideslip angles,  $\delta h = 0^\circ$  (circles: data points, lines: function approximation).

dominant trends are captured with an acceptably close match. The explicit function approximation takes the form:

where  $\alpha, \beta, \delta h$  are all normalized variables. Hence only 16 coefficients are needed instead of the original 297 measurements, i.e. a 95% compression.

$$\begin{aligned}
 C_n(\alpha, \beta, \delta h) = & (-0.000070 + 0.000705T_1(\alpha) - 0.001146T_2(\alpha) + 0.000330T_3(\alpha)) + (0.028628 + 0.008793T_1(\alpha) \\
 & - 0.009567T_2(\alpha) - 0.001553T_3(\alpha))T_1(\beta) + ((-0.000104 + 0.000080T_1(\alpha) + 0.000477T_2(\alpha) \\
 & - 0.000229T_3(\alpha)) + (0.001626 - 0.000306T_1(\alpha) + 0.000897T_2(\alpha) - 0.001006T_3(\alpha))T_1(\beta))T_1(\delta h)
 \end{aligned} \tag{20}$$



**Fig. 9.** Yawing moment coefficient  $C_n$  as a function of angle of attack for different sideslip angles,  $\delta_h = 25^\circ$  (circles: data points, lines: function approximation).

#### 4 CONCLUSION

This project demonstrated the feasibility of using Chebyshev polynomials for reducing the size of a multivariable aerodynamic database. The resulting function approximation can capture the dominant trends in the data and the procedure is suitable for the reduction of wind-tunnel measurements. Up to now ADMOP has been applied to the aerodynamic and thrust database for other types of aircraft and the results in terms of data compression, model construction and time saving are encouraging. The author may be contacted through the internet at mpelsb @ nus.sg for further discussion.

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