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University of Pittsburgh, Ph.D., 1971
Geophysics

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This dissertation was presented

by

Henry L. Pollak

It was defended on

May 25, 1971

and approved by

Dr. M. D. Fuller

Dr. E. Strick

Dr. B. Hapke

Dr. M. S. Kanefsky

Dr. R. D. Wyckoff Chairman

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El cerebro mio se convirtió en una legumbre y hojas verdes cosquillean el interior de mi cabeza.

Un filosófo profundo.

<u>TO THE MEMORY OF MY FATHER</u>

who patiently guided a small boy's interest

and first steps in science.

<u>TO THE MEMORY OF MY MOTHER</u>

who seconded these efforts and who passed

away during the very final stages of this work.

... and the main reason for my enjoyment of conversation with Nature is that she is always right and any mistakes made can only be my own ... .

Johann Wolfgang Goethe.

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PREPENDIX.

TIDAL COMPONENTS ANALYZED FOR IN THIS WORK.

Component:	Excitation:	Period: hrs/cycle	Frequency: cycle/hours
	semidiurnal waves.		
K_2	lunisolar	11.97	0.0835
S_2	solar	12.00	0.0833
L_2	lunar	12.18	0.0821
M_2	lunar	12.42	0.0805
N_2	lunar	12.66	0.0790
	diurnal waves.		
K_1	lunisolar	23.93	0.0418
P_1	solar	24.07	0.0415
S_1	solar(implied)	24.00	0.0417
O_1	lunar	25.82	0.0387

1.0. INTRODUCTION.

1.1. GENERAL INTRODUCTION AND HISTORICAL SURVEY.

A good historical survey of Earth tide investigations will be found in LOVE (1911 and 1967) and, more recently, in MELCHIOR (1966). The unavoidable repetition here, for the sake of completeness, will be confined to the absolute minimum.

The first observation - indirect - and mention of Earth tides comes in the ancient times from water wells in Spain and is mentioned by Pliny. It is however only in the last century and a half that the actual theoretical foundations of Earth tides were laid down, while technological limitations have generally delayed observational work into this century. The real boost to systematic Earth tide work arrived with the initiation of IGY in 1957. The recognition that here is the only known instance in earth science when the magnitude of the exciting force is accurately known, is probably responsible to a large extent for its present day popularity. Nevertheless, such is the nature of our planet that even with the simplicity of input, the output from the system is vastly complicated, although some essentially simple parameters, such as Love numbers or their combinations, can be put forward.

The Royal Astronomical Observatory in Bruxelles, Belgium, is acting as the world's data and information bank.

LORD KELVIN observed that the oceanic tides are reduced by approximately one third of their amount by the existing Earth (bodily) tides. An equilibrium theory was used for both tides, which however is inaccurate for oceanic tides. The reason for this is overshooting of the equilibrium position by the individual water particles which are enjoying a considerable freedom of movement as opposed to the solid Earth particles which are bound by rigidity and where therefore the equilibrium theory is much more applicable. Nevertheless, a dynamical theory was applied to the bodily tides as well, for different reasons, as follows:

(i) a more accurate picture is obtainable for an Earth model which is part liquid in the interior;

(ii) application by LOVE (1911, 1967, Chapter V) in his attempt to obtain a picture of possible inertial effects on the corporeal tides (verdict: such influence is less than observational error).

Another important observation came from astronomy. CHANDLER observed that the variations of latitude had a periodicity of 427 days instead of the theoretically predicted 306 days. This deviation was correctly interpreted by NEWCOMB as being due to the tidal yielding of the Earth.

From the geophysicist's point of view the subsequent development was probably the most important one, viz. the introduction of the concept of Love numbers.

G. H. DARWIN was the first to attempt a systematic reduction of the tidal data by his own system of harmonic analysis. The development of this type of analysis is outlined in another chapter. The advent of the electronic computer in recent times has caused a number of workers to employ spectral analysis, which is more suited to modern computational technology.

The hope that Earth tide research can go a long way towards elucidation of the Earth's interior appears to be the spiritus rector behind this type of human endeavor.

1.2. PURPOSE OF OUR INVESTIGATION.

The investigation started as an attempt to open an avenue of research into Earth tides for a University department on a very restricted budget, especially a University department whose academic enthusiasm increases, but financial resources decrease with the passage of time. An investment in a Lacoste - Romberg gravimeter preceded the arrival of the author. Computer time was available and maintainance was provided by teaching obligations in the department. The cost of a few routine items, such as some office equipment and stationary, was provided by the department. Otherwise no financial assistance was obtained for the pursuit of this research. We firmly believe that such avenues were opened by our investigations, that further research in Earth tides can be engaged upon with practically no financial resources and that know-how to direct such researches is obtainable within the department and within the University.

The original effort concentrated around the determination of the δ value. The author has, throughout this work, enjoyed a remarkable measure of freedom. As work progressed, it became evident that some clarifying work could be entered upon in the future in quantitative noise removal by filters from a long tidal series. We have furthermore, developed a system of componental analysis by frequency domain interpolation rather than by filtering employing narrow Gaussian density functions. On a mere hunch the author became suspicious that perhaps too much emphasis is being laid on such componental analysis. The great variability of the δ values, and of the published phase differences from component to component <u>within one frequency band</u>, has run counter to our ideas of physically proper behavior. The results of these investigations are published in the concluding chapter. It would appear that some ideas on modulation in the resultant tidal wave might be obtained by a search for, and a study of, the sidebands resulting from such modulation. But of special importance we would consider work on theoretical models with <u>lateral</u> variations, to account for the persistent inversion in the δ values from our locality as well as from many parts of the world.

Only work which we consider to contain some inherent positive and definite conclusions, is published here. Other work, though extensive, is not availed on these pages. It involved work with Parzen filters to remove noise (needless in the shorter series); work with minimum phase functions in an attempt to discover whether a signal infinite in time (such as gravity), by acting on a physically passive medium such as the Earth (i.e. a medium whose system function is zero in the absence of an external signal) can be made causal; and attempts to discover energy irreversibility in tidal cycles.

1.3. ACKNOWLEDGEMENTS.

The research was originally suggested by Dr. Stoneley and Dr. M. D. Fuller. To Dr. A. J. Cohen the writer is indebted for many good turns during the initial steps at the University of Pittsburgh, as well as financial support as a reasearch assistant during his first year at the University. To Dr. William W. Johnson of ARCO Research in Dallas, Texas for a program of the Parzen filter as well as an explanatory write-up. To his brother Dr. Michael Pollak of the University of California at Riverside for discussions on filtering. Dr. E. Strick was always ready for a discussion and a willing travel companion on the occasional trips into the realm of metaphysics. Dr. W. Pilant provided patient instruction on the reader machine operation. Last but not least, the writer wishes to address his thanks to his academic advisor and senior partner, Professor Ralph D. Wyckoff. The association was at all times beneficial and stimulating to the writer. It can only be hoped that the writer's highly irregular habits have caused him only temporary annoyance, which hopefully was of the vanishing type only. The writer also wishes to express his appreciation to the other members of his Doctoral Committee, Drs. E. Strick, B. Hapke, M. D. Fuller and M. Kanefsky (Electrical Engineering).

The Department of Earth and Planetary Sciences of the University of Pittsburgh provided support in the form of a teaching assistantship and later a teaching fellowship in the Department, a duty which most most enjoyable and instructive to the writer. Computer time was provided, under an unsponsored research time facility, by the University of Pittsburgh at its own Computer Center on an IBM 7090 computer. Our thanks are due to Messrs. Bruce Godwin, Chuck O'Toole, Chester Page and Jay Muscaro for their everpresent willingness to help with program debugging. The writer enjoyed cordial relations with the staff of the Computer Center, but not necessarily with the computer itself.

The U.S. Naval Observatory in Washington, D.C. provided as a service the Ephemeris and Nautical Almanac on IBM data cards for the required calendar periods.

Interaction with the faculty, staff and fellow students at the Department will be a pleasant life-long memory.

1.4. DESCRIPTION OF THE SITES WHERE TIDES WERE STUDIED.

Both sites are geologically located on the Allegheny plateau; the geological substrate consists of typical Mississipian sediments of this region, such sedimentary pile being of very considerable thickness. The two sites are approximately 11½ miles apart and in all probability the main differences between these two sites are as follows:

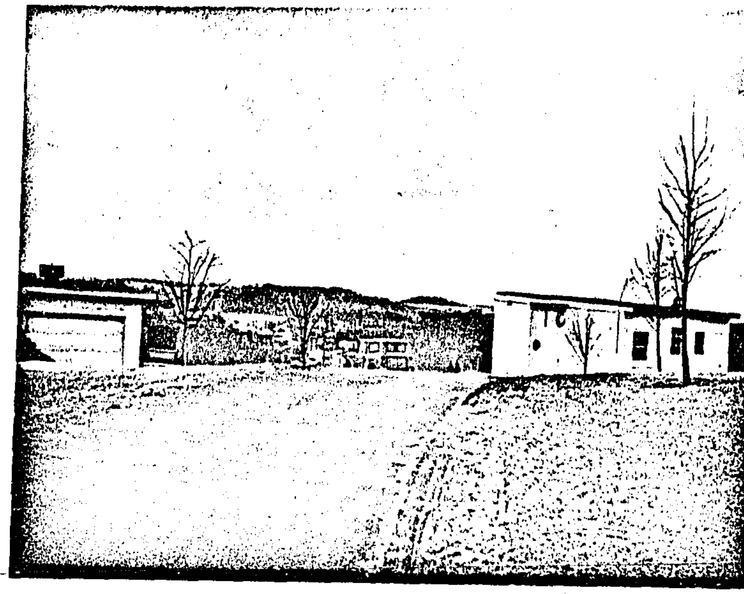
(i) The Cathedral of Learning site is in reasonable proximity of two large rivers, the Monongahela and the Allegheny, and hence the water table conditions might be somewhat different.

(ii) The Nike site is perhaps somewhat more rugged topographically and tends to be located at the highest point in its immediate area. In the vicinity of the Cathedral of Learning, on the other hand, "Cardiac hill" is towering several hundred feet over the Cathedral site.

(iii) The positioning of the instrument was on more solid foundations in the Cathedral of Learning (on an isolated seismic pier).

It is unlikely that any of these differences exert an influence of importance.

I. Nike site, Monroeville, Penna.



Geographical data:

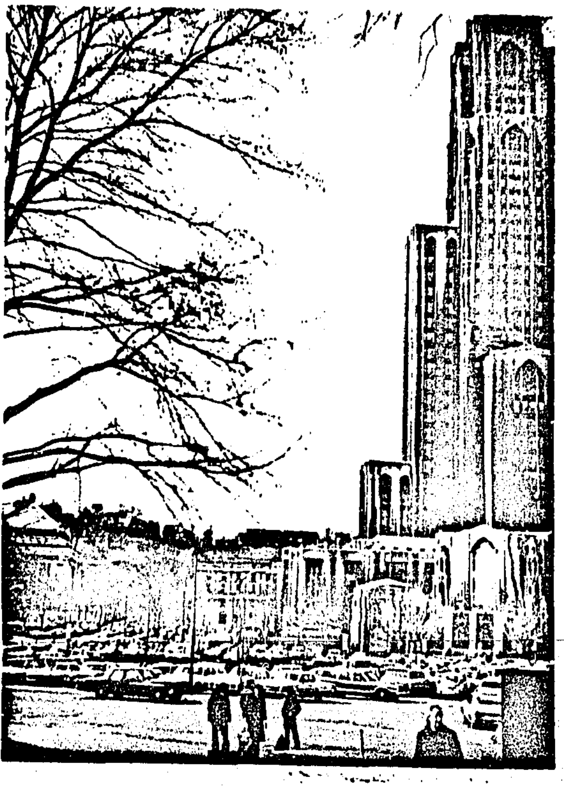
40° 28.0' N latitude;

79° 43.8' W longitude;

36720 cms elevation.

Instrument located in the corner of a cement floor in the abandoned Nike missile control building.

II. Cathedral of Learning, University of Pittsburgh, Oakland Boro, Pittsburgh, Penna.



Geographical data:

40° 27.0' N latitude;

79° 57.2' W longitude;

27300 cms elevation.

Instrument located in Room #24 in the basement on a seismic pier.

2.0. INTRODUCTION INTO THE THEORY OF TIDES.

2.1. THE TIDAL FORCE.

2.1.1. Dynamic view.- The deformation produced by the attractive forces between two bodies essentially consists of two bulges on the opposite sides of the Earth. The bulge adjacent to the disturbing body is easily explained by the attraction, but the opposite bulge can only be explained in terms of a dynamical theory. For orbital stability around a central locus, the smaller the orbital radius, the higher must be the linear orbital velocity of an orbiting point. If a discrete body orbits, such as a sphere, the cohesion prevents the orbital stability considerations to come into play and the reverse actually happens. To counterbalance this and reestablish orbital stability, the extrema tend to move away from the center of gravity, thus producing the two tidal bulges. We should like to point out here that since the diameter of the Earth is much smaller as compared to the Earth - Sun radius vector than as compared to the Earth - Moon radius vector, the two opposite bulges due to the Sun will not be significantly different in size, whereas in the case of lunar deformation, the proximal bulge will be significantly larger.

2.1.2. Static view; the gravitational and tidal potentials.- A brief summary of the gravitational and tidal potentials will be given in this section. Although a close relation exists between the two, there are nevertheless important differences. The most important question, in our opinion, is the relation between the magnitude of the forces descended from these potential functions and the distance between the particular celestial bodies under our consideration.

The gravitational potential is a scalar function from which the pure attractive forces are derived by forming the gradient of this function. The development is familiar from elementary physics textbooks and only the result will be stated here:

Let R = distance between the centers of two celestial bodies, then the potential function U will be proportional to 1/R. The derivative is taken with respect to R, so that the gravitational acceleration will be proportional to $1/R^2$.

The tidal potential is developed somewhat differently. (For a complete development see MELCHIOR (1966), pp. 13 - 15). The tidal force is not a pure force of attraction but a difference in either the forces of attraction (at a point) or of attraction and centripetal acceleration, depending on the depiction and the statement of the problem. This resultant force, the tidal force, can be mathematically descended by forming the gradient of the potential function W_2, where

$$W_2 = \frac{fm}{2} \frac{a^2}{R^3} \left(3 \cos^2 z - 1\right) \tag{1}$$

and

f = gravitational constant;

m = mass of the disturbing body;

R = defined as before;

a = radius of the disturbed body (we are considering surface tides only);

z = local coordinates of disturbing body.

This static view is fundamental for the illustration of the action of the tidal forces and in the first instance is applicable to both rigid and yielding Earth models. The tide-generating potential W_2 on the surface of the Earth will be a resultant of (i) the potential of the Sun, (ii) the potential due to the Moon, (iii) the potential of the Earth, all at the particular point under consideration and neglecting the influences of any other celestial bodies.

The <u>radial component of the tidal force</u> (i.e. the tidal change in gravitational acceleration g) is represented by forming the derivative with respect to <u>a</u> (with a negative sign) and not with respect to R, as was the case with gravitational acceleration.

$$dg = -\frac{\partial W_2}{\partial a} = -f m \frac{a}{R^3}\left(3\cos^2 z - 1\right) \qquad (2)$$

The important implication is that the radial tidal force will be inversely proportional to the cube of the centric distance between the two celestial bodies. (This applies to the lowest harmonic, which contributes about 98% of the total potential. The proportionality of the higher harmonics will successively grow by a term 1/R. These higher harmonics are not considered in the present work).

Equation (1) is inconvenient for practical consideration, since it necessitates a clumsy local co-ordinate system. It is therefore generally redeveloped in terms of astronomical co-ordinates and represented in the following form:

$$W_2 = G\left(\frac{c}{R}\right)^3 \Big\{ \cos^2\phi \cos^2\delta \cos 2H + \qquad (\underline{A})$$

$$+ \sin 2\phi \sin 2\delta \cos H + \qquad (\underline{B})$$

$$+ 3\left(\sin^2\phi - \tfrac{1}{3}\right)\left(\sin^2\delta - \tfrac{1}{3}\right)\Big\} \qquad (\underline{C}) \quad (3)$$

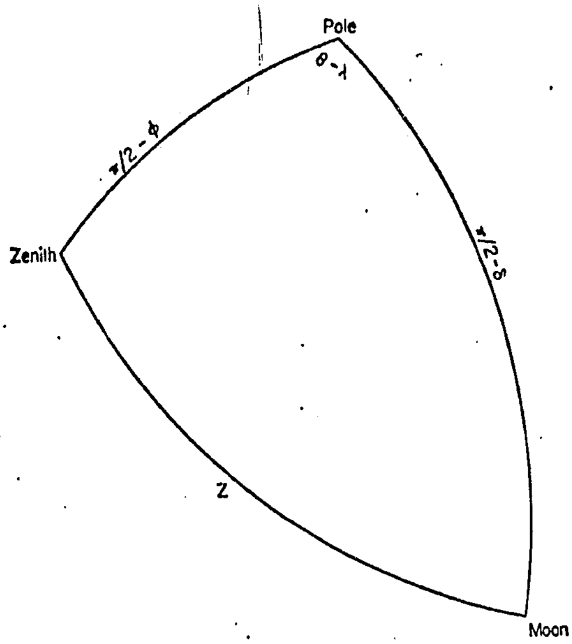


Fig. 1 - The astronomical spherical triangle (after MELCHIOR (1966)).

where (see Fig.1.)

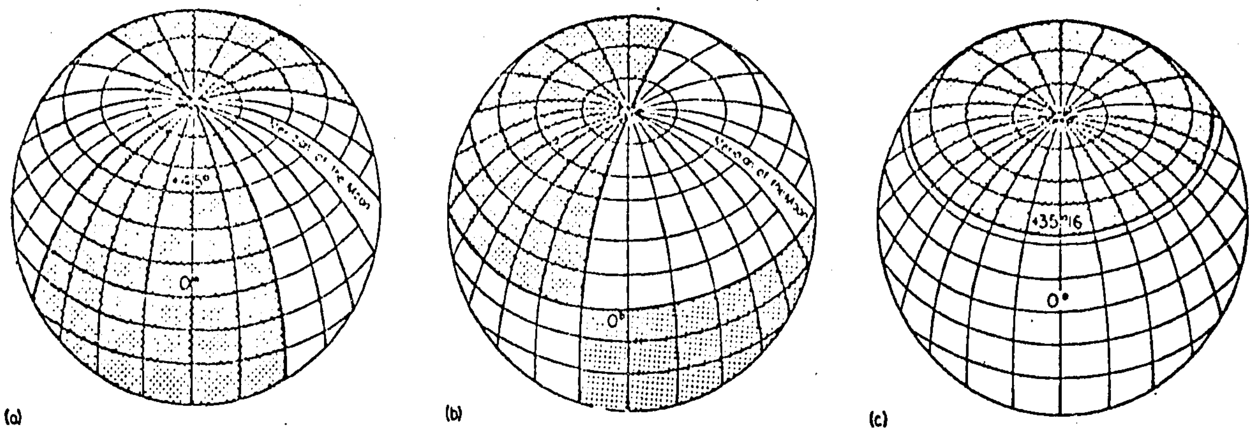
H = hour angle; δ = declination (equatorial co-ordinates)

ϕ, λ = astronomical co-ordinates of place of observation

G = Doodson's constant (= 26.206 $cm^2.sec^{-2}$)

c = half-major axis of the orbit under consideration.

The fraction c/R is thus a measure of the instantaneous deviation from orbital circularity, It also should be pointed out here that our important term $\underline{a}$ is hidden in Doodson's constant, the latter being of the form $G = G(a)$; however, for surface tides such as we consider in this work, $G(a)$ becomes a constant.



sectorial tesseral zonal

Fig. 2 - Types of tides in terms of spherical harmonics (MELCHIOR (1966)).

The picture is best obtained by means of <u>spherical harmonics.</u> These are a solution of the Laplace equation in spherical co-ordinates (and, we should note, that any solution of the Laplace equation is harmonic). The essential part of the solution is the polynomial coefficient P_l^m, generally multiplied by a weighting polynomial, where m = = order of differentiation and l = order of the polynomial. If m = 0 the result is a <u>zonal harmonic</u>, fundamental to all; all others are derived from it by differentiation and are known as <u>tesseral harmonics</u>, such that $m < l$. A special case holds for the condition m = l, known as a <u>sectorial harmonic.</u> There are 2l + 1 tesserals possible. For the 2nd harmonics l = 2. Of paramount importance are the zeros of the polynomials which produce nodal lines. <u>l</u> produces nodal latitudes and <u>m</u> nodal longitudes. Besides, there may be nodal poles. The number of nodal lines in this manner determine the order of harmonic and their distribution its type. The treatment of the potential is basic to tidal concepts (Fig. 2.).

(a) Sectorial tides.- The nodal meridians are at 45^{o} to the disturbing body. They are represented by the term A in equation (3). The term 2H shows that they are semidiurnal and they have a maximum amplitude at the equator. Variations in the distributions of the masses (i.e. yielding) do not modify the pole of inertia, nor the moment of inertia, on which the speed of the Earth's rotation depends.

(b) Tesseral tides.- These are represented by the term B in equation (3). They are always zero at the equator and their maximum is reached at 45^{o} latitude. Yielding affects the pole of inertia but not the moment of inertia; dissipation of energy is therefore possible. The term H shows that these tides are diurnal.

(c) Zonal tides.- These are longer period tides which depend only on the latitude as inspection of the term C will reveal the absence of the expression H there. The nodal lines are the parallels $+35^{o}16'$ and $-35^{o}16'$. They are only affected by the declination of the celestial disturbing body, therefore they are fortnightly for the Moon and 6-monthly for the Sun. Yielding of masses does not affect the pole of inertia but affects the moment of inertia and hence the speed of the Earth's rotation is affected with the corresponding periodicities. This tide will depress the level surface approx. 28 cms at the pole and raise it about

14 cms at the equator.

The gravitational component of the sectorial and tesseral tides are of immediate concern to us in this work. Due to the length of records studied, no work was possible on the zonal tides.

A brief summary of the genesis of the tidal waves is here appended:

Tide:	Cause:
diurnal and semi-diurnal	rotation of the Earth and inclination of the axis;
fortnightly and monthly	variation in the declination of the Moon;
semi-annual and annual	revolution of the Earth around the Sun.

It is of interest to note, at this stage, that the tide generating forces are the only geophysical forces whose magnitude and direction are known a priori.

2.2. THE TIDAL COMPONENTS.

To obtain a purely harmonic development, DOODSON has introduced 6 independent variables:

(1) mean lunar time;

(2) mean longitude of the Moon;

(3) mean longitude of the Sun;

(4) longitude of the lunar perigee;

(5) longitude of the ascending node of the Moon;

(6) longitude of the perihelion.

This makes possible the separation into a very large number of tidal waves (components), of which only some are important.

2.2.1. Semidiurnal (sectorial) waves.-

M_2 lunar; period (P) 12h25m14s; due to fictitious Moon describing a circular orbit in the plane of the equator with a velocity equal to the mean velocity of the real Moon.

N_2 lunar (P = 12h39m) and L_2 lunar (P = 12h11m) due to the Moon's elliptical orbit.

K_2 lunar (P = 11h58m) due to the Moon's orbit not being in an equatorial plane.

S_2 solar (P = 12h00m) and K_2 combine together into a single lunisolar wave.

The Sun introduces important perturbations in the Moon's orbit (evection and variation) which in turn produce tidal waves, but we shall not consider these here.

2.2.2. Diurnal (tesseral) waves.-

There is no wave corresponding to M_2, i.e. M_1. The lunar waves are only elliptical and declensional. Important are:

K_1 lunisolar wave with a period of exactly one sidereal day 23h56m04s.

O_1 lunar, P = 25h49m10s.

There is again no fundamental wave S_1, since the declination of the Sun at the equator is zero. The lunisolar wave K_1 has the same period as the theoretical solar wave S_1 and is therefore an implied S_1 wave. The component S_1 in our work refers to this wave K_1.

2.2.3. The long period (zonal) waves.-

These waves are unimportant from our point of view, since our records are too short for their detection. They are fortnightly and monthly (lunar), since $d = 27.321$ mean solar days period; also solar annual and semiannual. There is also a 18.61 years lunar wave due to the revolution of the nodes of the lunar orbit.

It is of great importance to stress that these waves are not harmonics of each other. Some important considerations will arise from this in connection with analytical procedures employed.

2.3. THE YIELDING EARTH.

2.3.1. Up to now no attention was paid to the actual behavior of the Earth. It can be shown - but by no means will be done here - that on a completely rigid Earth the tidal effect will be as follows:

(i) changes in the gravitational acceleration of about 0.2 miligals (1 gal = 1 $cm.sec^{-2}$ acceleration);

(ii) an instantaneous deviation from the vertical (0.04").

On a yielding Earth (i) will be enhanced, because the instrument will move with the swell, thus increasing the effect due to its alternately greater/smaller distance from the center of the Earth. The ratio (observed change in gravity)/(change in gravity on rigid Earth) = gravitational magnification factor δ, which is an important quantity whose variation in different geological environments is one of the major objects of Earth tide research efforts.

The effect of (ii) on the other hand will be a decrease in the deflection of the vertical. No studies along these lines were made in this work and the theory will not be elaborated on here in this respect, although work along these lines is becoming very popular with numerous Earth tide investigators.

The Earth's free oscillations are of the order of 1 hour period and the shortest 2nd harmonic tidal period is

12 hours. We therefore can use an equilibrium tidal theory, since no resonance can occur. It should be noted here that any elastic parameters here might be different to those from seismic data where the periods are much, much shorter. Furthermore the type of deformation, i.e. elastic, plastic, visco-elastic etc. remains completely unspecified here. No doubt, all are present to some extent. The elastic effects will preponderate in the shortest periods, whereas the others will be increasingly felt with increasing periods.

2.3.2. Love numbers. LOVE gave simple expressions to these amplification factors in terms of parameters (Love numbers) which are related to the yielding properties of the Earth and its density. Each type of elastic deformation can be described in terms of these numbers (the underlaying theory is the Herglotz equation, a differential equation relating the distribution of the rigidity moduli and densities through the Earth).

LOVE's basic postulate and assumption is that, since the disturbing potential is closely enough represented by a 2nd order spherical harmonic, the produced disturbance should likewise be expressible by 2nd order spherical harmonics, employing proper coefficients, which are the Love numbers, or combinations thereof.

Let ξ = radial shift

D = cubical dilatation

V = potential due to the deformation,

then

$$\xi = \frac{H(r)\, W_2}{g} \tag{2}$$

$$D = \frac{F(r)\, W_2}{g} \tag{3}$$

$$V = K(r)\, W_2 \tag{4}$$

The changes then will be expressed as

$$V = \frac{L(r)}{g} \frac{\partial W_2}{\partial \theta} \tag{5}$$

in the meridian

$$W = \frac{L(r)}{g \sin\theta} \frac{\partial W_2}{\partial \lambda} \tag{6}$$

in the prime vertical.

At the surface $r = a$, and

$$\left.\begin{aligned} H(a) &= h \\ K(a) &= k \\ L(a) &= l \\ F(a) &= f \end{aligned}\right\} \quad \text{Love numbers}$$

Hence $\xi = \dfrac{h\, W_2}{g}$, $h = \dfrac{\xi g}{W_2}$

where W_2, the tidal potential = height of static oceanic tide at the surface and ζ = height of the earth tide

and similarly for the others, which this work is not concerned with.

The interpretation of the Love numbers is as follows:

h = ratio of the Earth tide to the height of the corresponding ocean tide;

k = ratio of additional potential, produced by this deformation, to the deforming potential;

l = ratio of the horizontal displacement of the crust and the corresponding static oceanic tide at the surface;

f = the ratio of the cubic expansion and of the height of the corresponding static tide at the surface.

The Love numbers are involved in all types of tidal deformation in arithmetical linear combinations as follows:

(1) $\gamma = 1 + k - h$; the deflection of the vertical with reference to the Earth' crust. Also γ is the ratio (Earth tide)/(static oceanic tide), i.e. $\gamma < 1$.

(2) $\delta = 1 + h - 3/2\ k$; the gravitational magnification factor, the object of our investigations. We have always $\delta > 1$.

(3) The variation in the speed of the rotation of the Earth is a function of the Love number k. This number, therefore, can be determined from Chandler's wobble. Non-coincidence of the instantaneous axis of rotation and of the axis of inertia creates a stress field which causes a wobble whose

period should be 306 days (rigid Earth), but is 427 days (deforming Earth).

(4) $\Lambda = 1 + k - l$; represents the changes in the vertical with reference to the Earth's axis.

Individual Love numbers can be obtained by making two different tidal determinations such as γ and δ , each determination giving a pair of Love numbers in linear combination. Love numbers can also be obtained by calculating out various Earth models and in this manner they enable us to match the real Earth to one of its models.

2.3.3. Nonideal elasticity. So far we have investigated tides on a rigid Earth and on a perfectly yielding Earth. We shall now pay some attention to a "nonideally" yielding Earth. Such yielding we shall define as being one in which the stress - strain relationship is time dependent, i.e. the response is not instantaneous and the maximum response is reached some time after the maximum impulse.

This implies that the maximum height of the tide will not be at the line joining the centers of the Moon and the Earth, but the maximum bulge is delayed and thus will be carried forward by the rotation of the Earth. As a consequence of this the following events happen:

(i) Because of the late response to the signal, the gravitational pull on the bulge is assymetrical to the line of centers and this gives rise to a torque on the Earth and an equal and opposite torque on the Moon.

(ii) One component of the torque tends to accelerate the Moon and retard the rotation of the Earth, provided that the angular velocity exceeds the orbital velocity of the Moon, i.e. the bulge is carried forwards, otherwise the opposite will happen.

(iii) The other component of the torque tends to tip the orbital plane of the Moon and the Earth's axis of rotation.

(iv) The torques transfer angular momentum. A loss of mechanical energy from the Earth - Moon system also ensues. The kinetic energy of rotation thus partly dissipates and partly is converted into kinetic and potential energy speeding up the Moon in its orbit.

A purely radial tidal component would not transfer angular momentum, since no torques would develop. If the angular velocity of the Earth's rotation is equal to the angular velocity of the Moon's revolution, no torques are established, but energy is still dissipated in the bulges if the elastic response is nonideal.

The frictional effects of the terrestrial tides are supplemented by the effects of the lunar tides, which will not be developed here. The situation becomes more complex still if the Sun's effects is introduced as well. For detailed analysis see MCDONALD (1964).

An important consequence of nonideal elasticity, and hence tidal friction, is that no matter whether angular momentum is transferred or not, mechanical energy is lost from the Earth - Moon system. This produces numerous quantitative and semiquantitative speculations on the past and future of the Earth - Moon system, exhaustively treated by MCDONALD. It is of interest to note that, since the tidal force is inversely proportional to the 3rd power of the distance, any decrease in the distance of the exciting celestial body could produce a substantial heating effect by tidal dissipation. The time lag of the arrival of the nonideal bulge is astronomically calculated at about eight minutes of time (MACDONALD).

There is another interesting question in connection with tidal dissipation, namely, where as a function of depth in the Earth is the sink?

2.4. TIDAL EARTH MODELS.

2.4.1. A complex system such as the Earth can only be studied effectively by matching it to some idealized model with respect to the properties which are being under investigation. A brief survey of some of the more important models will be given here.

(1). The first tidal Earth model was proposed by LORD KELVIN, who assumed homogeneity, isotropic elasticity

(Hooke's Law), incompressibility (i.e. $\lambda = \infty$) and he obtained the following final solutions

$$h = \frac{5}{2}\left(1 + \frac{19\mu}{2g\rho a}\right)^{-1}$$

where μ = Lamé's constant (rigidity) and k = 3/5 h, the guiding assumptions being of course, grossly oversimplified.

(2). HERGLOTZ introduced a much more elaborate theory in which the rigidity and density are a function of the radius, but the Earth is still assumed to be incompressible; the result is a 6th order partial differential equation (the Herglotz equation), expressing the above relationship and which reduces to a Clairault type equation (which relates flattening to the radius in a <u>fluid</u> spheroid of rotatation), taking $\mu = 0$. Numerous methods were proposed for the numerical integration of the Herglotz equation, but the equation is not geophysically very realistic because of some of the original assumptions.

(3). Later models were worked out by TAKEUCHI with a more rational approach (he does not assume incompressibility) and further improvements still were made by MOLODENSKY.

<u>2.4.2. The dynamic effect of the liquid core.</u> We have seen that the tesseral part of the lunisolar potential forms the couple responsible for the precession and nutation phenomena (precession = "spinning top" gyration; nu-

tation = wobble). Thus, for each precession-nutation term there is a corresponding component of diurnal (tesseral) tide. To the lunisolar precession corresponds the lunisolar wave K_1, the semiannual solar nutation corresponds the solar wave P_1 and the fortnightly lunar nutation corresponds to the lunar waves O_1 and OO.

JEFFREYS pointed out that in case of deformations producing a shift in the small axis of inertia - such as is produced by the tesseral tides - a static theory (model) does not express the conditions at the mantle - core boundary and a dynamic theory must be resorted to. Workers such as POINCARÉ, HELMHOLTZ, etc. worked before him on problems of liquid cores, but did not propose any models. JEFFREYS picked up POINCARÉ's theory and pointed out the following geophysical significancies:

(i). A model consisting of two elastic media gives too large a value for the Chandler wobble period.

(ii). The model with the liquid core and rigid mantle reduces to the Chandler period.

(iii). The free period of the fluid globe would be infinite.

JEFFERYS and VICENTE introduced a model using TAKEUCHI's model for the elastic mantle and POINCARÉ's theory for the liquid core; the numerical integration leads to conflict with observation; a newer model introduced by MOLODENSKY agrees well with the observations. He has shown that the k/h ratio is almost unaffected by dynamic effects in the core and always close to 0.5 for all semidiurnal and diurnal waves. (JEFFREYS & VICENTE find k/h = 0.493 for the semidiurnal and 0.412 for the diurnal waves and this discrepancy cannot be explained). It is also of interest to note that virtually all models give for the gravitational magnification factor δ the result that δ should increase with the period of the tidal wave.

Survey of some models.

Worker:	Basic postulate:	k/h:
KELVIN	homogeneity	0.600
PREY	Roche's density law, rigidity law (3.38)	0.518
PREY	" " " " (3.39)	0.545
BOAGA	Trinomial density law " 3.38a	0.497
JEFFREYS	Wiechert's constitution 2 homo. media	0.549
ROSENHEAD	" (more up to date data)	0.531
TAKEUCHI	Bullen's density law; rigidity and compressibility according to seismological results	0.457 - 0.494
MOLODENSKY	various models	0.500
OBSERVATIONS	k/h = 0.414 or 0.479, depending on δ .	

2.5. SOME FURTHER CONSIDERATIONS OF TIDAL FRICTION.

The brief discussion under this heading addresses itself primarily to an outline of its importance on the eluciadtion of the elastic parameters of the Earth. We shall not dwell here on the very important aspects of the influence of tidal dissipation on orbital mechanics, etc., as was briefly outlined already.

The energy dissipation is frequently referred to as Q, such a quantity being mentioned essentially in connection with two processes in planetary bodies:

2.5.1. Tidal friction Q. This represents the dissipation of tidal energy and it has been determined already for several planets from orbital characteristics; this general term refers to loss processes within the planet or near its surface. In the absence of an atmosphere and ocean such a loss can be totally attributed to various mechanisms of dissipation within the planet and can in this way be considered a measure of the physical state and the temperature in the interior of the particular planet. On this basis, there may be two groups of planets (LAGUS *et al.* (1968)).

(i) Low Q planets, with a Q value range 10 - 100; this group comprises the terrestrial planets.

(ii) High Q planets with a Q value of the order 10^5; comprises the Jovian planets.

<u>2.5.2. Seismic Q.</u> This refers to the attenuation of the Earth's free oscillations excited by a major earthquake. This value of Q is in fact obtained from free oscillation determinations.

If there were no oceans and if Q is frequency and amplitude independent (which it is for supposedly low strain values (LAGUS *et al.* , *op. cit.*)), then these two Q's should have very similar values.

Q is defined as

$$Q^{-1} = \frac{1}{2\pi} \frac{\Delta E}{E}$$

where E = maximum energy stored in a cycle;

ΔE = energy dissipated in a cycle.

It was further shown that for a spherically symmetrical gravitating body, such as most Earth models are,

$$Q^{-1} = \tan \delta$$

where δ = phase lag, the major sources of inaccuracies being the effects of the ocean tides as well as instrumental characteristics.

Different harmonics of tides sample the depths in the Earth in a different manner, since the elastic energy is stored in different ways with depth.

From the preceding formula it can be easily seen that the phase determination is of great importance in the elucidation of the elastic properties of the Earth with possible additional information obtainable on the layering of the Earth. However, it should be clearly understood that the frequency/amplitude dependence of Q is different for different types of solids (e.g. Kelvin solids, etc.) (SLICHTER (1963)). In addition, according to some workers (MUNK & MACDONALD (1960)) not all dissipation is referrable to nonideal elasticity. According to these workers, energy is also dissipated in the grinding of continental blocks.

The problem of Q will not be elaborated on in this work, our sole purpose here being the focusing of the reader's attention on the importance of phase determinations in the elucidation of the Earth's elastic properties. We also might point out fleetingly that we prefer the term "arrival time difference" to phase difference for reasons which will become more evident in the Chapter on phase results (7.0). It will be shown there that many weaknesses still exist in the determination of this particular quantity.

3.0. INSTRUMENTATION. THE LACOSTE - ROMBERG GRAVIMETER.

This instrument is designed to measure the variation in gravity at any given location with an accuracy of better than 1 microgal (μgal). The salient features are shown in Figure 3.

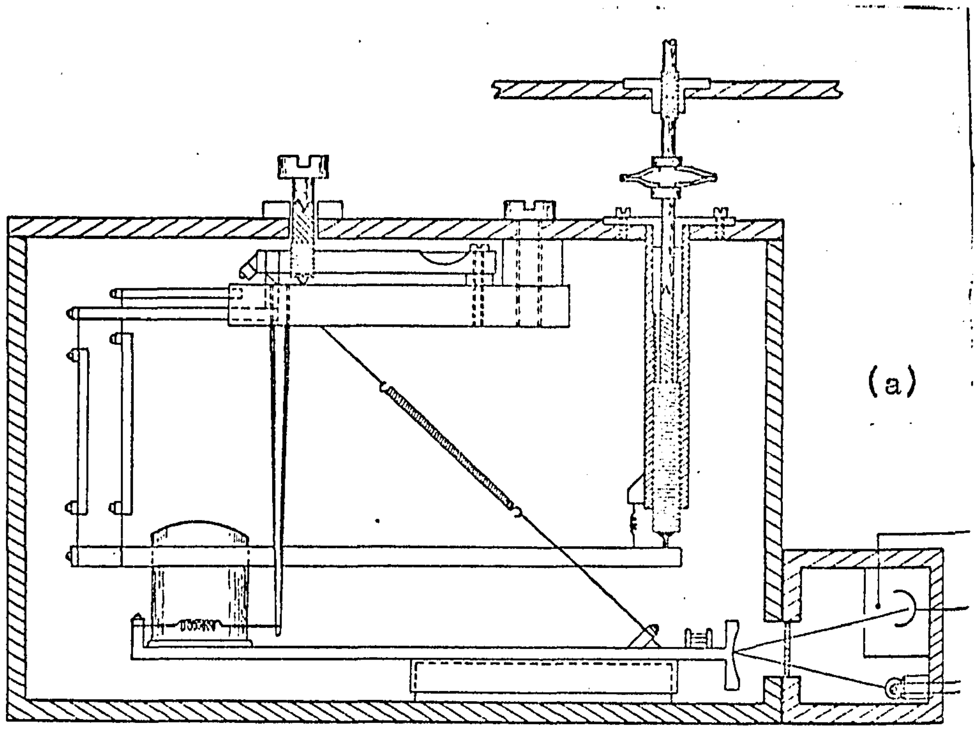
<u>3.1. Description of the instrument.-</u> The heart of the instrument is the gravity sensing system, essentially an astatized system of a weight supported by a "zero length" spring. Light reflected from the mirror on the weight is converted into an a.c. current.

The gravity sensing device is held to within 10^{-7} in. of its null position by a servo system stabilized by a tachometer and the ensuing time lag is corrected by another servo system.

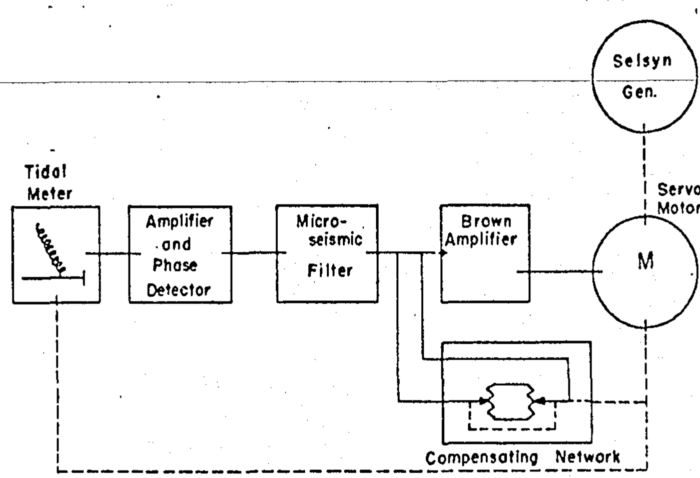
The converted a.c. current is amplified and phase corrected in the control box. The sign and magnitude of the resulting d.c. voltage corresponds to the position of the gravity sensing system with respect to the null position - - i.e. the voltage is proportional to the beam deflection.

A measuring screw is geared to a synchro transmitter which transmits a signal to the strip chart recorder.

The temperature inside the instrument is held constant to within $.001^{\circ}C$ at $39.7^{\circ}C$.



(a)



(b)

Fig. 3.

(a). LaCoste-Romberg gravity meter with photoelectric reading system.

(b). Control system for tidal gravity meter employing drift free photoelectric optical system.
(CLARKSON & LACOSTE (1956, 1957)).

<u>3.2. Theory of the instrument.-</u> The fundamental papers for comprehensive information are by CLARKSON & LACOSTE (1956), CLARKSON & LACOSTE (1957) with amplifications by HARRISON, NESS <u>et. al.</u> (1963). The last named authors provide the most up-to-date treatment with some amplifications on CLARKSON & LACOSTE.

One problem is that the gravimeter may be unstable if the response of the nulling system is too rapid, in which case the servo system can enhance the oscillations; this tendency is combated in the design of the instrument by a feedback system.

For the sake of completeness many of the equations of motion of the instrument as well as some transfer functions will be given here, albeit without their derivations. For greater details the reader is referred to the above-mentioned papers.

Let θ_i = input to the system

θ_o = output from the system

ϵ = difference signal

θ' = motor output

J = inertia of motor gear train

f = viscous damping

K_1 = motor gain constant

K_2 = gear gain constant

m = mass of mechanical oscillator

k = stiffness constant

c = damping factor

K = gain of the system.

The relevant equations of motion are

$$\epsilon = \Theta_i - \Theta_o$$

$$J\ddot{\Theta}' + f\,\Theta' = K_1 K_2 G \epsilon$$

$$m\ddot{\Theta}_o + c\dot{\Theta}_o + K\Theta_o = k'\Theta'$$

in Earth tides $J\ddot{\Theta}' \lll f\dot{\Theta}'$

hence

$$\Theta' = \frac{K_1 K_2 G}{f}\int \epsilon\, dt$$

giving the transfer functions for the elements

$$Y_1(j\omega) = G, \quad Y_2(j\omega) = \frac{K}{j\omega}, \quad K = \frac{K_1 K_2 G}{f}$$

$$Y_3 = \frac{\omega_o^2}{(j\omega)^2 + 2\kappa j\omega + \omega_o^2}, \quad 2\kappa = \frac{c}{m}, \quad \omega_o^2 = \frac{K}{m}$$

A lead network is built in to provide extra stability so that the total transfer function of the servo loop will be

$$Y(j\omega) = \frac{K\omega_o^2}{j\omega\,\frac{jT\omega + \alpha}{jT\omega + 1}\left(\omega_o^2 - \omega^2 + 2j\kappa\omega\right)}$$

where ω_o = angular natural frequency of the system;

α = ratio of the sum of resistors to shunting resistor;

$T = \frac{1}{\omega_o}$ = time constant of parallel condenser and resistor.

The measuring system is photoelectric. Such a system is normally subject to drift caused by changes in the photocell and the illumination source.

The improved tidal gravimeter has overcome the following difficulties:

(i) the drift in the photoelectric optics;

(ii) improvement in the servo loop which was hitherto overly sensitive to gain and phase adjustment.

CLARKSON & LACOSTE (1957) discuss the requirements needed to obtain a drift free photoelectric system; a transfer function for the improved servo loop is likewise discussed. It could also be noted here that the automatic control system is a speed proportional servomechanism.

The stability of the system can be studied by plotting transfer functions against frequency or phase. (CLARKSON & LACOSTE (1956)).

Another method of studying the stability of a system was done by HARRISON et al. (1963) by considering the equation of motion of the whole system

$$\dddot{\phi} + 2\omega_0 \ddot{\phi} + \dot{\phi} + \omega_0^2 KL\phi = Kg' \qquad [K = K_1 G/f]$$

where ϕ = angular position of the measuring screw;

L = constant relating the linear distance of deflection to ϕ;

G = constant gain factor;

f = a frictional term.

These authors came to the conclusion that the servo system still lags behind the Earth tide and that it also underestimates the tidal amplitude, however, with rather small magnitudes. The phase lag is given by the authors as being 5×10^{-2} (no units are mentioned) and the amplitude is underestimated by a factor of 0.9990 - 0.9998.

By a study of the mentioned equation of motion, the authors show that the servo control can make the gravimeter unstable when the response of the nulling system is too rapid. This situation is avoided by a feedback in the servo system in the improved gravimeter, as was already noted.

4.0. DISCUSSION OF NOISE AND ITS REMOVAL.

4.1. THE SEARCH FOR NOISE.

Throughout this work, unless otherwise availed, the term "noise" refers to the total unwanted signal; it is not necessarily random or wholly random, and indeed the most preponderant part, as we shall see, is linear.

At the onset of the work two ways were employed to search for noise:

4.1.1. Search for noise in the time domain - detection of noise by means of the autocorrelation function. (Goldman (1953)). The following is the rationale underlaying this type of detection:

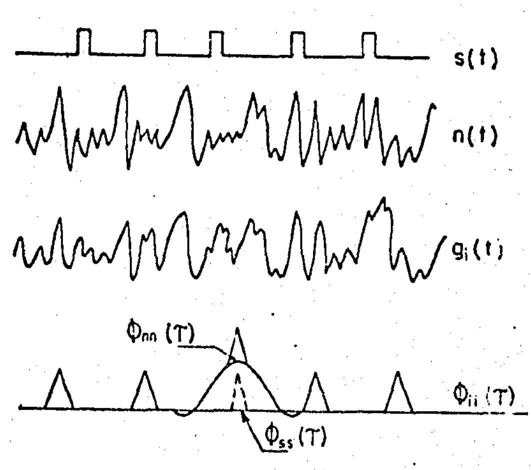


Fig. 4 - Use of the autocorrelation function to discover a signal in noise background (after GOLDMAN (1953)).

Let there be an input signal $g_i(t)$ such that

$$g_i(t) = s(t) + n(t)$$

where $s(t)$ = repetitive signal;

$n(t)$ = noise

then the autocorrelation function $\phi_{ii}(\tau)$, τ = time lag

$$\phi_{ii}(\tau) = \int_{-\infty}^{\infty} g_i(t)\, g_i(t+\tau)\, dt$$

$$= \phi_{ss} + \phi_{ns} + \phi_{sn} + \phi_{nn}$$

$= \phi_{ss} + \phi_{nn}$, because $\phi_{ns} = \phi_{sn} = 0$, since the signal and noise are incoherent (by "coherence" we mean dependence, linear or otherwise).

We could expect $\phi_{ii}(\tau)$ to have the appearance of the type as shown in Figure 4. A pre-existing knowledge of the pure signal is a necessary enabling condition to make this method operative.

At the beginning of the work, and employing rather short series, no discernible noise was discovered with this method. It was abandoned as confidence was gained with the growing number of analyzed time series records, and it became evident that all noise can be accounted for in the very low frequency bands employing method 4.1.2. Although it would appear that nothing is to be gained by autocorrelation analysis in tidal problems, it is nevertheless our belief that at the onset of an investigation it is a good policy to look for noise in the time, as well as in the frequency domain. E.g. method 4.1.2. could not detect noise which is not square integrable.

4.1.2. Search for noise in the frequency domain - - detection by means of the Fourier transform. The theory of this method is described in Chapter 5.0, since it is an integral part of the determination of delta. The operational heart is the computer program FOURIER.FILTR.

Inspection of the noise in the frequency domain will show the amount of adversity under which delta was determined. The noise removal must be done prior to the actual operation of the Fourier transform and is, in fact, done in the subroutine PRELIM. Any noise removing system can be built into this subroutine. The reader will find all aspects of noise interference and noise removal of importance to this investigation discussed in the subsequent paragraphs.

4.2. NATURE OF THE NOISE.

Our term "noise", i.e. the undesirable parts of the total signal, comprises mainly the following items:

(1). Instrumental drift.- This is inherent in every gravimeter. The instrument employed in this investigation was considerably less offending than many others whose records are in the published literature.

(2). The migrating zero.- This is due to the fact that neither of the time series employed, i.e. the theoretical and observed tides are zero mean, nor can they be made zero mean because of the inconstancy of the mean for each period. It can be shown that this is a consequence of digitization.

(3). Fluctuations of the ground.- These are referable mostly to temperature and (atmospheric) pressure changes of sufficiently low frequencies to be detected by the Lacoste - Romberg gravimeter. The noise contribution due to tidal ocean loading, normally of great importance, can be considered unimportant in our region. The reasons for this are discussed in Chapter 8.0.

This study does not concern itself with the relative amounts of the individual contributions but only with the complete removal of noise as far as possible.

4.3. REMOVAL OF THE NOISE.

4.3.1. In the preceding paragraphs we have acquainted ourselves with the methods for noise detection. We have especially noted that for the purposes of our investigation at any rate, the frequency domain detection appears to be much more sensitive. We also noted the individual contributors to the noise. Even without a very thorough noise analysis, it is possible to build up very rapidly the impression (and confirmed fact) that a very definite linear trend overwhelms in magnitude all other noise components. This observation will prove of very great importance in connection with our selection of a noise removal method.

Tables 2 and 3 in Chapter 6.0 will show that while the amount of noise influences the amplitudes of the tidal signals somewhat, the same influence is much greater on the value of delta. The noise must be removed virtually completely, if our determination of delta is not to suffer unduly. In the succeeding paragraphs methods of removal are discussed, some used by this author, other methods used by other authors. However, first we consider it of some importance to use a little space and dwell slightly on the time and patience of the reader, to discuss the frequency domain of the noise.

4.3.2. The frequency domain of the noise. The frequency domain in our investigation is confined to the physically meaningful part, i.e. from zero frequency to folding frequency. Due to various approximations generally involved in a physically realizable system, the values are not necessarily very significant at the extrema of the physically realizable range. We find that in our work the near zero frequency noise amplitudes give reasonable values but the actual spectral frequency assignment is not necessarily absolutely correct. Thus in our case the constant linear trend should give a considerable noise amplitude at zero frequency, whereas in the actual analysis the amplitude which should belong to this zero frequency will actually appear at the next digitized frequency station. It follows from this, that while the magnitude of the noise can be determined from the amplitude peaks, it would be impossible to determine the actual shape of the noise by reinverting the noise band spectrum into the time domain by means of the inverse Fourier transform. This operation will generally be unnecessary, however, since we are primarily concerned with the amount of noise and its complete removal and not with its appearance in the time domain.

4.3.3. Noise removal procedures. Since the value of delta is greatly influenced by the amount of noise, it is essential that the offender be removed as much as possible. The only way to ascertain removal is by inspection in the frequency domain. It is this writer's experience that disturbing amounts of noise are not necessarily revealed in the time domain. It would appear from scanning published data that the importance of quantitative noise removal is not necessarily fully realized and is not shown in the published results. In this work we have arbitrarily allowed a spectral noise peak of 10 μgals as the upper limit. Above this noise level the value of delta found is not considered as being acceptable and records from which a higher residual noise level was irremovable were not used for the tabulation of relevant investigative results.

The noise must be removed prior to conversion of the time domain into the frequency domain. The latter domain will then automatically reveal not only the presence or absence of noise residue, if any, but will also show the tidal signals, and the values of delta at each frequency station within the signal bandwidth.

The following noise removal routines offer themselves:

4.3.3.1. Methods prior to analysis.

4.3.3.1.1. Linear detrending.- This is the simplest of all the available methods and is the procedure adopted in this work. It was not encountered elsewhere in the literature and it is the writer's opinion that this rather simple correction owes entirely its existence to the very low drift characteristics of the gravimeter in our use. It was found that noise was virtually completely eliminated from a substantial number of approximately one month long records. Experience with this method indicates that it is unusable for:

(a) instruments with high drift characteristics;
(b) records of longer duration than one month.

The method consists essentially of selecting one or several integer periods which must closely approximate a realistic tidal period. In our work detrending periods of

12 and 24 hours were selected (which gave much better noise reducing results than 25 hours). The numerical average was taken in the first period and in the period which was the largest number of integer multiples away from the first period and still be wholly contained within the series.

Let

T_1 = first period in the series;

T_e = last possible full period in the series, n times away from T_1, i.e. $T_e = nT_1$;

a_{T_1} = average in period T_1;

a_{T_e} = average in period T_e,

then $a_{T_e} - a_{T_1}$ = drift

The first digit in period T_e will be nT (where T = number of integer digits per period) digits away from the equivalent point in period T_1; taking the average of each terminal period as dwelling in the middle of such period, we obtain for the drift per hour, i.e. per interval between two adjacent digitized stations

$$(a_{T_e} - a_{T_1})/nT \quad \text{(gravity units employed).}$$

This drift per hour is then incremented pro rata hour-by-hour and applied to the observed tidal series hour--by-hour in the opposite sense to the drift found, in order to remove the latter. This operation is, of course, carried out in the time domain.

After inversion into the frequency domain, the frequency spectrum is inspected for noise residue. Depending on the amount of such residue, the result is either accepted or rejected. The admissible upper limit was arbitrarily selected as a noise spectral amplitude peak of 10 μgals. If a higher amplitude is obtained, there are some simple modifications possible before the result is rejected, or a more complicated noise removal method attempted, such as the use of filters. All these modifications will basically somewhat alter the value of the applied correction:

(a). Remove a few leading digits in period T_1. This will change the scope of periods T_1 as well as T_e (since these are an integer multiple number of stations apart) and a slight modification of the drift value can thus be obtained.

(b). Remove some trailing stations in the record so as to cut into period T_e. The period T_e will thus become incomplete and the computer program will automatically discard it and use the period T_{e-1} for the correction calculation. It was found that this method will generally produce a gentler correction than the preceding one.

(c). Change the digit value of the numerical period, e.g. use T = 11 instead of T = 12, etc. It was generally found that the change in the correction value is too drastic with this procedure..

It would at first glance appear that there is another way to do the linear detrending, slightly different from the previous one, namely as follows: The first integer period is taken as before, but the last integer period T_e is taken as the last number of digital stations corresponding to the length of the selected detrending period. It will be noted that in this case there is no multiple integer relationship between periods T_1 and T_e; the periods in question will therefore not be positionally concatenated and consequently the relative position of T_e and T_1 will impose itself much too importantly on the evaluation of the corrective value.

4.3.3.1.2. Methods of moving mean.- These are the oldest methods and were used in conjunction with "harmonic" analysis. In our opinion, these methods are obsolete, although still in use. A good survey and description of these is given in NAKAGAWA (1962). They employ the concept of a moving mean which is based on some integer hour period, viz.:

(a). 25-hour (oldest method); cf. remark for this period length in the "linear detrending method";

(b). 30-hour method ("Admiralty method", DOODSON);

(c). 15-hour method (PERTSEV); this method became popular because it was labor saving as compared to the others.

We shall not dwell on these methods here. We consider them to be of historical interest and the reader searching for details is referred to NAKAGAWA's paper (above) as well as to MELCHIOR's book.

4.3.3.1.3. Digital filter methods.- This is a powerful method of getting rid of noise, especially so if the random component becomes rather important. It therefore might be the only method truly available for long tidal records, but obtaining the right filter for the right job can be a time consuming business. A brief outline of filter theory would therefore be in order on these pages:

From an available frequency bandwidth $F(\Omega)$ we wish to cut out a smaller bandwidth $F(\omega_2-\omega_1)$. We can achieve this by multiplying this function by some specification such as

$$K(\omega) = 1, \qquad \omega_1 < \omega < \omega_2$$
$$= 0 \quad \text{otherwise;}$$

so that the result

$$X(\omega) = F(\omega) \cdot K(\omega) \tag{1}$$

will give a value in the bandwidth $(\omega_2-\omega_1)$ only.

We call the specification $K(\omega)$ the systems or transfer function of the filter $k(t)$. Transforming the systems function into the time domain, we obtain

$$k(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} K(\omega)\, e^{j\omega t}\, d\omega \tag{2}$$

and we note that this equation holds strictly only with the integration performed over the total frequency universe. Individual k(t)'s (digitized) will give us filter weights in the time domain.

We can likewise transform equation (1) into the time domain and obtain the final result

$$x(t) = \int_{-\infty}^{\infty} h(\tau) f(t-\tau)\, d\tau \qquad (3)$$

which is a convolution integral. It is this convolution operation which has to be performed for our filter operation in question (and not the much simpler multiplication in the frequency domain, since we must obtain the latter in its "expurgated" state).

We notice from equation (2) that for the actual execution the positive and negative frequency bands must both be employed, although only the former has physical significance. Using this fact, together with our former bandwidth limits, we obtain

$$K(\omega) = 1, \qquad \omega_1 < |\omega| < \omega_2$$
$$= 0 \qquad \text{otherwise;}$$

and hence

$$h(t) = \frac{1}{2\pi}\left[\int_{-\omega_2}^{-\omega_1} e^{j\omega t}\, d\omega + \int_{\omega_1}^{\omega_2} e^{j\omega t}\, d\omega\right]$$

$$= \frac{1}{\pi t}\left(\sin \omega_2 t - \sin \omega_1 t\right)$$

in angular frequencies (4)

$$= \frac{1}{\pi t}\left(\sin 2\pi f_2 t - \sin 2\pi f_1 t\right)$$

in non-angular frequencies

where k(t) are the filter weights in the time domain and for a digitized system

t = -M, -M+1, ... 0, ... M-1, M, i.e. we shall have 2M+1 weight values of which M+1 will be independent.

Since we use a digitized system, equation (3) will in fact become

$$X(t) = \sum_{\tau=-M}^{M} k(\tau) f(t-\tau) \tag{5}$$

We also note, in passing, that

$k(0) = f_2 - f_1$

$k(-1) = k(2) = \frac{1}{\pi}(\sin \pi f_2 - \sin \pi f_1)$

$k(-2) = k(2) = \frac{1}{2\pi}(\sin 2\pi f_2 - \sin 2\pi f_1)$

i.e. the filter is even and in theory therefore should not produce a phase shift.

The above basic equations can be modified to suit special needs. Thus for instance for a lowpass filter $f_1 = 0$ and f_2 = cutoff frequency, we have

$$k_L(t) = (\sin \pi f_2 t)/\pi t.$$

For our particular noise problem we need a low frequency rejection filter which can be obtained as follows:

To build a band rejection filter, let us have a function I(t) such that

$$I(t) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j\omega t}\, d\omega = \frac{1}{\pi}\,\frac{\sin \pi t}{t}$$

and the band rejection filter weights $k'(t)$ are given by

$$k'(t) = I(t) - k(t).$$

We note that

$$I(0) = 1, \quad t = 0$$
$$I(t) = 0, \quad t = \pm 1, \pm 2, \ldots$$

hence

$$\begin{aligned} k'(0) &= 1 - k(0), & t &= 0 \\ k'(t) &= -k(t), & t &= \pm 1, \pm 2, \ldots \pm M \end{aligned} \tag{6}$$

In developing equation (2) we have found that the relationship between $k(t)$ and $K(\omega)$ holds strictly only for a frequency band from $-\infty$ to $+\infty$. Any infringement of this, such as of necessity must arise in a physically meaningful system, will produce undesirable wriggles in the response. Such wriggles can be greatly reduced or smoothed by the employment of a tapering window function.

A considerable number of such tapering window functions are available in the literature, depending on the shape of the taper. Our work was done with a PARZEN window (W. W. JOHNSON, personal communication).

Let $u = t/M$ and let us have three functions such that

$$h_0(u) = 1\,, \quad |u| \leqslant 1,$$
$$= 0, \quad |u| > 1;$$

$$h_1(u) = 1 - |u|\,, \quad |u| \leqslant 1,$$
$$= 0, \quad |u| > 1;$$

$$h_2(u) = 1 - 6u^2 + 6|u|^3, \quad 0 \leqslant u < \tfrac{1}{2}$$
$$= 2(1 - |u|)^3, \quad \tfrac{1}{2} \leqslant |u| \leqslant 1$$
$$= 0, \quad u > 1;$$

so that in the actual calculation, instead of the filter weights k(t) alone, we use values such as

$$h_0(t) \cdot k(t)$$
$$h_1(t) \cdot k(t), \text{ etc.}$$

On the whole h_1 and h_2 produce a smoother filter with less sideband than h_0. h_2 gives lower sidebands than h_1, but h_1 is better for filters employing a small number of weights.

With the extremely good separation of the peaks in the frequency spectrum of the tidal signal, it would appear that the use of a low frequency rejection filter should be a very simple matter. In actual practice, the requirement of quantitative noise removal imposes considerable restrictions on our filter. The problem of distortion, on the other hand, is not very vexing, since we assume that both tidal signals will have equal distortions, not being significantly different in their geometry. Our experiments in this work with the digital filter were not very impressive.

It will be noted that a filter contains a considerable number of variable parameters, such as shape of the window, number of weights to be used, etc. To obtain the desired result the optimum combination has to be found.

It is our belief that for a truly quantitative noise removal each tidal series has to receive individual consideration with a tailor made filter suited to itself. It would appear rather unlikely that the requirement for each series individually could be satisfied by a general procedure.

The above considerations apply to a longer tidal series, in which random noise increases in importance, assuming that the recording gravimeter is a low noise instrument. We have found that a month's record, employing the Lacoste - Romberg gravimeter, good noise removal can be obtained by means of the linear correction only; this correction, however, was unable to handle the job satisfactorily once the record length was about 1000 hours or more (on a few occasions even a month's record could not be satisfactorily handled in this manner). With the employment of one month long records, the use of digital filters as described here would indeed be tantamount to attacking a mosquito with a sledgehammer.

4.3.3.2. Methods contemporaneous with analysis.

These methods, which are all based on some mathematical and computational assumptions of noise approximation or collocation (and we might mention here that any methods involving least square approximations can never collocate), have enjoyed, and indeed still do enjoy considerable popularity in some circles. They address themselves primarily to workers who endeavor to obtain their results by "harmonic" analysis. They are labor saving and hence were of considerable interest prior to the advent of electronic computers. For this reason we shall only briefly outline them here, rather than engage in an exhaustive survey. And again, with all the methods under this heading, no direct inspection of the noise left behind can be made. The interested reader is referred for details to NAKAGAWA's paper (1962) as well as MELCHIOR's book.

The following are some of the more interesting methods used:

(1). Lecolazet's method (LECOLAZET (1956)).- This method employs a low order approximating polynomial.

(2). The Doodson - Lennon method.- (DOODSON & LENNON (1958)). This method applies a linear transformation to hourly values.

(3). Suthons' method.- (SUTHONS (1958)). This is a graphic method of noise removal and is applicable to records of a short length, such length being approximately coextensive with the record length in our employ.

5.0 METHODS OF ANALYSIS.

5.1 SAMPLING CONSIDERATIONS.

For the convenience of the reader, we shall introduce this section with the definition of a few terms.

(1). Degrees of freedom.- Several meanings can be read into this term, even in statistics alone. Unless otherwise defined, we shall take the number of degrees of freedom as refering to the number of independent variables of the signal, i.e. the number of sampling points. The number of degrees of freedom can be less than this value in case of "intersymbol influences" - i.e. when a given station is not independent of its neighbors (Markov chain).

(2). Coherence.- This term implies a complete dependence between two sets x_i and y_i, not necessarily linear. Linear coherence is known as correlation.

(3). Fundamental frequency (elementary frequency bandwidth).- Let $2T_n$ = record length in time units, then the above is defined as

$$\Delta f = 1/2T_n \quad \text{cycles/unit time}$$

and represents the frequency incrementing done in the frequency domain. This is a consequence of the Fourier integral concept, but the mathematical details will not be discussed here. It is, however, important to realize that in order to obtain numerically meaningful Fourier amplitude coefficients by means of spectral analysis, it is imperative to perform the frequency incrementing in terms of the fundamental frequency.

(4). Shannon's numerical sampling theorem.- If the Fourier transform of a time series f(t) is zero above a maximum frequency Ω, then a digitized equidistant time series can be obtained from the spectrum in terms of Ω. The relationship which SHANNON derived is

$$\Omega = \frac{\pi}{\Delta t}$$

, where Δt = sampling interval in units of time and Ω is an angular frequency. Substituting

$$\Omega = 2\pi f_N$$

and

$$2\pi f = \omega$$

and inserting these into the original equation, we obtain

$$f_N = \frac{1}{2\Delta t} \quad \text{cycles/unit time,}$$

where f_N is known as the Nyquist or folding frequency and reversing the above argument somewhat, it can be shown that it represents the highest possible frequency obtainable from a digitized series. In our work Δt = 1 hour, so that f_N = .5 cycles/hour.

(5). Aliasing (folding).- If the original record contains frequencies higher than f_N, such as $f_{(N+i)}$, then the frequency spectrum $0 \geqslant f \geqslant f_N$ obtained from a digitized record will be contaminated by the frequencies $f_{(N+i)}$

which will appear in the spectrum as $f_{(N-i)}$ - i.e. fold back around an ordinate at f_N as a folding axis. In our work f_N = .5 cycles/hour and the highest frequency tidal component of interest to us, viz. K_2 has a frequency of .0835 cycles/hour. Our results, therefore, are completely uncontaminated.

Preceding items (1), (3), (4) and (5) are not inherent in Fourier analysis, but are a direct consequence of sampling, i.e. digitization.

The following considerations were taken into account for a successful sampling technique of the tidal series:

(1). The <u>sufficiency in the time and frequency domains</u> will be discussed in greater detail in connection with the "sinc function" ($\sin\omega t/(\omega t)$). For the present we shall confine ourselves to the fact that the sufficiency of sampling in the time domain of a function G(t) which has the highest frequency Ω, the sampling at $1/2\Omega$ time units apart is the minimum necessary and sufficient to define the series G(t) throughout the time domain. In our case $\Omega = K_2$ = .0835 cycles/hour, which gives Δt = 6 hours. We are thus well within defining our tidal time series by using Δt = 1 hour; in fact the tidal series is statistically overdefined, so that strictly speaking not every digit will be a degree of freedom and the system will have, statistically speaking, "intersymbol influence". We note, incidentally, that the employment of Δt = 6 hours would move the Nyquist frequency just into the spectral region of interest and therefore introduce the risk of contamination.

(2). The next step is to discuss <u>the scanning of the frequency spectrum by the fundamental frequency</u> and their relation to the degrees of freedom. We shall, to simplify matters, assume that all digitized stations in the time domain represent the number of degrees of freedom and that there is thus no "intersymbol interaction".

Intuition would guide us into the belief that there are two ways to increase the number of degrees of freedom:

<u>(i)</u> Leave the sampling interval the same and increase the record length.

Let

$2\,T_n$ = record length (in time units);

$2\,T_n$ = new record length (in same units);

Δt = sampling interval (same units again);

n = number of sampling digits in record T_n;

n' = number of sampling digits in record T_n';

f_N = Nyquist frequency;

f_F = fundamental frequency;

and we have automatically the relations

$$2\,T_n = n\,\Delta t$$

$$2T_n' = n'\,\Delta t$$

(a). Record T_n:

$$f_N = \frac{1}{2\Delta t}$$

$$f_F = \frac{1}{2T_n} = \frac{1}{2n\Delta t}$$

and we define the density as

$$\frac{f_N}{f_F} = \frac{\frac{1}{2\Delta t}}{\frac{1}{2n\Delta t}} = n$$

(b). Record T_n':

$$f_N' = \frac{1}{2\Delta t} \qquad \text{i.e. unchanged}$$

$$f_F' = \frac{1}{2T_n'} = \frac{1}{2n'\Delta t}$$

density = n';

We note that for the same folding frequency in both cases, i.e. f_N, since $n' > n$, we increased the density of elementary frequency bands and hence the resolution. A statistically significant change was therefore made.

(ii) Leave the record of the same length and change the sampling interval.

Here $T_n = T_n'$. As an example let us decrease the sampling interval by a factor of 5, i.e.

$\Delta t'$ = new sampling interval;

Δt = old sampling interval;

We also have $\Delta t' = \Delta t/5$,

whence

$$2\ T_n = n\,\Delta t = 2\ T_n' = n'\ \Delta\,t' = n'\ \Delta t/5$$

and $n' = 5n$.

By arguments similar to previous, we find that for the old series

$$f_N = \frac{1}{2\Delta t}$$

and for the new series

$$f_N' = \frac{1}{2\,\Delta t} = \frac{5}{2\,\Delta t}$$

We see that although the Nyquist frequency band is five times wider, since $n' = 5n$ there will also be five times as many elementary frequency bands scanning it, therefore the density remains the same and there is no statistical change at all. We incidentally note that in case (ii) the fundamental frequency remains the same.

We thus see that with the two possibilities in existence, viz.; (i) Nyquist frequency constant, fundamental frequency changes; and (ii) Nyquist frequency changes and fundamental frequency constant, <u>only (i) produces a statistical change.</u>

It should be impressed upon the reader that these are purely statistical considerations. Method (ii) would be quite appropriate if some physical phenomenon existed whose time characteristic would be smaller than the selected interval Δt.

For the purposes of our work these considerations are exhaustive. The interested reader will find a good discussion in HUANG (1966). The reader is also warned that further considerations are frequently necessary in the analysis of stochastic series, such as the number of digital stations necessary to obtain meaningful phase increments (not necessarily applicable to tidal work), Δt necessary to eliminate wideband extraneous noise, etc. A discussion can be found in BLACKMAN & TUCKEY (1958).

5.2. CALCULATION OF THE THEORETICAL TIDES.

The calculation of the vertical component of the theoretical (rigid Earth) tides is basic to the whole problem, since the observed gravitational component of the tides has to be matched to it in order to obtain the value of delta.

Several methods are available for this calculation, depending on the time interval desired between the individual digitized stations in the time series. Method (1), which was used by the author, will be discussed in some detail while the others will be only briefly mentioned.

5.2.1. PETTIT's method. (PETTIT (1954)). This method provides for the calculation of the vertical component of the tidal acceleration anywhere on a rigid Earth, the tidal force being due to both the Moon and the Sun. The relevant paper, which is well written, is completely self-explanatory and details will not be discussed here. The paper draws the necessary astronomical data from the American Nautical Almanac and in one case, viz. the Earth - Sun radius vector, from the Ephemeris, for the execution of the relevant calculations. The time spacing of the American Nautical Almanac thus automatically enforces an hourly digitization of data. In addition, the geographical co-ordinates of the location in question, as well as the elevation (in metric units), are necessary.

The American Nautical Almanac as well as the Ephemeris are available on punched IBM cards from the U.S. Naval Observatory in Washington, D.C. and this deck can be fed directly into the computer as a $DATA deck. Use of this service was made by the author to the full extent needed. The resulting tidal force is calculated in the program TIDAL. .FORCE specially written for this purpose and filed with the National Oceanographic Data Center in Washington, D.C.

5.2.2. LONGMAN's method. (LONGMAN (1959)). This method, which has the same astronomical rationale as the preceding one, is much more complicated to program for a computer. The main difference is that it computes its own Nautical Almanac and Ephemeris and is therefore extremely useful when digitization based on a different time interval is required.

5.2.3. Other methods still can be found in the literature, completely untested by this writer, such as HEILAND (1940) (arbitrary time intervals), DARWIN (1907), SCHUREMAN (1940), ELKINS (1943) and others.

5.3. PROCEDURES OF TIDAL ANALYSIS.

5.3.1. Essentially there are two fundamental methods of analysis available, of which we shall call one harmonic and the other spectral. At the onset of the discussion we should like to point out that the former term is, strictly speaking, a misnomer. Inasmuch as harmonic analysis refers to the determination of the Fourier coefficients, the term in actual fact applies to both of the procedurs. Nevertheless, for lack of better terminology, we shall apply it to a certain method of numerical procedures (now mad outdated by the electronic computer) as described in the sequel.

The two methods mentioned here were compared by several workers as will be seen later.

However, prior to the description of the two methods, it might be of some profit to look briefly at some aspects of Fourier series and the Fourier Integral Theorem. To be sure, no attempt will be made here to show mathematically how one can be derived from the other, which is done in appropriate textbooks. It is assumed, nevertheless, that the reader might be interested in the conditions of validity of these concepts. (For a thorough discussion see e.g. PAPOU-

LIS (1962)). These validity ranges are as follows:

(1). Fourier series.-

(a). a representation of certain functions over a limited range, such as $(-\pi, \pi)$ or $(-L, L)$.

(b). representation of a periodic function over an infinite range.

(2). Fourier transform.-

generally represents a non-periodic function over an infinite interval. Physically this means resolving a single pulse or a wave packet into sinusoidal waves, which process is the theoretical foundation for the spectral analysis undertaken in this work.

The Fourier transform is defined as

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega \qquad (1)$$

or, in its digitized form

$$f(t) = \sum_{n=-\infty}^{\infty} \alpha_n\, e^{jn\omega_0 t} \qquad (2)$$

where $\omega_0 = \frac{2\pi}{T}$ = fundamental frequency

T = period;

and

$$\alpha_n = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\, e^{-jn\omega_0 t}\, dt \qquad (3)$$

reminiscent of the formula for the calculation of the coefficients of a Fourier series. At the same time, equation (3) is another Fourier transform, the inverse of equation (1).

The independent variables $\underline{t}$ and ω are arbitrary and need not have a physical meaning. Such a meaning, however, is frequently attached to them, in which case, if $\underline{t}$ denotes the time domain, ω will denote the frequency domain.

Equations (2) and (3) will reveal the following facts which are of importance:

(i) the Fourier coefficients and the digitized amplitude spectrum are identical if the incrementing is done in terms of the fundamental frequency ω_0;

(ii) the value of the spectrum has to be multiplied by a factor 1/T to obtain a numerically significant quantity.

We also note that

(i) the Fourier integral is valid everywhere;

(ii) the infinite limits of the integral/summation are not physically attainable, which usually will create a slight disturbance;

(iii) the only conditions necessary for the existence of the transform are

(a) square integrability, i.e.

$$\int_{-\infty}^{\infty} |\alpha(\omega)|^2 d\omega < \infty$$

(b) ω must be real.

It could also be profitably pointed out here that the method of analysis described here as "harmonic" is directly related to the Fourier series (1b), whereas the spectral analysis is related to the Fourier transform (2).

5.3.2. Harmonic analysis. These are the oldest methods which, in our opinion, were made obsolete by electronic computers. For this reason their detailed description would be out of place in this work. For a survey the interested reader is referred to MELCHIOR's book (1966) and papers by NAKAGAWA (1962), PETTIT *et al.* (1953), while PARISKII *et al.* (1967) describes in great detail the very popular Pertsev method. A number of authors compared the harmonic analysis methods to the spectral ones, an especially thorough study being due to BARSENKOV (1967). None of the workers found a significant difference between the two methods; the selection of the appropriate methods is essentially a consideration of technical limitations and economy of labor or computer time. In the handcalculating days, the latter was of paramount importance and the harmonic analysis was the much more feasible one; on the other hand, a Fourier transform, especially the simplified "Fast Fourier Transform" for a large number of data is meat to an electronic computer. The basis of all methods under this heading is the obtaining of the Fourier coefficients indirectly from linear combinations of constants which can be arrived at by various assumptions, or manipulations, or constraints on the governing equations, depending on the method immediately at hand.

5.3.3. Spectral analysis. The foundations of this method were already discussed in the preceding paragraphs. It is not our intention to build the whole theory from scratch on these pages, but rather to provide some amplifications and details which might be of particular interest in connection with this research. Apart from standard textbooks discussing the principles of the Fourier transform pairs, a thorough discussion as applied to Earth tide investigations will be found in papers by NAKAGAWA *et al.* (1966), MIKUMO *et al.* (1968), BARSENKOV (1967) and others.

We wish to warn the reader at this stage that a number of comments on the phase component of the spectrum will be made by this investigator at a later stage (see Chapter 7.0).

We have seen, in the section on noise, that nothing is to be gained in tidal analysis by the employment of the autocorrelation function; the Fourier transform of this function - the power spectrum - can, of course, likewise detect periodicities. Some further reasons for not using the autocorrelation - power spectrum pair in our analysis might be mentioned as follows:

(i) there would be no simple way to reinvert the power spectrum into a tidal time series;

(ii) the ratios between the two tidal waves would be in terms of δ^2 rather than δ.

There are several ways in which the Fourier transform can be represented:

(1) As a combination of sine and cosine transforms plotted separately.

(2) As a combination of amplitude and phase spectra plotted separately.

(3) As a continuous 3-dimensional curve whose ordinates are the frequency (or time) values and the sine and cosine transform values for this particular frequency.

(4) The above-mentioned 3-dimensional plot reduced to two dimensions, representing an "end on" view of the resultant vector of representation (3).

Representation (2) was selected for our work.

Given a Fourier transform pair

$$f(t) \longleftrightarrow F(\omega)$$

and

$$F(\omega) = P(\omega) + jQ(\omega),$$

then the Fourier amplitude spectrum $|F(\omega)|$

$$|F(\omega)| = \sqrt{P^2(\omega) + Q^2(\omega)}$$

and the Fourier phase spectrum $\Phi(\omega)$

$$\Phi(\omega) = \arctan \frac{Q(\omega)}{P(\omega)}$$

where $P(\omega)$ is the cosine transform representation, and

$Q(\omega)$ is the sine transform representation.

The above-indicated operation is performed within the computer program FOURIER.FILTR. The preceding representation, which might be described as the trigonometric form, is more digestible to the IBM 7090 computer than the complex form.

It is of importance to discuss and mention some points in connection with the outlined analysis:

(1) No preparatory corrections were used, as done by some workers, such as the Filon quadrature, windows, etc.

(2) Because of the absence of numerical windows, "leaks from the peaks" are established in the frequency domain - i.e. the peak contents are somewhat more broadened near the base at the cost of some amplitude loss; the maximum possible value for a spectral peak is thus not quite attained.

(3) Because of the spectrally identical character of the theoretical and observed tidal waves (apart from the low frequency noise) and because the amplitude values are not very different, it is assumed that the error due to item (2)

affects both waves identically and hence the value of δ - - a far more important investigative target than tidal amplitudes - will be unimpaired, since δ is a ratio.

We note that the factor 1/T in equation (3) makes the integral an averaging integral for the whole fundamental period, i.e. the period equalling the length of record. It is of importance to realize at this stage that the amplitudes obtained by this method, as well as the values of δ derived therefrom, represent an average for the whole tidal record under immediate investigation.

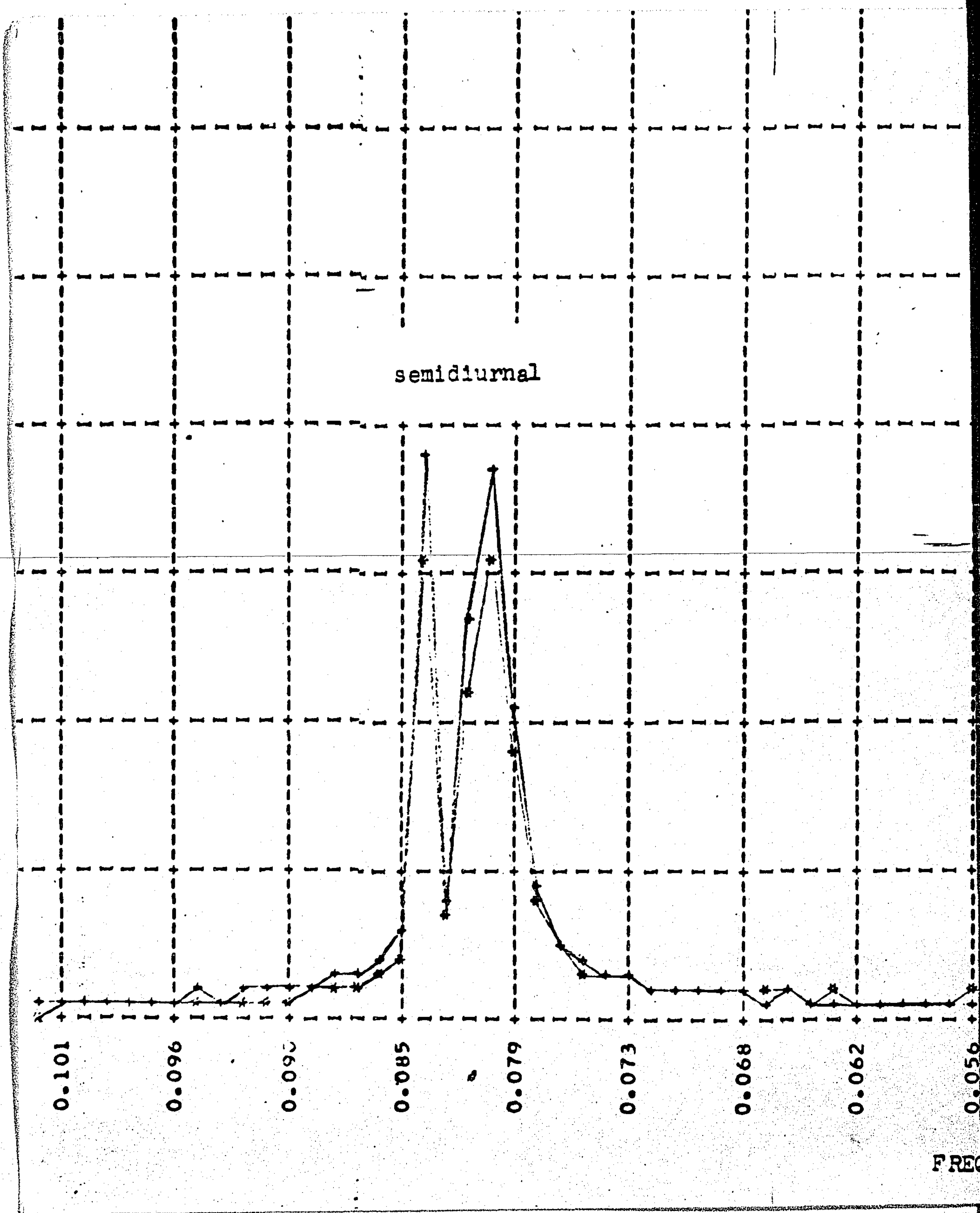
Figures 5 and 6 show the spectra obtained from an 887-hour long tidal record, with and without noise removal respectively. The corresponding numerical computer output is presented in Appendix VI. * denotes the theoretical tide and + the observed tide spectra. The connecting lines are drawn merely to "guide the observer's eye". They do not imply continuity, since the spectrum is digitized in terms of fundamental frequency.

The following facts are apparent from the spectra:

(i) the signal bandwidths, including the undesirable signal ("noise") are distinctly separate;

(ii) the largest part of the undesirable signal is essentially a zero frequency (linear trend) with some low frequency random noise;

(iii) some impression of the tidal components on the two ti-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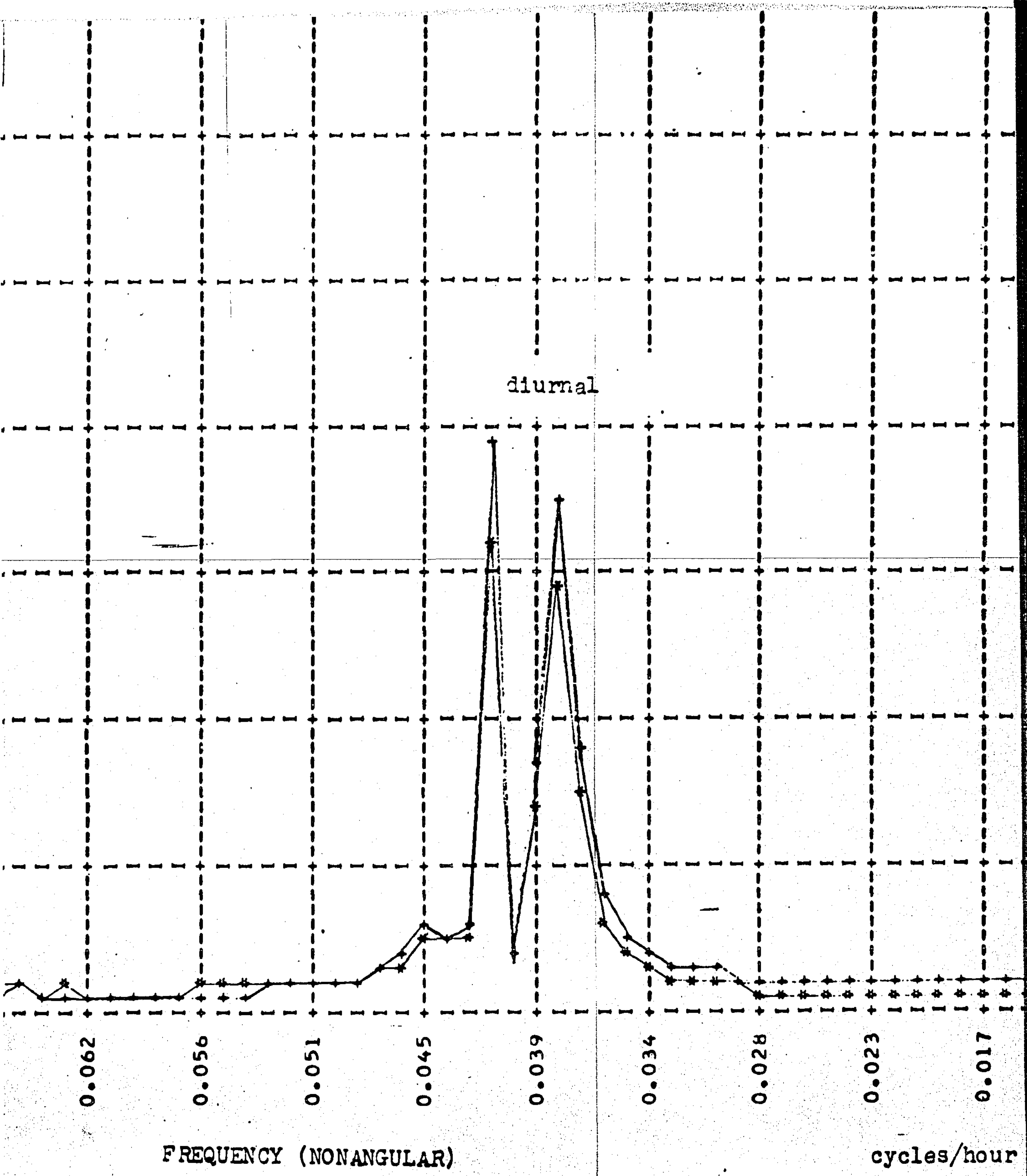
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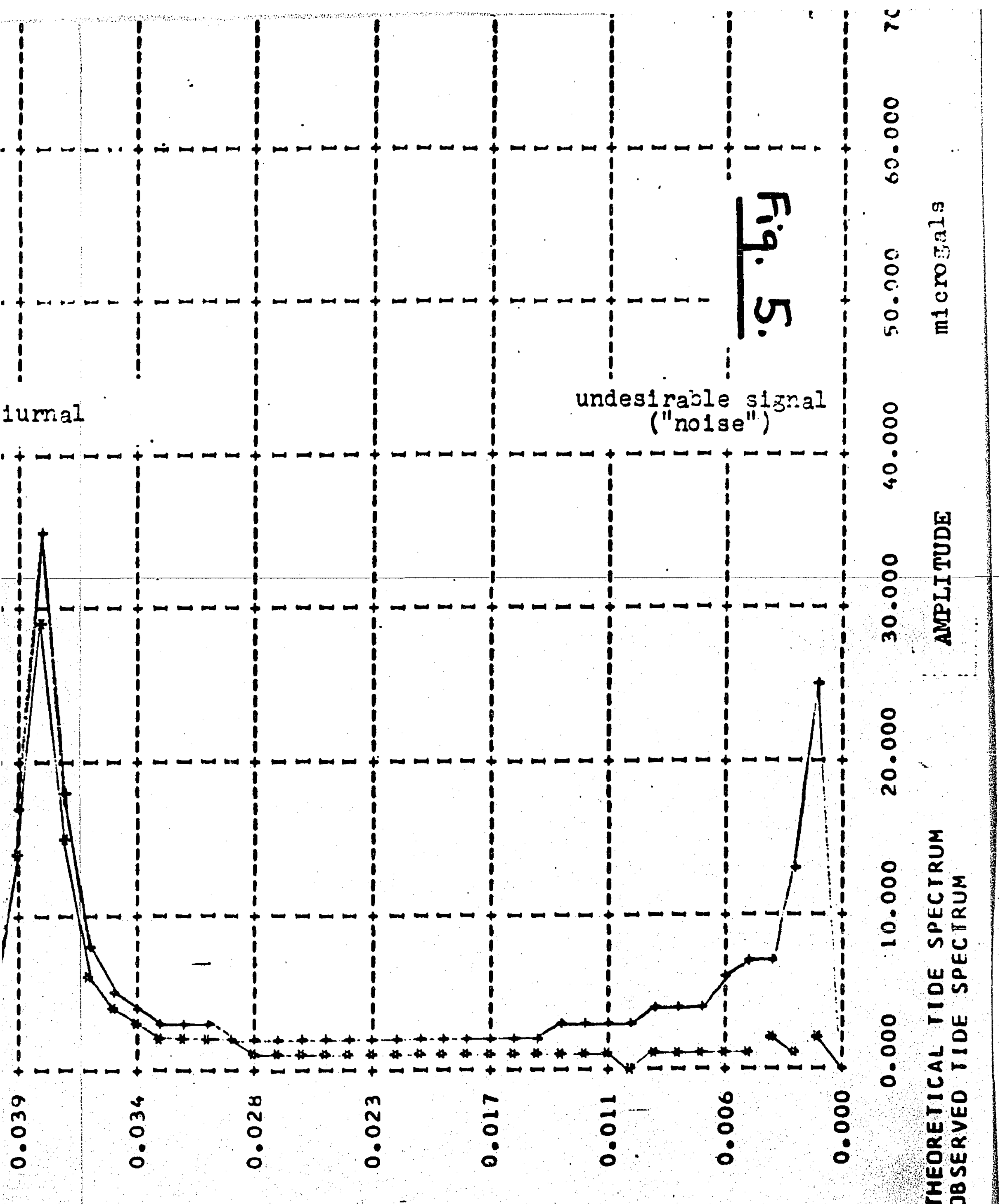
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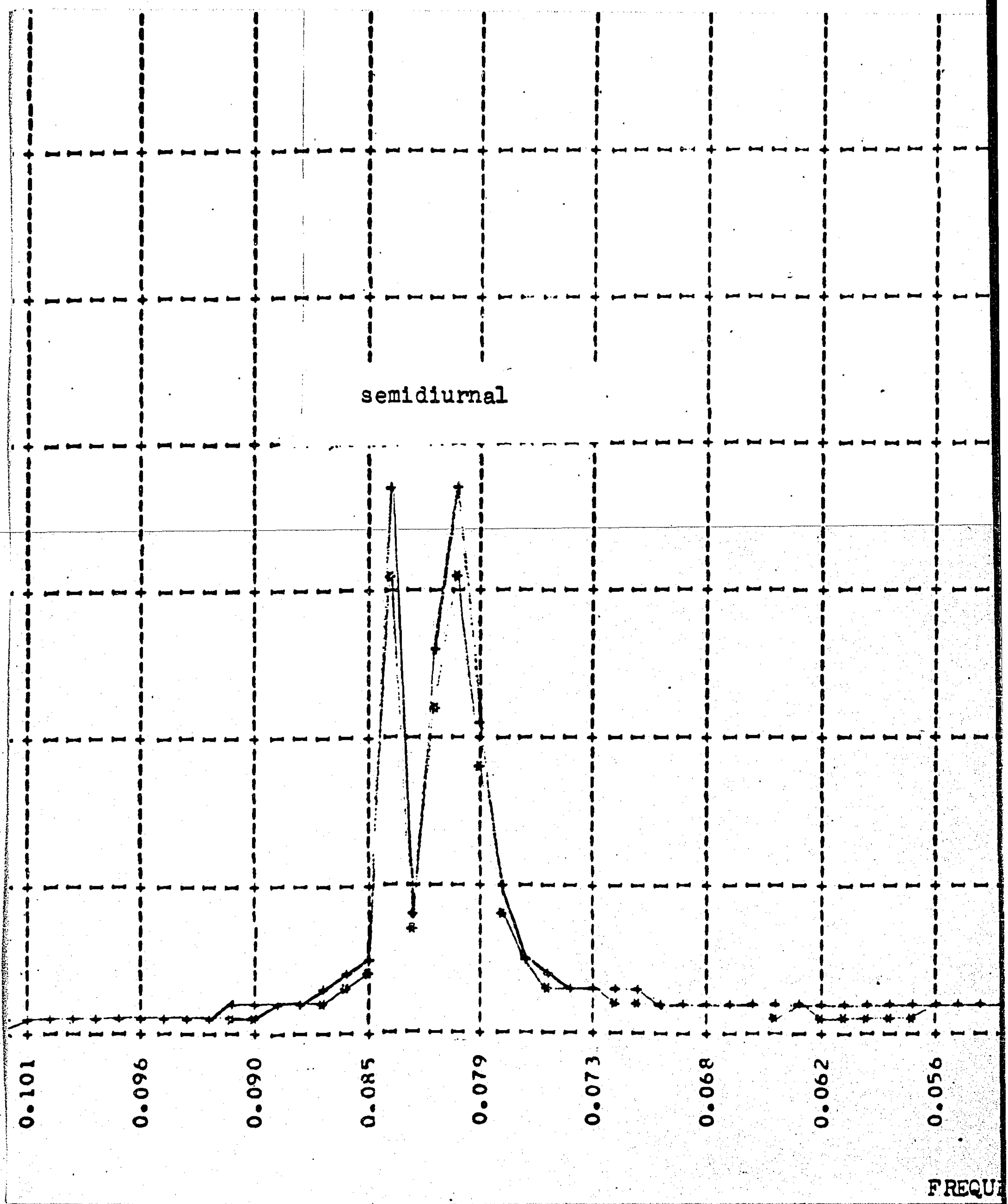
* THEORETICAL TIDE SPECTRUM
+ OBSERVED TIDE SPECTRUM

Fig. 5.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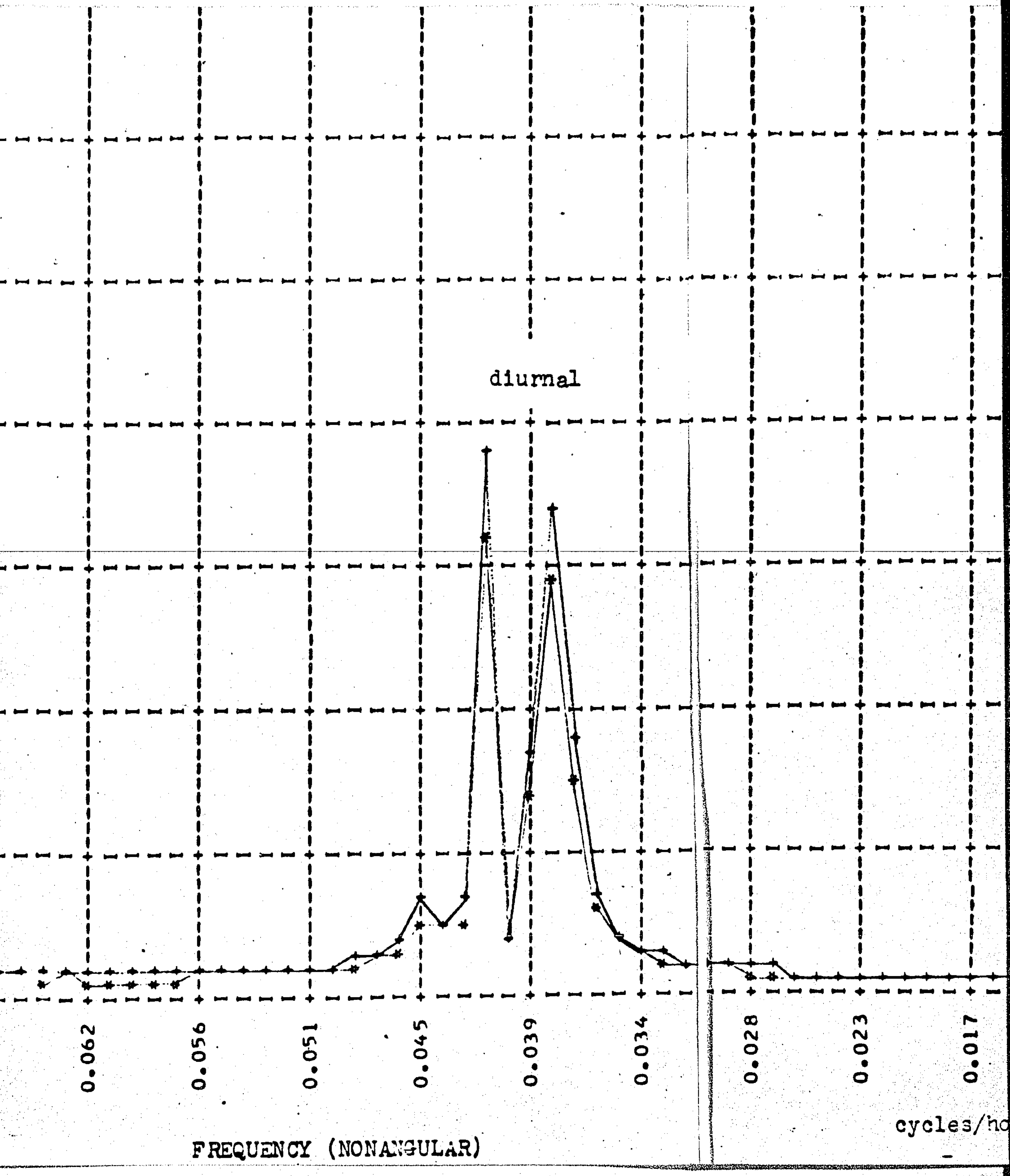
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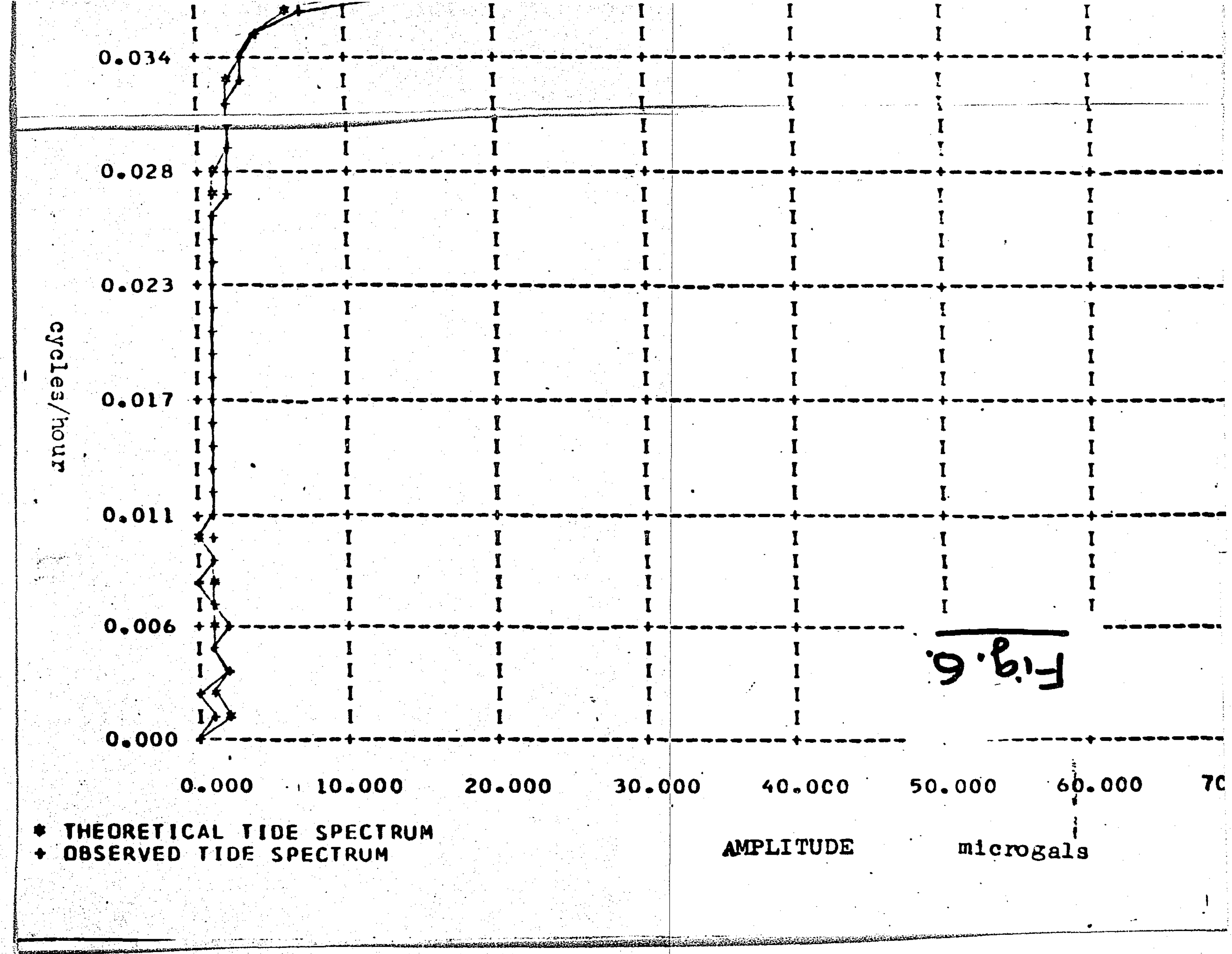
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* THEORETICAL TIDE SPECTRUM
+ OBSERVED TIDE SPECTRUM

Fig. 6.

dal bandwidths is evident in the splitting of the bandwidth peak;

(iv) the undesirable signal is effectively removed by the linear detrending procedure.

It would appear from pure theory that no noise removal would be necessary, due to the complete separation of all the signals in the frequency domain. Yet it will be shown later in this work (6.4) that what can be interpreted as a definite improvement results from the application of such a removal procedure. It also will be noted that, surprisingly, the linear correction effectively removes the low frequency random noise.

These refinements defy theory and we are in no position to explain them. It will suffice to mention that in a high accuracy determination, such as attempted here, the final touches are an art (as in virtually any discipline), even when the theory in the general context and broader outline holds (as undoubtedly it does in our case here, since otherwise the whole analytical procedure would be wrong).

Detailed procedural descriptions are reserved for Chapter 6.0 dealing with the study and results of the amplitude component of the tides. Essentially results are obtained in the following terms:

(1) δ separately for the diurnal and semidiurnal tides;

(2) δ for the total tidal signal;

(3) δ for the (nine) individual components.

Inasmuch as results (3) are obtained on the basis of an entirely different procedure than in use up to now, we intend to dwell here to a moderate extent on the theory of the method adopted, as well as on the comparison to the existing method(s).

The method essentially adopted by other workers consists of the employment of a very narrow bandpass Gaussian filter centered on the frequency of the particular component in question. The interested reader will find a discussion of filters in the paper by MIKUMO et al. (1966), since it is not intended to dwell on the details here. The inverse of the filter is convoluted with the tidal signal in the time domain and the result is Fourier-transformed into the frequency domain, employing both the amplitude spectrum as a measure of the tidal amplitude and the phase spectrum as a measure of phase difference between the rigid and yielding tides. A discussion of our results of phase analysis will be found in Chapter 7.0 with conclusions somewhat different to other workers.

We adopted the method of interpolation in the frequency domain by means of a weighted function whose kernel is the well known "sinc function" or "Lanczos function". A discussion of the operation of this function will be found in Appendix I and its relation to the number of degrees of freedom put in as well as put out is discussed in Appendices II and III. The general appearance of the function is

$$F(\omega) = \sum_{n=-\infty}^{\infty} F\left(\frac{2\pi n}{T}\right) \frac{\sin\left(\frac{\omega T}{2} - n\pi\right)}{\left(\frac{\omega T}{2} - n\pi\right)}$$

where $F(\omega)$ = value of the function at any arbitrary frequency ω interpolated for;

$F(\frac{2\pi n}{T})$ = value of the function at successive fundamental frequency stations.

The theory as well as the derivation of the interpolating function is extremely well done in GOLDMAN (1954). We shall merely point out the following salient features:

(1). The function is perfectly analytical, derived from its axioms without any assumptions whatsoever.

(2). The kernel of the function, whose graph can be seen in Appendix I, shows that the value of the ordinate dwindles very rapidly away from the maximum value.

(3). Since it is in terms of physical reality impossible to perform the summation over the infinite universe, it follows from (2) and (1) that the only errors introduced by the employment of the interpolation function are truncation errors, but the values cut off are negligibly small.

(4). The information contained in Appendices II and III shows that nowhere in our work was any offense given to complete statistical determinancy, i.e. the resulting system is still completely defined in terms of the number of degrees of freedom.

It is necessary to point out that no information as to the phase spectra can be obtained by this procedure. It will be shown, however, in Chapter 7.0 that such information is unreliable by Fourier transform methods.

The described interpolation operation is carried out in the program SINC-FCTN.FR.

Comparing the interpolation method to the filtering procedures, we believe that the following points make the former superior to the latter:

(1). The filter "slices out" a value centered around the frequency desired. It cannot be made infinitely narrow, whereas the interpolation function is automatically so, since it calculates the value for the required arbitrary frequency.

(2). The actual tidal amplitude at an arbitrary frequency can be higher than at the two adjacent fundamental frequency stations; in fact it can be higher than the maximum amplitude in the peak attained at a fundamental frequency station. No filter can exceed such a value, but the interpolation function can do so, since it employs summation.

(3). It was shown that we are employing a tidal series whose digitization is such that it is statistically overdefined. This implies that neighboring stations within a certain radius are not independent of each other. The interpolation function which samples the entire frequency band is, of course, a respecter of the neighborhood. A filter, on the other hand, cuts brutally across all the neighbors.

In the light of these facts, we may conclude that the failure to obtain reasonable results for some tidal components is due not to the inherent weaknesses of the method adopted, but rather due to the length of the tidal series employed. A 1-month record has insufficient information imbedded for the detection of a substantial number of tidal components. However, it is of sufficient length to allow the extraction of respectable δ values for both bandwidths and some interesting conclusions will be arrived at from this information alone at a later stage (see Chapter 6.0). Among others, the sufficiency of the length of the series in the time domain is shown by the narrowness of the peaks in the frequency domain (by the well known "reciprocity principle"). The record length limitation is imposed by the storage capacity of the IBM 7090 computer used in the analysis of the results.

This work does not pay too much attention to the δ values obtained for the individual tidal components. Nevertheless, it behooves to mention methods adopted for checking reasonableness of the componental results obtained in this manner. The operating program SINC-FCTN.FR calculates the amplitudes of both the rigid and the yielding tidal waves and obtains the value of δ in the usual manner by dividing the latter into the former. An obtained value is deemed to be reasonable if the rigid tide amplitude for a given component compares reasonably to an amplitude obtained from astronomical calculations.

5.3.4. "Handwork" analysis. (WYCKOFF, personal communication; also WYCKOFF, unpublished report to the International Earth Tide Center at Brussels, Belgium (1970)). This work was undertaken by R. D. WYCKOFF as a separate investigation, concurrently with the work of this writer. The work is extremely tedious and its main value rests in the fact that the whole process of analysis unfolds before the operator's eyes while engaged in the work. It is probably the only analysis of its type in existence and its main salient features may be pointed out as follows:

(1). This is the only method where the actual drift curve is obtained (as opposed to the mathematical simulation or residue).

(2). It enables us to arrive at an overall δ value for the total tidal signal only.

(3). It shows no detectable phase lag between the two tidal waves, in respect of the gravity component of the tides, as shown in the time domain. For further discussion of this result by different methods in the frequency domain, see Chapter 7.0.

The results obtained by this method and by spectral analysis will be compared in Chapter 6.0. As will be seen, numerous and varied safeguards were brought into operation to prevent any errors of the blunder class while preparing and processing the data by spectral analysis. Therefore, while it is totally unwarranted to expect a perfect match of numbers obtained by two entirely different analytical procedures, a considerable inconsistency would nevertheless have to be interpreted either as an unsufficiency of the spectral method itself, or of some manner in which it was employed. (As will be seen in Chapter 6.0, the agreement is embarrassingly close).

The method is best described by WYCKOFF in his own words as follows (WYCKOFF, personal communication):

"... It is essential that noise must be removed and the common source of noise is drift of the gravimeter. I have "removed" this drift by an uncommon and laborious method which I hope is good.

This method is as follows:

(i). The theoretical tidal amplitude was multiplied by an approximate (trial and error) δ factor on a transparent velum strip hour-by-hour to obtain a plot which matched the observed tidal curve. The approximate δ (amplification) was obtained (now and then) by trial and error.

(ii). The above hour-by-hour plot was then matched with the observed record and the zero line punched through to the record. This (I hope) represents the instrument drift, hour-by-hour. The amplitude (ordinates) measured from this point then represents the observed tidal amplitude each hour.

It will be noted that this drift includes every - thing, except an averaged out (inspection) tidal curve. The overall curve shows on the record and any rather long period variations may be noticed on the record.

This is rather different than machine analysis to obtain the drift.

I am assuming that the hour-by-hour averaging will effectively eliminate the small unavoidable errors in delta over the length of the record.

Again, it is obvious that I cannot obtain any (tidal) components. In any event, deltas for various components are incompatible with the type or averaged delta I obtain.

To repeat, the observed tidal amplitudes are obtained by measuring the observed curve from the above-derived drift curve and then divided by the theoretical amplitude hour-by-hour. This gives the hourly δ or amplification factor. These values are then averaged over the duration of the record, thus giving the average for the duration. I then take the weighted average, thus taking the duration of the records into account, and obtaining, for example, the Pittsburgh average which is 1.19596 or 1.1960 ...".

5.3.5. Summary of analytical methods employed in this work to obtain some of the Earth tide parameters:

(1). The δ values are obtained from the amplitude spectra in the frequency domain, the series having been inverted by means of a Fourier transform; the application of corrections preceded the transform.

(2). Although on theoretical grounds alone these corrections are unnecessary, because the spectral peaks are separate, there is nevertheless a strong suggestion that results are improved by suppressing signals existing in the near-zero frequency range to an amplitude value below 10 μgals.

(3). It was found that one month record is sufficient to give δ values for the total tides as well as the diurnal and semidiurnal bandwidths. Results can be improved by the employment of several records per location (not necessarily of the same length) and taking weighted averages of the results, the weighting being done with respect to the record length.

(4). The bandwidth deltas are obtained by weighted averaging of all δ values from within one band, the weighting being done with respect to the amplitude and the limiting bounds of each band being taken arbitrarily at 10 μgals. However, stations within a band having an amplitude of less than 10 μgals count as signal. To obtain δ for the total tide, all fundamental frequency stations which have occured within a bandwidth are used for the weighted averaging.

(5). δ values for individual tidal components are obtained by making a frequency interpolation for the desired componental frequencies with respect to the observed and rigid tides. δ is then obtained by taking the ratios at these required frequencies. The δ's thus obtained are values independent of those calculated in the Fourier spectra. The value of the rigid tide amplitude is used as a check of reasonableness of the result, since it can be matched to a value calculated from astronomical data independently.

(6). While results for the phase spectra, and particularly their difference, were obtained, those values in their present state are not given any great significance. Some of these values together with their meaning are discussed in Chapter 7.0. It is only fair to admit that no great effort was channelled in this work towards an elucidation of tidal phase differences.

To the best of our knowledge, except for the use of the Fourier transform, no other worker was using methods described above. For this reason attempts to compare these methods to existing ones are made, whenever relevant, in the body of the text.

6.0. STUDIES AND RESULTS OF THE AMPLITUDE SPECTRUM OF THE TIDES.

As opposed to the preceding chapters dealing with the theory of the analytical procedures adopted, this and the following chapter, will be devoted to some operational details, where necessary, as well as the sequence of steps taken, the location of the steps with respect to the existing computer programs and also the steps taken to monitor possible blunders. This chapter will confine itself to the results obtained from the amplitude spectra of the tides. The results will be expressed along the following lines:

(i) δ value from the diurnal and semidiurnal bandwidths;

(ii) δ value for the complete tidal signal;

(iii) δ value for some of the tidal components obtained by the interpolation method.

6.1. SEQUENCE OF OPERATIONS.

6.1.1. Securing the data.- The tidal record obtained from the gravimeter was read every hour in a reader manufactured by the Benson - Lehner Corporation of Los Angeles, California, and directly connected to an IBM 24 card-punch. The cross-hair was held in register with the center of the ink line at all reading stations. The results are obtained in arbitrary reader units. In order to translate these into rational gravity units (microgals), the width of the active part of the recording paper had to be standardized against a tidal gage (manufacturer's calibration) and against the reader unit. All shifts in the tidal record were likewise recorded in reader units and recalculated into microgals. The flagging system of the computer program will automatically nullify these.

6.1.2. Preparation of the series for inversion into the frequency domain.- This operation consists of two steps:

(1) Deterending on the basis of a selected integer period as described in Chapter 4.0. This operation is carried out in the observed tidal series only, with the purpose of reducing the noise to a tolerable level.

(2) Reduction to zero mean. Strictly speaking this operation is a fiction, since the tidal series cannot be made zero mean (see 4.2). The operation consists of calculating the average value for the whole effective tidal series and subtracting it from each member of the series. This operation is performed on both tidal series independently of each other. Theoretically this procedure is not necessary, but it was found helpful to operate if for the purpose of plotting and visual inspection of the two series in alignment.

6.1.3. Inversion into the frequency domain.- This is done by means of the Fourier transform. A renewed description can add nothing to the information already given in other parts of this work.

6.1.4. Calculation of delta for the diurnal, semi-diurnal and total tide.- The δ values for the two spectral peaks are calculated independently by weighted averaging within each peak. The beginning and termination of the peak is arbitrarily set at 10 microgals of amplitude of the rigid tide. Let a_i be such an amplitude at station i, and δ_i be the δ value at station i, these stations being fundamental frequency stations. Then the weighting factor r_i

$$r_i = \frac{a_i \delta_i}{\sum a_i}$$

and $$\delta_{result} = \sum r_i$$

where the summation is carried out over each peak separately. To obtain δ for the total tide, this summation is carried out over both peaks. It is important to note that the latter value is not obtained by averaging the δ values for each peak. The reason for this procedure lies in the frequently uneven bandwidth of the two peaks and such a bias must be included in the total value.

6.1.5. Interpolation in the frequency domain.- This operation is done to obtain the amplitudes and δ values for selected tidal components by procedures described elsewhere.

6.2. THE SEQUENCE OF COMPUTER PROGRAMS AND THEIR BRIEF DESCRIPTION.

(1). Program TIDAL FORCE.

This program calculates the theoretical gravitational component of the tidal force on the surface of the rigid Earth. The calculation is done from the Nautical Almanac and Ephemeris cards supplied by the U.S. Naval Observatory in Washington, D.C.

(2). Program PLAYBOY.

This program essentially transforms numbers obtained in the reading machine into units of microgals. A flagging system is built into the program which automatically annuls register shifts, positive or negative, regardless of the reason for such a shift and up to six shifts can be accomodated per record. If desired, it can also make remarks against values obtained from a tidal record which was locally disturbed by the registration of an earthquake.

(3). Program FOURIER.FILTR.

The main part of the program computes both the frequency and the phase spectrum of the input tidal series separately, as was discussed elsewhere. The subroutine PRELIM., which is operative prior to the calculation of the Fourier transform, performs the linear detrending and also executes an operation which is the nearest thing to reducing to zero mean (see text). Both of these operations are optional in the program. Should it be so desired, another filter could be substituted into subroutine PRELIM. The actual Fourier transform is executed in the subroutine TRANSF.

(4). Program SINC-FCTN.FR.

This program calculates the values of δ , as well as the rigid and yielding tidal amplitude values for selected tidal components, by interpolation in the frequency domain. Up to 15 components can be selected by presetting the freuency values to be interpolated for.

These individual programs were compounded into a single superprogram PING.PONG. This superprogram performs a complete "cradle-to-grave" tidal analysis, from the supply of raw data into the presentation of the complete analytical results, which comprise the following items:

(a) printout of the tidal series, detrended and brought to "zero mean";

(b) printout of the amplitude spectrum and corresponding values;

(c) printout of the phase spectrum;

(d) amplitudes and δ for selected tidal components;

(e) a plot of the amplitude spectrum.

6.3. BLUNDER MONITORING SYSTEMS AND PRECAUTIONS.

Throughout the work a monitoring system was set up whose purpose was to give warning of illegitimate errors, so that corrective measures could be brought into force. These precautions cannot, of course, detect errors of the systematic or random classes which are inherent in any physically measured and computed system.

(1). While reading the record, the thickness of the ink line was taken into consideration and care was exercised to hold the crosshair in the middle of this line when reading the value.

(2). Where earthquakes were present in the record, a median line was interpolated within the disturbed area of the record. A warning flag was built into the computer program processing the data, which signals in the output the numerical area which was read over an earthquake. In case of a severe earthquake, the record was interrrupted and a new record started after the earthquake, provided that such a remainder was of sufficient length.

(3). The raw data which were transformed into rational gravity units (i.e. the output of the program PLAYBOY), were subjected to a plotting subroutine and inspection of the plot will reveal misread elements of the time series. On the very few occasions that this happened (twice altogether), the erratic member was "eyeballed" back into line and a correction made on the relevant punched card.

(4). The date and hour on the Nautical Almanac and Ephemeris decks obtained from the U.S. Naval Observatory were used as counting vectors by a checking computer program, designed to detect out of order data cards.

(5). Every punched card computer output also produced a counting vector in the end columns of the punched cards. Such a counting vector served the following purposes:

(i) out of order arrangement was searched for by a small specially written computer program;

(ii) the deck could be sorted on the IBM sorting machine, if accidentally dropped.

(6). The veracity of the computer program TIDAL.FORCE had a thorough checking in WYCKOFF's work.

(7). The frequency interpolation program SINC.FCTN-FR. was checked by handcalculation.

6.4. PRESENTATION OF THE RESULTS.

The following tidal series were read and used in the calculations of the relevant values. (See Table 1). (The location terms "Monroe" and "Nike site" are identical).

Table 1.: The tidal series employed.

Batch #:	Location:	Dates:	Length (hours):
A	Monroe.	Jul 20/65 - Aug 13/65	583
B	Monroe.	Aug 15/65 - Aug 29/65	356
1	C.L.	Aug 30/67 - Oct 06/67	887
2	C.L.	Jun 17/67 - Jul 22/67	858
3	C.L.	Mar 30/67 - Apr 29/67	706
4/II	Monroe.	Aug 29/65 - Oct 14/65	1105
5	C.L.	May 16/68 - Jun 20/68	835

Table 2 contains the essential results with tolerable noise residues. Table 3 contains the same results with an unacceptable noise residue. It is emphasized here again that the values in the latter table were not taken into account. It is appended merely to show an interested reader the values obtained from a system where the residual noise is considered as disturbing, although not yet overwhelmingly so.

The following points can be gleaned from this numerical tabulation:

(1) On the whole, the δ values for the semidiurnal waves show remarkable stability, the exception being the de-

Table 2.: Tabulation of Earth tide analyses results. Low level noise residue.

Batch #:	Location:	Legth: (hours)	Detrend. period:	Irremov. noise residue:	delta diurnal:	delta semidiurnal:	delta total tide:
A (14)	Monroe	583	24	4.89	1.18561	1.20679	1.19407
B (24)	Monroe	356	24	4.744	1.18915	1.21002	1.19946
4/II (9)	Monroe	583	24	5.054	1.20188	1.20652	1.20390
4/II (26)	Monroe	835	12	7.478	1.21764	1.20855	1.21292
1 (2)	CL	887	24	1.131	1.18340	1.20447	1.19471
1 (6)	CL	867	12	3.897	1.17622	1.20039	1.18670
1 (18)	CL	835	25	7.958	1.19153	1.20732	1.19776
1 (23)	CL	887	25	2.324	1.18229	1.20421	1.19404
2 (4)	CL	709	24	3.997	1.17934	1.20588	1.18768
2 (5)	CL	709	12	3.931	1.17957	1.2059	1.1877
2 (22)	CL	709	25	3.819	1.17962	1.20588	1.18769
3 [illegible]	CL	690	24	3.332	1.19245	1.20681	1.19998

2 (5)	CL	709	12	3.931	1.17957	1.2059	1.1877
2 (22)	CL	709	25	3.819	1.17962	1.20588	1.18769
3 (15)	CL	690	24	3.332	1.19245	1.20681	1.19998
3 (16)	CL	690	12	7.024	1.19375	1.20722	1.20177
3 (17)	CL	705	24	4.905	1.18642	1.20575	1.19471
3 (25)	CL	705	25	5.252	1.18654	1.20561	1.19470
5 (11)	CL	583	24	4.473	1.19109	1.21317	1.19826
5 (12)	CL	583	12	8.36	1.18957	1.21715	1.19850
5 (21)	CL	835	25	7.958	1.19153	1.20733	1.19776

Table 3.: Tabulation of Earth tide data results. High level noise residue.

Batch #:	Location:	Length: (hours)	Detrend. period:	Irremovable noise residue:	delta diurnal:	delta semidiurnal:	delta total tide:
A (13)	Monroe	583	12	11.31	1.18556	1.20521	1.19340
4/II (29)	Monroe	835	24	13.540	1.21409	1.20899	1.21143
4/II (28)	Monroe	819	24	13.609	1.23288	1.20875	1.22306
4/II (10)	Monroe	575	12	11.07	1.21118	1.20679	1.20896
4/II (19)	Monroe	583	12	18.962	1.19910	1.20594	1.20210
1 (8)	CL	887	12	25.926	1.16773	1.20123	1.18570
1 (7)	CL	871	12	11.720	1.17470	1.20180	1.18680
1 (3)	CL	887	887	24.743	1.19960	1.20760	1.20390
1 (1)	CL	835	12	21.325	1.19090	1.20472	1.19635
3 (20)	CL	705	12	36.186	1.19461	1.21387	1.19582

termination for Batch #B and the values in Batch #5. The fluctuations in the δ values for the diurnal waves are much greater. The reader might recall that the results were obtained from an averaging integral. Since twice as many waves are sampled in the semidiurnal case as in the diurnal one, better results for the semidiurnal wave are to be expected. In this light the deviant result in Batch #B can be explained as being due to too short a series. It also can be argued on similar lines that a 2-month long record might be necessary to stabilize the diurnal values.

(2). The semidiurnal deviation of the δ value in Batch #5 determinations seems to be inherent true values. This contention is supported by the fact that all δ values for the total tide are very close in spite of the variations in the bandwidths.

(3). The semidiurnal δ values are consistently higher than the diurnal ones. This result is considered significant and important and will be discussed in Chapter 8.0 where the conclusions from this work are summarized.

(4). The only exception to (3) is found in one determination of Batch #4/II. It is in all probability only due to the random variability of the diurnal δ determinations.

Table 4.: Comparison of averaged spectral analyses values (total tide).

Record:	Location:	Spectral value:
4/II	Nike site	1.20841 ± .00451
1	C.L.	1.19330 ± .00660
2	C.L.	1.18769 ± .00010
3	C.L.	1.19779 ± .00309
5	C.L.	1.19817 ± .00041
A	Nike site	1.19407

Figure 7 shows the result of the componental analysis using the interpolation function for the nine components. The overall value, diurnal and semidiurnal, for this record (887 hours long) as well as the final values for the geographical location, are likewise displayed in the figure. More detailed componental results for a number of tidal series are displayed in Table 5.

It will be seen that the δ's for the individual components remain tolerably well clustered around the diurnal and semidiurnal values respectively. (Fig. 7). There is also a good consistency of numerical separation of the diurnal components from the semidiurnal ones. Finally, it will be observed that the semidiurnal values have a smaller spread than the diurnal ones; this is to be expected, since the former have twice the number of cycles, per record, and hence the sampling is better.

Fig 7. The δ values for some of the tidal components by frequency interpolation and their relation to established diurnal and semidiurnal values.

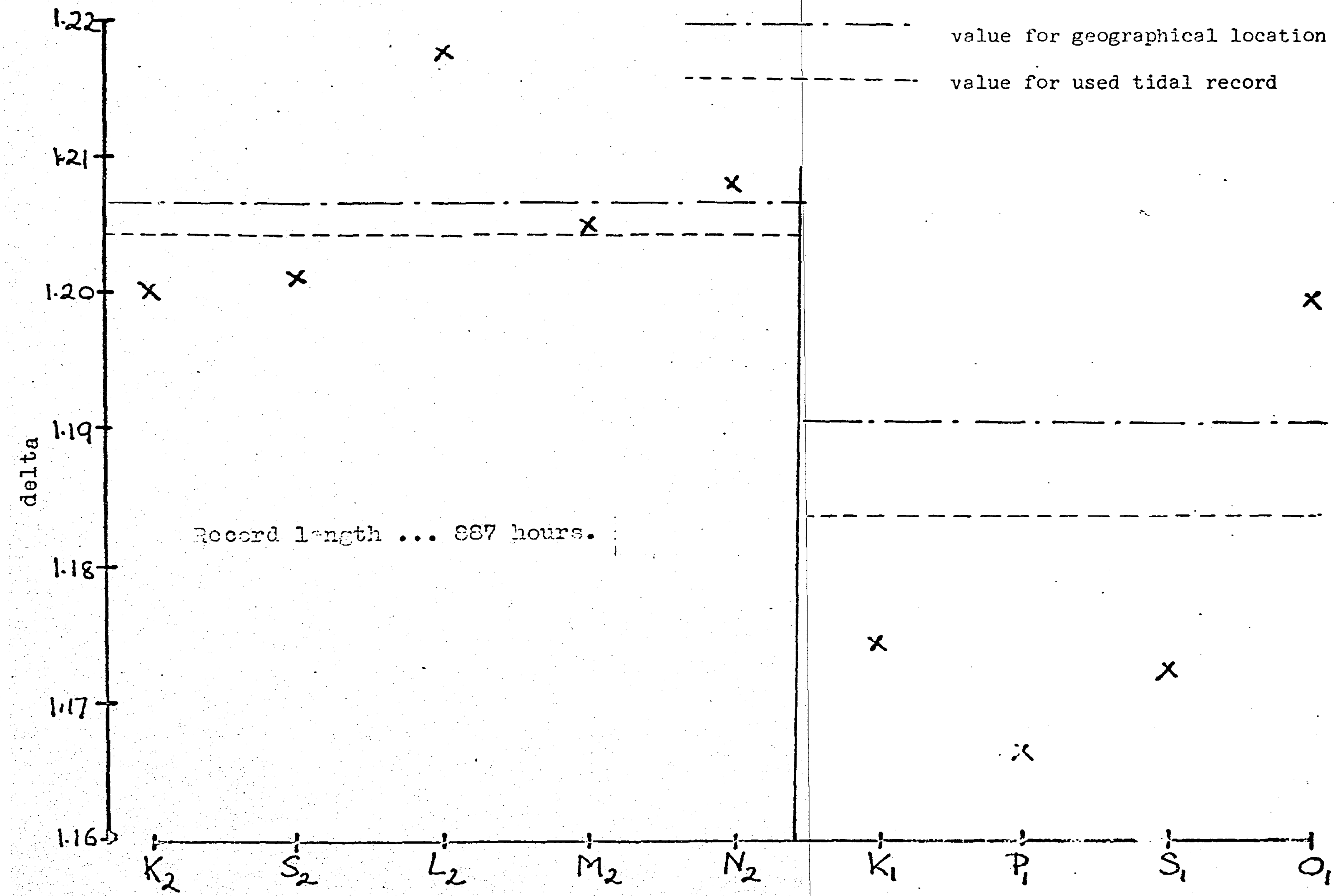


Table 5.: Tabulation of the componental analyses of some tidal series.

Amplitude listings refer to the rigid tides only.

Series		K2	S2	L2	M2	N2	K1	P1	S1	O1
Theoret. Amplit.		7	21		45	9	45	15		32
1	A	30.98	29.37	4.33	33.85	17.67	32.40	29.27	32.16	29.68
(2)	δ	1/2003	1.2009	1.2175	1.2051	1.2078	1.1740	1.1659	1.1716	1.1990
1	A	27.61	27.71	9.54	38.42	-1.49	29.31	28.06	29.39	26.62
(6)	δ	1.1930	1.1951	1.2085	1.2033	1.1086	1.1763	1.1586	1.1709	1.2031
1	A	12.60	14.11	10.03	35.65	6.98	52.71	47.21	51.72	30.69
(18)	δ	1.1961	1.1920	1.1723	1.2181	1.2154	1.1760	1.1796	1.1771	1.2101
2	A	15.02	13.58	2.83	42.00	6.91	40.94	38.91	40.36	24.41
(4)	δ	1.1980	1.1981	1.2330	1.2061	1.2078	1.1742	1.1750	1.1746	1.2005
2	A	15.03	13.58	2.83	42.00	6.91	40.95	38.91	N.D.	24.42
(22)	δ	1.1980	1.1979	1.2353	1.2060	1.2070	1.1744	1.1751		1.2004
3	A	17.48	17.90	24.84	31.66	14.29	36.26	27.67	33.72	30.74
(15)	δ	1.2035	1.2015	1.2009	1.2113	1.2065	1.1912	1.1932	1.1917	1.1814
3	A	17.48	17.90	24.84	31.66	14.29	36.26	27.67	33.72	30.74
(16)	δ	1.2044	1.2022	1.1968	1.2116	1.2134	1.1922	1.1915	1.1919	1.1844
3	A	18.24	15.60	17.34	37.96	2.91	28.31	25.48	27.60	28.55
(17)	δ	1.2048	1.2013	1.1975	1.2085	1.2091	1.1823	1.1715	1.1792	1.1987
3	A	18.23	15.60	17.34	37.96	2.91	28.31	25.48	N.D.	28.56
(25)	δ	1.2051	1.2013	1.1968	1.2083	1.2119	1.1824	1.1717		1.1990
4/II	A	12.81	10.88	16.60	36.74	4.75	25.28	28.50	26.38	23.15
(9)	δ	1.1935	1.1906	1.2014	1.2160	1.2459	1.2105	1.2158	1.2126	1.2048
5	A	6.05	4.96	16.50	39.94	7.37	39.17	39.24	39.39	15.25
(11)	δ	1.1584	1.1323	1.2010	1.2224	1.1725	1.1600	1.1564	1.1586	1.2189
5	A	6.05	4.96	16.50	39.94	7.37	39.17	39.24	39.39	15.25
(12)	δ	1.1460	1.1172	1.2078	1.2273	1.2051	1.1480	1.1433	1.1460	1.2247
5	A	12.60	14.11	10.03	35.65	6.98	52.71	47.21	N.D.	30.69
(21)	δ	1.1961	1.1920	1.1723	1.2181	1.2154	1.1760	1.1796		1.2101

It will be apparent that the values are much more reasonable than those published in the literature as obtained by means of Gaussian filters, as evidenced by the much greater consistency within a frequency bandwidth. Nevertheless, it behooves to mention that the tidal record is considered to be of insufficient length for the establishment of really significant componental values.

The interpolation method was not tried for the phase spectra.

It will be immediately evident from Table 5 that the series are not of sufficient length to contain the information necessary for a componental analysis. Nevertheless, it will be seen that a number of important tidal components have persistently reasonable values. Others, such as P_1 are consistently too high. Weaknesses are also detectable with the K_2 and S_2 components. In spite of all this, it is of interest to note that here again is a marked tendency for higher δ values in the semidiurnal components.

In case a componental analysis is required, the simplicity of the interpolation method as compared to filtering procedures is undisputable.

The end of this Chapter is a convenient place to summarize briefly the numerical results of this Chapter. The weighted averages of δ for the Pittsburgh area are shown in Table 6, the weighting being based on the length of the series. Results from series with (i) the noise well removed, and (ii) the noise incompletely removed, are shown. The standard deviation is given in each case to show the spread obtaining to each expressed average. Finally WYCKOFF's determination, done by an entirely different methos, is included in this table.

Table 6.: Weighted averages: Tabulation of final results.

A. LOW LEVEL NOISE RESIDUE.

	δ - value:	standard deviation:
Diurnal	1.19024	$\pm$ .00916
Semidiurnal	1.20670	$\pm$ .00351
Total tide	1.19594	$\pm$ .00626

B. HIGH LEVEL NOISE RESIDUE.

	δ - value:	standard deviation:
Diurnal	1.19653	$\pm$ .01823
Semidiurnal	1.20644	$\pm$ .00349
Total tide	1.20055	$\pm$ .01100

C. OTHER METHODS.

	δ - value:
Total tide (WYCKOFF)	1.19596

As was shown in Section 5.3.3, there is <u>no</u> theoretical justification for the noise removal. Nevertheless, the above table is strongly indicative of improvement by the employment of such a removal. Analogous to the Second Law of Thermodynamics, which likewise is lacking a theoretical proof for its existence, such an "improvement" is impossible to prove. It is merely based on the facts that (i) the results obtained therefrom are in excellent agreement, indeed embarrassingly so, with results obtained by an entirely different analytical procedure, (ii) the standard deviation is much less, indicative of higher accuracy. Nevertheless, only the Law of Large Numbers could approach anything resembling proof (or large scale acceptance) in this case, i.e. if a large number of results were obtained in this manner, and a very large number of these gave results in the same sense ... and so forth ... .

Finally Figure 8 is appended at the end of this Chapter, to show the interested reader, without further comment, the location of some of the tidal components with respect to the spectral structure obtained from an 887-hour long record.

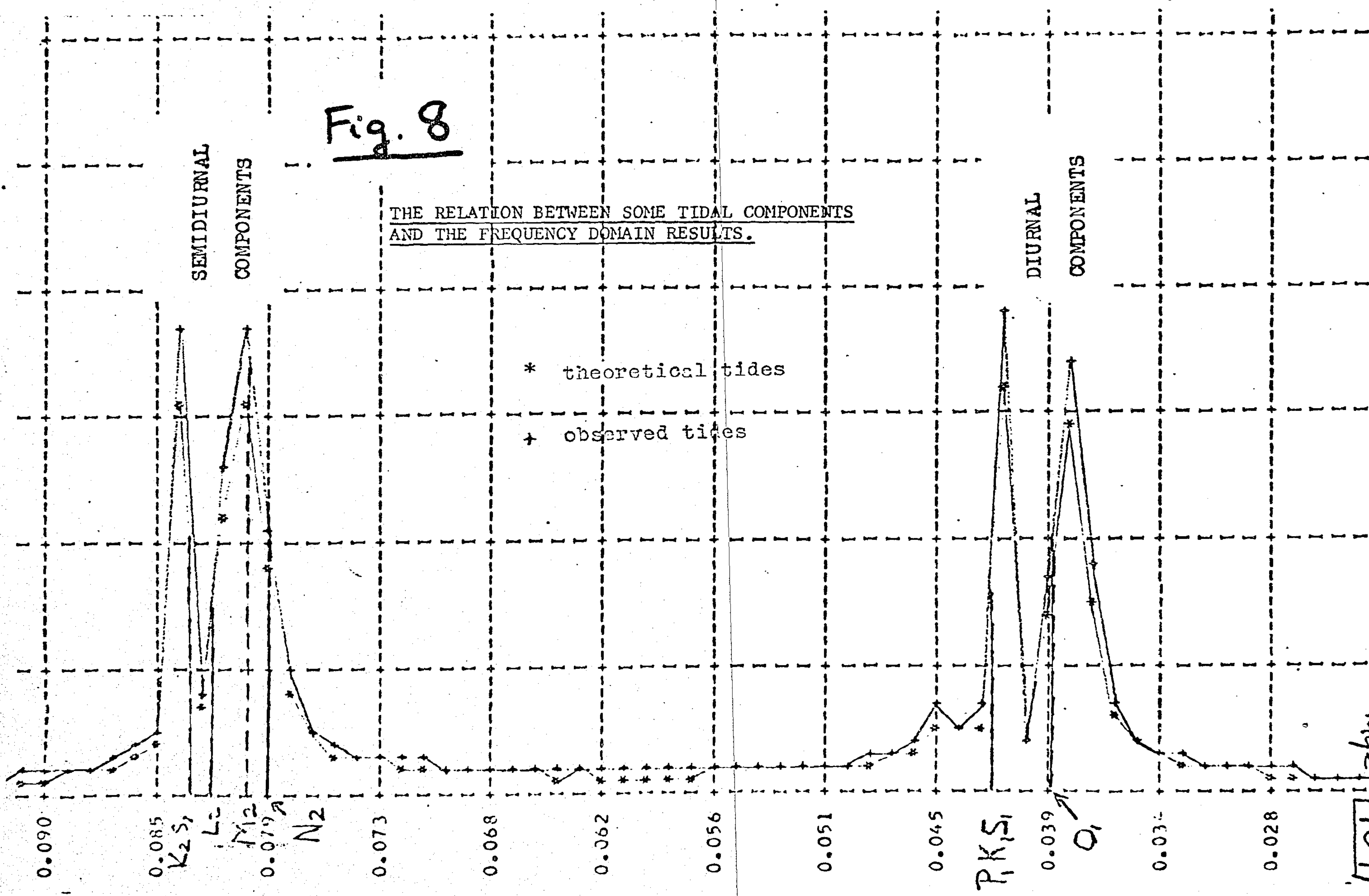


Fig. 8

THE RELATION BETWEEN SOME TIDAL COMPONENTS AND THE FREQUENCY DOMAIN RESULTS.

7.0. RESULTS OF THE PHASE SPECTRUM OF THE TIDES.

Phase studies were not an important concentration in this work. We shall merely state here the results together with some comments.

7.1. RESULTS.

Following a number of authors, such as NAKAGAWA <u>et al.</u> (1966), MIKUMO <u>et al.</u> (1968), BARSENKOV (1967) and others, we shall define a quantity $k(\omega)$ such that

$$k(\omega) = \phi_o(\omega) - \phi_r(\omega) \tag{1}$$

where

$\phi_o(\omega)$ = the phase spectrum of the observed tide;

$\phi_r(\omega)$ = the phase spectrum of the rigid tide,

and the results of equation (1) from Batch #1 are expressed in Table 7.

It is evident from this table that considerable variation exists.

Table 7 : Phase differences matched to the corresponding rigid tide amplitudes.

Frequency: (nonangular, cycles/hr)	Amplitude: (rigid tide, microgals)	Phase differ.: (angle radians)	Phase dif.: (min. time)
	diurnal tides		
.0383	27.43	-.0246	-5.64
.0395	17.91	.0397	9.99
.0407	17.34	-.0232	-5.32
.0419	52.32	-.0096	-2.20
	semidiurnal tides		
.0790	10.17	-.0256	-5.87
.0802	36.13	.0092	2.11
.0814	15.10	.0064	1.47
.0826	13.49	.0208	4.77

7.2. COMMENTS.

(1). We first of all wish to point out that absolutely no corrections were made for whatever instrumental influencing of the phase results might have occured. In particular, no adjustements to the results were made by considering the systems function of the instrument. Nor has this writer come across any work in the literature where such a correction was applied. It would appear that such work is definitely in order before committing the above tabulated values to a definite physical meaning.

(2). This writer is rather dissatisfied by the highly variable values even in tidal signals with very similar frequencies. (The closeness obtained in the semidiurnal tides is accidental and not repeated in other records). Especially disturbing is the change of sign; while physically not an impossibility (a phase advance implies that energy is flowing into the locally reacting system from elsewhere, the reverse being true for a phase delay), credibility is reduced when this happens in case of signals closely related by frequency.

(3). There is likewise a great variation from series to series, for a given frequency, without any apparent pattern in the general apparent randomness of the results.

(4). For these reasons it is felt that no great significance should be attached to the values in Table 7 as they stand as of present.

8.0 TIDAL MODEL OF AN EARTH WITH A LATERAL PETROLOGICAL FACIES CHANGE.

8.1. PURPOSE OF MODEL.

It was seen in section 6.4 that our δ results show a "crossover", i.e. $\delta_S > \delta_D$, where S denotes semidiurnal and D refers to diurnal values. This anomalous behavior - - anomalous with respect to all existing models - is reported from many other parts of the world, particularly from Central Asia. This model was designed to test the possibility that such a crossover might reflect lateral facies changes in the subjacent crust . The model used is perhaps the simplest possible one, together with a number of simplifying assumptions which will be pointed out in the body of the text as occasion arises. These simplifying assumptions will generally lead to simpler differential equations which can be solved analytically. It should be pointed out at this early stage that the model is not designed for absolute numerical accuracy of the tidal parameters. Since essentially it is a comparison of some diurnal and semidiurnal values, the principal aim is logical consistency, as well as consistency in the variations, as opposed to consistency in absolute values.

8.2. METHOD OF ATTACK.

8.2.1. Density and Lamé constants assumptions.- It is pointed out elsewhere in this work (Appendix VI) that the Lamé constants can be taken to vary sympathetically with density for naturally occuring rocks. Moreover, Poisson's ratio σ does not deviate greatly from the value .25 for such materials and that this is equivalent to asserting that $\mu = \lambda$ where μ and λ are the elastic Lamé constants for isotropic materials.

8.2.2. Appearance of the model.- The lateral facies change was simulated by assuming that the Earth is composed of two identical hemispheres of different materials, which in order to enhance the contrast were taken as granite (ρ = 2.619, μ = .317 x 10^{12} bars) and dunite (ρ = 3.267, μ = .681 x 10^{12} bars), where ρ = density and μ = Lamé constant. The geography of this composition was fixed in such a manner, with respect to the spherical coordinate ϕ, that the granitic region occurs from $\phi = 0^{\circ}$ to 180°, whereas the dunite lives at $\phi = 180^{\circ}$ to 360°. In the mathematical treatment the granitic region is identified by the superscript $^{(1)}$ and the dunite by the superscript $^{(2)}$, the brackets being introduced to prevent possible confusion with power indices.

8.2.3. Scaling.- The tidal force was assumed to be orbiting in the equatorial plane of the Earth and was due to one celestial body only (the Moon). The whole astronomical system was scaled down in proportion in such a manner that the radius of the Earth (a) was set at 1 cm. The following dimensions will then occur for our "Universe":

Moon radius = .2734 cms;

Earth radius = 1.00 cms;

Earth - Moon radius vector = 60.47 cms.

One great problem - if one aims at attainment of exact Love number values - is what value is to be assigned to the density of the Earth in the model? The total density of the Earth is 5.5, but averaging the values of granite and dunite the value of 2.993 is obtained, which was used by this author. We simply wish to inform the interested reader here that, in order to determine the Love numbers absolutely, and particularly the Love number (h), where the density value enters explicitly into the computational formula, one must assume some density law.

8.2.4. Discussion of approach.- The first and most essential step is to obtain a value for the elastic displacements under the application of a tidal force. These are obtained from Navier's equation of elastic motion which is particularized for the necessary boundary conditions and to which some simplifying assumptions were applied. These assumptions will detract from the absolutely correct values somewhat, but this is not disturbing from our point of view and the resultant equation can be solved analytically.

It is not possible to compare the diurnal and semidiurnal displacements directly - which procedure would greatly simplify the task on hand - because of the following reasons:

(i) the mode of vibration of the two tides is entirely different, the semidiurnal tides having sectorial and the diurnal tides tesseral nodes respectively;

(ii) it will be shown that for the tidal force in a particular location, the geographical distribution of the maximum displacements of each of the two tidal types will be quite different.

There are essentially two ways to tackle the problem on hand:

(1). Obtain the (radial) displacements (henceforth called u_r); from this compute the change in gravity dg; then obtain the value of Δg which is the change in gravitational acceleration due to the rigid tide. Note that dg is an effect additional to Δg.

We define the gravitational magnification δ

$$\delta = \frac{\Delta g + dg}{\Delta g}$$

The u_r's are being determined separately for semidiurnal and diurnal tides, these different tides being obtained - as will be shown later - by the employment of the appropriate forcing function on the RHS of Navier's equation of motion.

(2). The method adopted in this model is as follows:

The displacements (diurnal and semidiurnal) are determined as above. The gravitational magnification is then obtained from the Love numbers (h) and (k) by the relation

$$\delta = 1 + h - \tfrac{3}{2} k$$

The Love numbers must be computed from the following relations:

$$h = \frac{\xi g}{W2} \qquad (1)$$

where ξ = displacements and whatever the vectorial character of ξ, we shall assert that $\xi = u_r$.
W2 = tidal potential (evaluated at the Earth's surface only).

Furthermore

$$k = \frac{V}{W2} \qquad (2)$$

where V = potential due to the deformation.
The equation for this has to be derived, which will be done at the appropriate stage.

As will be shown in the proper place, a certain order has to be followed; in particular the solution for u_r has to be done first, since these values will be needed in the determination of (h) as well as V.

8.3. DEVELOPMENT OF THE INDIVIDUAL STAGES.

8.3.1. Boundary conditions.- In this section we shall pay particular attention to possible boundary conditions which might be used to particularize Navier's equation of motion. Some other boundary conditions, especially those applicable to other equations in this system, will be discussed in other parts of the text as the occasion might arise.

The first essential boundary condition to be stablished is to insure that only elastic deformation contributes to the resultant displacements. Considering a purely elastic system, the following displacements are possible as being due to

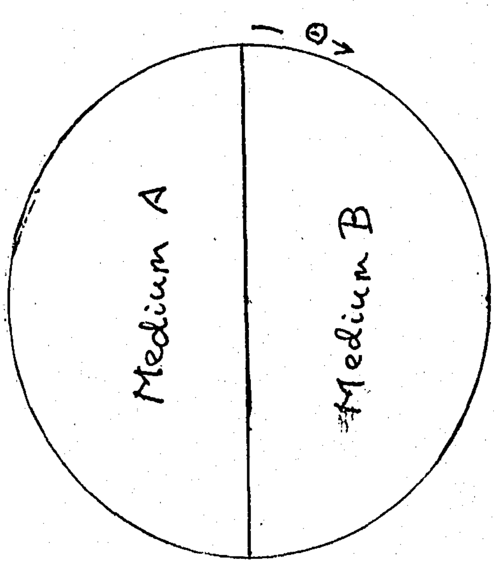
(i) translation;

(ii) rotation;

(iii) elastic deformation.

We have to eliminate (i) and (ii).

The basic model we shall call Model A.



Take radius = 1 and x = y = 0 for the origin, then the surface of separation (in spherical coordinates) is

$$\rho^2 \sin^2 \Theta = r^2 = 1$$

Fig. 9 : Earth Model A

There is another related Model B, perhaps geologically more relevant, which will have somewhat different boundary conditions and which will not be solved in this work.

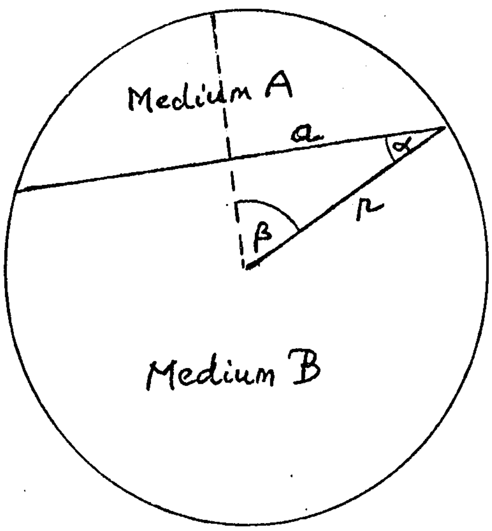


Fig. 10: Earth Model B.

Sphere of unit radius, r = 1; circle of separation will be here a small circle of radius (a), where

$a = r \cos\alpha$

The formula for the small circle of separation with the origin (x,y,z) of the sphere at (0,0,0) in spherical coordinates is

$$\rho^2 \sin^2\Theta = r^2 \cos^2\alpha$$
$$\rho^2 \sin^2\Theta = \cos^2\alpha$$

Paying attention to Model A only, we shall assert that at the contact of the two media $^{(1)}$ and $^{(2)}$ the contacts are welded, i.e.

$$u_r^{(1)} = u_r^{(2)} \qquad (3)$$

To prohibit translation it is sufficient to forbid any displacements in the center of the sphere, thus at the center (0,0,0) we have

$$u_r^{(1)}(0,0,0) = u_r^{(2)}(0,0,0) = 0 \tag{4}$$

To prohibit rotation we must have the condition that the rotation tensor $\omega_{ij} = 0$. Using Cartesian coordinates, we have

$$\omega_{zy} = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) = 0 \tag{5a}$$

$$\omega_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = 0 \tag{5b}$$

$$\omega_{yx} = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0 \tag{5c}$$

where u, v and w are the displacements along the x, y and z axes respectively.

Since the tidal force orbits in the x - y plane, it can produce only a possible rotation tensor ω_{xy}, so that the only equation we have to worry about is

$$\omega_{yx} = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0 \tag{5}$$

Now, using general subscripts i, j instead of x, y we have

$$\omega_{ij} = \frac{1}{2}\left(u_{i/j} - u_{j/i}\right)$$

where u = displacements and the slash denotes the (covariant) derivative.

Remembering that by definition the rotation tensor is antisymmetric, i.e.

$$u_{i/j} = -u_{j/i}$$

we have, moreover, inherent in the condition $\omega_{ij} = 0$,

$$\omega_{ij} = \frac{1}{2}\left(u_{i/j} - u_{j/i} \right) = 0$$

i.e. $u_{i/j} = u_{j/i}$ - a symmetric tensor, but this is the characteristic of the strain tensor (infinitesimal strain theory). It follows, therefore, that the prohibition of rotation equations are unnecessary. It is of interest to note that for a tensor to be symmetrical and at the same time antisymmetrical everywhere in a given plane, we have

$$u_{i/j} = -u_{j/i} = u_{j/i} = 0$$

i.e. for a given plane the displacements will be the same everywhere (this original condition will, of course, be altered by other imposed boundary conditions).

In actual practice, the elimination of the rotation tensors is accomplished when the stress (or strain) tensor satisfies the necessary compatibility equations.

From the general equation of displacement within some region (P_oP) we have

$$u_j(x_1, x_2, x_3) = u_j^{o} + \int_{P_o}^{P} e_{jk}\, dx_k + \int_{P_o}^{P} \omega_{jk}\, dx_\ell$$

where u_j^{o} represents rigid motion of translation,
e_{jk} is deformational (strain tensor) and
ω_{jk} is rigid body motion of rotation

the compatibility equations will take care of ω_{jk}, but u_j^{o} has to be supressed, as was done in equation (4).

Another necessary boundary condition

$$\tau_{ij}\, \nu_j = T^{\nu_i} \tag{6}$$

where T^{ν_i} is the surface traction (our tidal force) and ν_j is the normal-vector. For work in the interior of the Earth the functions τ_{ij} must obey everywhere the Beltrami-Mitchell compatibility equations to insure that the τ_{ij}'s are single--valued continuous functions. For the type of forces considered in our system, this will happen whenever the biharmonic equation

$$\nabla^4 \tau_{ij} = 0$$

is satisfied. We shall omit this extremely tedious procedure, since our work is concerned with the surface only, but it is of interest to note that this author has seen this nowhere checked in models applying themselves to the interior regions.

(τ_{ij} are the stress tensors).

Since the displacements are very small (compared to the size of the Earth) and the time taken to travel from one displacement extremum to the other one is very large (i.e. the velocity will be very small), we can, if this will prove necessary, make the approximation

$$\frac{\partial u_r^{(1)}}{\partial t}(r) = \frac{\partial u_r^{(2)}}{\partial t}(r) \tag{7}$$

to be another boundary condition.

8.3.2. Reduction of the Navier equation.- The basic equation is

$$\mu \nabla^2 u_i + (\lambda + \mu)\, \vartheta/_i$$

the steady state equation, ~~to which later another term will~~ be added.

ϑ = strain invariant = e_{ii}

in spherical coordinates this is

$$\frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{2u_r}{r} + \frac{1}{r\sin\theta}\frac{\partial u_\phi}{\partial \phi} + \frac{u_\phi \cot\theta}{r} \tag{8}$$

We are interested only in the u_r displacements (by nature of the gravity tidal variation) and only on the surface at that - hence we shall consider only the θ and ϕ variation.

Using $\lambda = \mu$ as described before, we get

$$\mu(\nabla^2 u_i \quad + \quad 2\vartheta/_i) = 0$$

Part I Part II

Part I:

$\nabla^2 u_1 = \nabla^2 u_r$, by our considerations, which in spherical coordinates is:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_r}{\partial r}\right) + \frac{1}{r^2 \sin\Theta}\frac{\partial}{\partial\Theta}\left(\sin\theta\frac{\partial u_r}{\partial\theta}\right) + \frac{1}{r^2\sin^2\Theta}\frac{\partial^2 u_r}{\partial\phi^2}$$

discarding the r-dependence and performing the necessary differentiations, we obtain

$$\frac{1}{r^2\sin\Theta}\left(\cos\Theta\frac{\partial u_r}{\partial\Theta} + \sin\Theta\frac{\partial^2 u_r}{\partial\Theta^2}\right) + \frac{1}{r^2\sin^2\Theta}\frac{\partial^2 u_r}{\partial\phi^2} \tag{9}$$

Part II:

$$2\vartheta/_i = 2\nabla\vartheta \tag{10}$$

and we are interested only in the Θ and ϕ variations of the gradient.

From the point of view of our usefulness (8) reduces to

$$\vartheta = \frac{\partial u_r}{\partial r} + \frac{2u_r}{r} \tag{11}$$

using equation (11) in (10) we have (in spherical coordinates)

$$\nabla = \frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial}{\partial\Theta} + \frac{1}{r\sin\Theta}\frac{\partial}{\partial\phi}$$

hence

$$2\left\{\underbrace{\frac{\partial}{\partial r}\left(\frac{\partial u_r}{\partial r}+\frac{\partial}{\partial r}\left(\frac{2u_r}{r}\right)\right)}_{\boxed{A}}+\frac{1}{r}\left[\frac{\partial}{\partial\theta}\left(\frac{\partial u_r}{\partial r}\right)+\frac{\partial}{\partial\theta}\left(\frac{2u_r}{r}\right)\right]+\right.$$

$$\left.+\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\phi}\left(\frac{\partial u_r}{\partial r}\right)+\frac{\partial}{\partial\phi}\left(\frac{2u_r}{r}\right)\right]\right\}$$

where the term marked "A" does not concern us and hence the above equation simplifies to

$$2\left\{\frac{1}{r}\left[\frac{\partial^2 u_r}{\partial r\partial\theta}+\frac{2}{r}\frac{\partial u_r}{\partial\theta}\right]+\frac{1}{r\sin\theta}\left[\frac{\partial^2 u_r}{\partial\phi\partial r}+\frac{2}{r}\frac{\partial u_r}{\partial\phi}\right]\right\} \quad (12)$$

Using (12) and (9) to recombine into the original equation, and remembering that for our Earth r = 1, we obtain

$$\mu\left\{\frac{1}{\sin\theta}\left(\cos\theta\frac{\partial u_r}{\partial\theta}+\sin\theta\frac{\partial^2 u_r}{\partial\theta^2}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2 u_r}{\partial\phi^2}+\right.$$

$$\left.+2\left[\frac{\partial^2 u_r}{\partial r\partial\theta}+2\frac{\partial u_r}{\partial\theta}\right]+\frac{2}{\sin\theta}\left[\frac{\partial^2 u_r}{\partial\phi\partial r}+2\frac{\partial u_r}{\partial\phi}\right]\right\}=0 \quad (13)$$

(13) is a very formidable equation indeed, which has to be solved if correct values for (h), (k) and δ are required everywhere on the surface of the sphere. However, since we are only interested in comparing the diurnal and semidiurnal values at a given parallel on the sphere, we can obtain considerable simplification of the equation by neglecting the θ-variation.

Accordingly, we obtain from (13)

$$\mu\left\{\frac{1}{\sin^2\theta}\frac{\partial^2 u_r}{\partial\phi^2}+\underbrace{\frac{2}{\sin\theta}\frac{\partial^2 u_r}{\partial\phi\partial r}}_{\text{Term } B}+\frac{4}{\sin\theta}\frac{\partial u_r}{\partial\phi}=0\right.$$

We shall now use in Term B the boundary condition (4) and we shall further assert that at the surface, i.e. at r = a, $u_r = u_r$ and that the increase is linear along the whole radius from 0 to u_r, i.e.

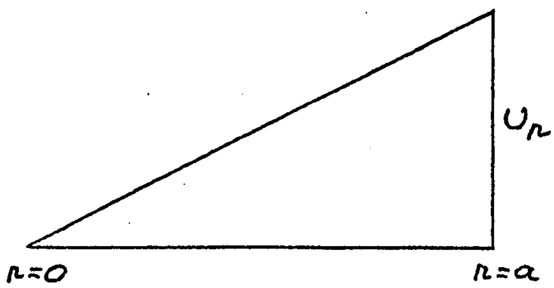


Fig. 11 : $u_r = u_r(r)$

then , from geometry

$$\frac{\partial u_r}{\partial r} = \text{slope} = \frac{U_r}{a} = (U_r)_{\text{surface}}$$

since a = 1;

hence

$$\mu\left(\frac{1}{\sin^2\theta}\frac{\partial^2 U_r}{\partial\phi^2} + \frac{2}{\sin\theta}\frac{\partial U_r}{\partial\phi} + \frac{4}{\sin\theta}\frac{\partial U_r}{\partial\phi}\right) = 0.$$

Since a force is acting on our system, we cannot ignore the body forces within the sphere and to which hitherto no attention whatsoever was paid. Without going here into proofs and derivations, the body forces (arrived at by invoking d'Alembert's principle) $= -\rho\,\frac{\partial^2 U_r}{\partial t^2}$

Hence the complete elastic equation of motion awaiting solution is:

$$\frac{1}{\sin^2\theta}\frac{\partial^2 U_r}{\partial\phi^2} + \frac{6}{\sin\theta}\frac{\partial U_r}{\partial\phi} - \frac{\rho}{\mu}\frac{\partial^2 U_r}{\partial t^2} = 0. \qquad (14)$$

In the section dealing with the solutions of the developed differential equations, we shall see that we need only the particular integral for a forcing function on the RHS, which will be the tidal force, diurnal or semidiurnal.

8.3.3. Development of the deformation potential equation.- The total disturbing potential R can be taken as being composed of

$$R = V_o + W + V$$

where

V_o = initial terrestrial potential;

W = external potential;

V = additional potential due to the deformation (where V = kW, k = Love number).

The variation of density at a given point is

$$\rho_o - \rho = \xi \frac{\partial \rho_o}{\partial r}$$

, where ρ_o = initial density and ξ = displacements, which we shall again equate with u_r, the above equation being valid for a model without a lateral density variation. For models with such a variation, the density change will be

$$\rho_o - \rho = \xi \left(\frac{\partial \rho_o}{\partial r} + \frac{1}{r} \frac{\partial \rho_o}{\partial \theta} + \frac{1}{r \sin\theta} \frac{\partial \rho_o}{\partial \phi} \right) \qquad (15)$$

and we shall again assert $\xi = u_r$.

The proposed model has no meridian density variation, since going along a given meridian ρ_o will always be the same. Furthermore, the r-variation is of no interest to us, since we concentrate on surface displacements only.

Under these conditions equation (15) will reduce to

$$\rho_0 - \rho = \frac{U_r}{\sin\theta}\frac{\partial \rho_0}{\partial \phi} , \qquad \text{(letting r = 1)}$$

from which we obtain the altered density ρ

$$\rho = \rho_0 - U_r\left(\frac{1}{\sin\theta}\frac{\partial \rho_0}{\partial \phi}\right) \tag{16}$$

Now, the following relations must hold for the potentials:

$$\nabla^2 W = 0 \qquad \text{(external - Laplace equation)}$$

$$\nabla^2 V_0 = -4\pi f \rho \qquad \text{(internal - Poisson's equtn.)}$$

where f = gravitational constant.

Using equation (16) to obtain the total internal potential together with the deformation, and remembering that the operators are linear,

$$\nabla^2 (V_0 + V) = -4\pi f\left[r - u_r\left(\frac{1}{\sin\theta}\frac{\partial \rho_0}{\partial \phi}\right)\right]$$

so that

$$\nabla^2 V = 4\pi f u_r \frac{1}{\sin\theta}\frac{\partial \rho}{\partial \phi} \tag{17}$$

where we have written $\partial \rho$ instead of $\partial \rho_0$, the difference being completely negligible.

Expanding the left hand side of equation (17) in spherical coordinates we obtain

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) +$$

$$+ \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2} = \cdots$$

and, since r = 1

$$\frac{\partial^2 V}{\partial r^2} + \frac{\sin\theta}{\sin\theta}\frac{\partial^2 V}{\partial\theta^2} + \frac{\cos\theta}{\sin\theta}\frac{\partial V}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2} = \cdots$$

To simplify this result, we shall again assert that $\partial V/\partial r$ is of no consequence to our problem. Furthermore, by arguments similar to those developed in the Navier equation, ~~we shall likewise ignore $\partial V/\partial\theta$. This is not strictly cor-~~ rect, of course, as Fig. 12 will show, where it is obvious that the individual points V_i do not have the same disposition with respect to the distribution of the mass, but we shall consider this approximation as being good enough, since we are interested in relative, rather than absolute values.

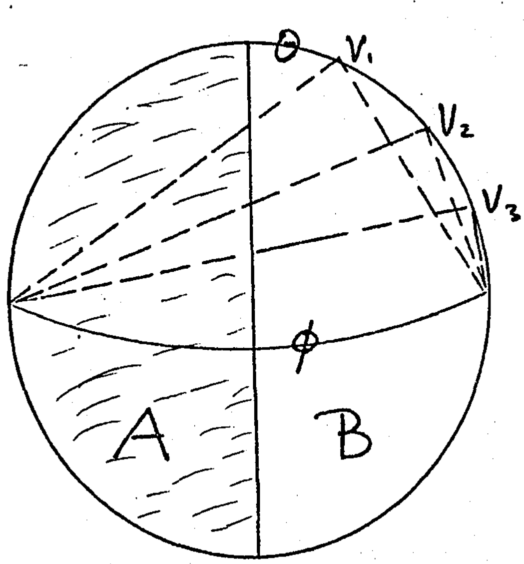


Fig. 12 : The relation of points V_i to the distribution of mass.

The required equation then becomes

$$\frac{\partial^2 V}{\partial \phi^2} = 4\pi f u_\mu \sin\Theta \frac{\partial \rho}{\partial \phi} \tag{18}$$

remembering the imposed limitations, especially the lack of functional Θ-dependence. For work at a given parallel we rely solely on the parameter sinΘ to provide the necessary variety.

Equation (18) in its present form still cannot be used, since it has two dependent variables.

This difficulty is overcome by assigning two ρ and μ values to the hemispheres, viz. $\rho^{(1)}, \mu^{(1)}$ and $\rho^{(2)}, \mu^{(2)}$ representing the two media considered and at the same time thus giving them a definite geographical orientation with respect to all integrations, etc., these values remaining unchanged throughout the life of the model. (See Fig. 13).

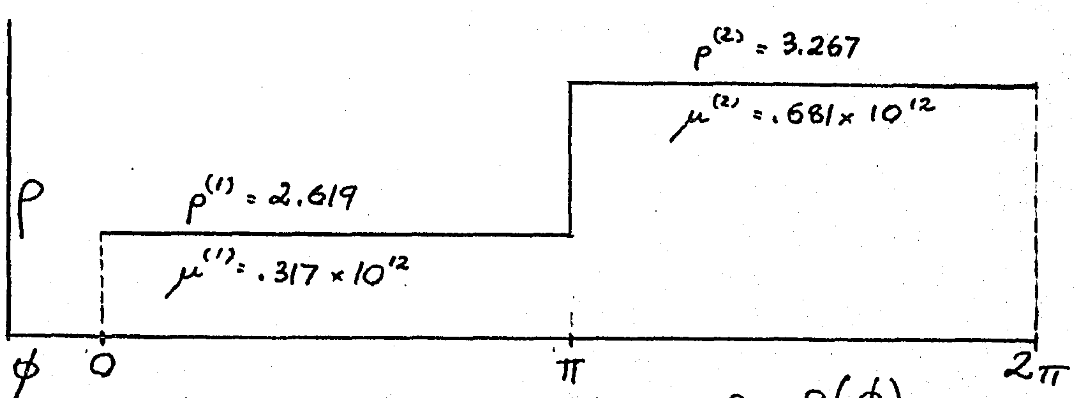


Fig. 13 : The distribution $\rho = \rho(\phi)$

This distribution, along any parallel, shown in Fig. 13 is approximated by a Fourier series expansion for $\rho = \rho(\phi)$, the result of which is

$$\rho(\phi) = 2.943 - \sum_{n} \frac{1.296}{\pi n} \sin n\phi \qquad (19)$$

n = 1, 3, 5 ... p

and

$$\frac{\partial \rho}{\partial \phi} = - \sum_{n} \frac{1.296}{\pi} \cos n\phi \qquad (20)$$

and shown in Figs. 14 and 15 . These figures also show that taking p = 11 or 13 will give sufficient accuracy.

The perusal of (20) in (18) amounts to a particularization of the differential equation to the boundary conditions.

We thus have the resultant equation

$$\frac{\partial^2 V}{\partial \phi^2} = -3.455 \times 10^{-7} U_r \sin\Theta \sum_{n=1,3,5...}^{p} \cos n\phi \qquad (21)$$

As an independent check on the correctness of our development, since the tidal force is the same everywhere (on a given parallel), it follows from the 1st Law of Thermodynamics that V must be the same everywhere on a given parallel, regardless of the values of ρ and μ , since otherwise it would be possible to either create or destroy energy.

8.3.4. Other equations considered in this system.- In order to define the system completely, we now have to produce the equation which determines the value of W2. We

Fig. 14 : Surface variation of density with ϕ and approximation by Fourier series.

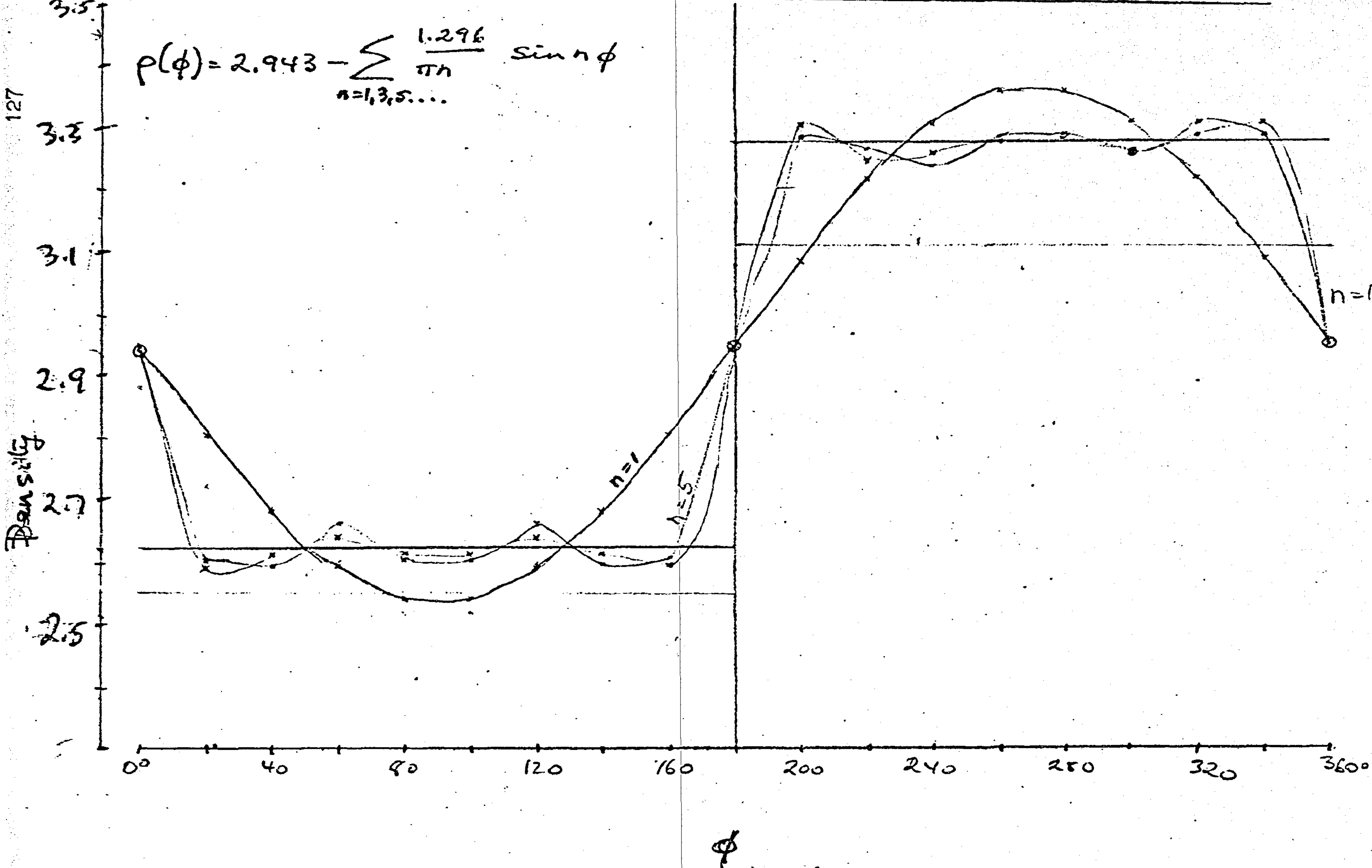
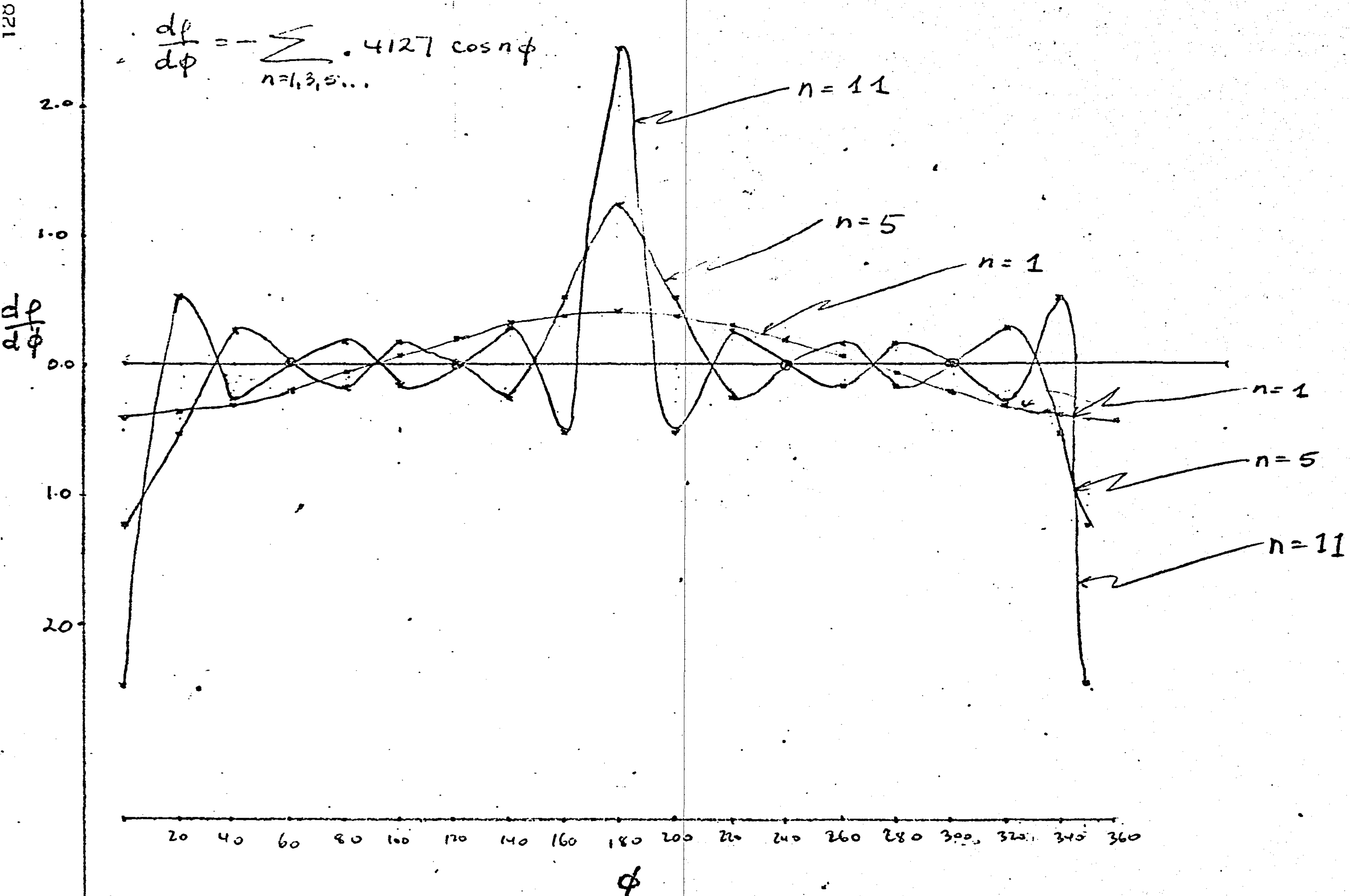


Fig. 15 : Derivative variation with ϕ.



shall consider only the sectorial and tesseral contributions to the potential. The zonal contributions (i.e. the long period phenomena) shall be ignored.

Under these conditions

$$W2 = fm\frac{a^2}{r^3}\left[\cos\theta'\sin\theta'\cos\phi + \sin^2\theta'\cos 2\phi\right]$$

where f = gravitational constant (6.673×10^{-8} cm^2 dynes gm^{-2});

m = mass of the Moon (to our scale: .2850 gms);

a = radius of the Earth (= 1 cm) ;

r = Earth - Moon radius vector (60.47 cms).

Hence

$$W2 = \frac{fm}{r^3}\left[\underbrace{\cos\theta'\sin\theta'\cos\phi}_{A} + \underbrace{\sin^2\theta'\cos 2\phi}_{B}\right] \quad (21)$$

where A represents the tesseral (diurnal) and B the sectorial (semidiurnal) part.

At this stage it is important to realize that θ' in the above equations is different from the previously encountered θ. The latter is the spherical coordinate whose origin is at the North pole. The former, on the other hand is the declination, whose zero is at the equator. The relation is $\theta' = 90^\circ - \theta$. We shall again encounter θ' in the forcing function. The final result therefore is

$$W2 = \frac{fm}{r^3}\left[\sin\theta\cos\theta\cos\phi + \cos^2\theta\cos 2\phi\right] \quad (22)$$

8.4. SOLUTION OF THE DIFFERENTIAL EQUATIONS.

8.4.1. The available choices.- As opposed to all the previously proposed models which were concentrating on the Earth's interior, the present model is concentrating on surface phenomena as a function of lateral facies changes. In order to produce a tidal response in different locations on the surface, it is necessary to employ in our model a relative motion between the Earth and the tidal force. In our model we shall assume that the Earth is stationary and that the tidal force is orbiting around it in an equatorial plane.

(1). To all appearances the simplest way to attain this would be to take the tidal force (represented by the surface traction) for a walk around the Earth by means of a line integral along a path given by the equatorial circle around the Earth. However, by the well known theorem on closed paths undertaken in a conservative force field, the result will be zero and we therefore would have to undertake homogeneous solutions for the relevant differential equations. Unfortunately such a solution could not discriminate between diurnal and semidiurnal values and hence another method has to be resorted to.

(2). We shall adopt a method of "instantaneous stretch" to enable us to evaluate diurnal and semidiurnal parameters. Since the tidal force is the same everywhere (on a given latitude), we do not need not worry about any variations on this count and we shall simply assume that the orbiting tidal force completes its orbit instantaneously, together with all the consequent tidal effects. We are thus effectively compressing the whole series of events into time $t = 0$. In doing this, it will only be necessary to take care of the following two conditions:

(i) The equations still have to be integrated properly, but time will cease to be a variable in the (final) evaluation;

(ii) by differentiating the diurnal and semidiurnal force functions with respect to ϕ and setting to zero, it will be seen that the tidal maxima of these two tides are separated by a 45° phase difference. Since the method of "instantaneous stretch" is based on the maximum displacements, this phase difference has to be honored. This will be nowhere evident in the solutions given in this section, but is fully allowed for computationally in the computer program.

8.4.2. The forcing functions.- We now must evaluate the forcing functions (i.e. the surface tractions) which will come to govern the appropriate equations of motion.

The well known "stress vector - stress tensor" equation (6) is one boundary condition to be imposed on the reduced Navier equation of motion (14). Equation (6) relates the surface tractions to the stress tensor in the interior region. Equations (6) and (14) form a Sturm - Liouville system of self-adjoint differential equations for reasons which will not be discussed here. It is one of the properties of such a system that it is possible to interchange the RHS of the equations belonging to this system. This enables us to move the force function (i.e. the surface traction) onto the RHS of the Navier equation. We note, in passing, although no need will arise for it in this work, that another property of a self-adjoint system is that each RHS function can be used once only. Thus for instance, should we be forced for some reason to use these two equations again, equation (6) must now be used with a zero on the RHS, since the T function is "used up". (Occasion for this would arise if work is done on the interior regions, which would need the evaluation of the τ_{ij}'s).

The forcing functions are descended from the tidal potential W2 by differentiation with respect to (a). The result is

for the diurnal tides: $8.468 \times 10^{-14} \cos\theta' \sin\theta' \cos\phi$

for the semidiurnal tides:

$$8.468 \times 10^{-14} \sin^2\theta' \cos 2\phi$$

where θ' = declination. With proper adjustment for the coordinate system in our use, as discussed before, we shall obtain

$8.468 \times 10^{-14} \sin\theta \cos\theta \cos\phi$ - diurnal

$8.468 \times 10^{-14} \cos^2\theta \cos 2\phi$ - semidiurnal

and these values will be used on the RHS for the appropriate tides. In the sequel we shall put $D = 8.468 \times 10^{-14}$ for the sake of simplicity of appearance and manipulation.

8.4.3. The particular integral of the Navier equation.-

(1). Diurnal tides:

The reduced Navier equation will now have the appearance

$$\frac{1}{\sin^2\theta}\frac{\partial^2 U_r}{\partial\phi^2} + \frac{6}{\sin\theta}\frac{\partial U_r}{\partial\phi} - \frac{\rho}{\mu}\frac{\partial^2 U_r}{\partial t^2} = D\sin\theta\cos\theta\cos\phi \qquad (23)$$

which is a linear equation in its coefficients which we shall temporarily set as

$A = 1/\sin^2\theta$ $\qquad$ $B = 6/\sin\theta$ $\qquad$ $C = \rho/\mu$

We shall first of all separate the variables and assert that a solution exists such that

$$U_r = \Phi(\phi) . T(t) \qquad (24)$$

To solve the T(t) part, we have

$$C\frac{\partial^2 T}{\partial t^2} = D\cos\theta\sin\theta\cos\phi$$

of necessity we also have in this connection $\phi = \phi(t)$, in particular $\phi = \omega t$, where ω = angular velocity of the tide raising celestial body (and not necessarily of the tide).

Hence

$$C\frac{\partial^2 T}{\partial t^2} = D\cos\theta\sin\theta\cos\omega t$$

integrating the 1st time

$$C\frac{\partial T}{\partial t} = \frac{1}{\omega}D\cos\theta\sin\theta\sin\omega t + a_1$$

and integrating the 2nd time

$$CT = -\frac{1}{\omega^2}D\cos\theta\sin\theta\cos\omega t + a_1 t + a_2$$

$$\therefore\ T = \frac{\mu}{\rho}\left(a_2 + a_1 t - \frac{D}{\omega^2}\cos\theta\sin\theta\cos\omega t\right)$$

Now, the boundary condition demands that T at t = 0 have the same value as at t = T'', where T'' = period, when $\phi = 360^\circ$.

Therefore $a_1 = 0$.

and the time function is

$$T = \frac{\mu}{\rho}\left(a_2 - \frac{D\cos\theta\sin\theta\cos\omega t}{\omega^2}\right)$$

that is

$$T = \frac{\mu}{\rho}\left(a_2 - \frac{D\cos\theta\sin\theta\cos\phi}{\omega^2}\right) \qquad (25)$$

The ϕ-dependence is to be handled as follows:

We have

$$A\frac{\partial^2 \Phi}{\partial \phi^2} + B\frac{\partial \Phi}{\partial \phi} = D\cos\theta\sin\theta\cos\phi$$

where A and B have values as stated before.

We are interested in the particular integral only and we shall assert that such an integral, using the method of undetermined coefficients is given by

$$\Phi = PD\cos\theta\sin\theta\cos\phi + QD\cos\theta\sin\theta\sin\phi$$

where P and Q have to be determined;

then

$$\frac{\partial \Phi}{\partial \phi} = -PD\cos\theta\sin\theta\sin\phi + QD\cos\theta\sin\theta\cos\phi$$

$$\frac{\partial^2 \Phi}{\partial \phi^2} = -PD\cos\theta\sin\theta\cos\phi - QD\cos\theta\sin\theta\sin\phi$$

and using the above relations in the original equation, we obtain

$$-PAD\cos\theta\sin\theta\cos\phi - QAD\cos\theta\sin\theta\sin\phi -$$

$$-PBD\cos\theta\sin\theta\sin\phi + QBD\cos\theta\sin\theta\cos\phi =$$

$$= D\cos\theta\sin\theta\cos\phi$$

Collecting coefficients and comparing, we get

$$P = -\frac{A}{A^2+B^2} \quad , \qquad Q = \frac{B}{A^2+B^2}$$

i.e.

$$P = -\frac{\sin^2\theta}{1+36\sin^2\theta} \quad , \qquad Q = \frac{6\sin^3\theta}{1+36\sin^2\theta}$$

using these values in the particular integral

$$\Phi = -\frac{\sin^3\theta}{1+36\sin^2\theta} D\cos\theta \cancel{\sin\theta} \cos\phi + \frac{6\sin^4\theta}{1+36\sin^2\theta} D\cos\theta \cancel{\sin\theta} \sin\phi$$

$$= \frac{1}{1+36\sin^2\theta}\left(6D\cos\theta\sin^4\theta\sin\phi - D\cos\theta\sin^3\theta\cos\phi\right)$$

Refering to equation (24), i.e.

$$U_r = \Phi(\phi) . T(t)$$

we obtain

$$\frac{\mu\left(a_2 - \frac{1}{\omega^2} D\cos\theta\sin\theta\cos\phi\right)}{\rho\,(36\sin^2\theta + 1)} \cdot$$

$$\cdot\left(6D\cos\theta\sin^4\theta\sin\phi - D\cos\theta\sin^3\theta\cos\phi\right)$$

from which the integration constant a_2 must be eliminated.

We shall do this by using the boundary condition $u_r^{(1)} = u_r^{(2)}$ at $\phi = 180^\circ$ (which can, of course also be stated at 360°).

Remembering the geography, i.e.

Medium$^{(1)}$ at $\phi = 0^o$ to 180^o

Medium$^{(2)}$ at $\phi = 180^o$ to 360^o, we have

$$\frac{\mu^{(1)}}{\rho^{(1)}}\left(a_2 - \frac{1}{\omega^2} D\cos\Theta\sin\Theta\cos\phi\right)\frac{1}{36\sin^2\Theta+1}\cdot$$

$$\cdot\left(6D\cos\Theta\sin^4\Theta\sin\phi - D\cos\Theta\sin^3\Theta\cos\phi\right) =$$

$$= \frac{\mu^{(2)}}{\rho^{(2)}}\left(a_2 - \frac{1}{\omega^2} D\cos\Theta\sin\Theta\cos\phi\right)\frac{1}{36\sin^2\Theta+1}\cdot$$

$$\cdot\left(6D\cos\Theta\sin^4\Theta\sin\phi - D\cos\Theta\sin^3\Theta\cos\phi\right)$$

which simplifies to

$$\frac{\mu^{(1)}}{\rho^{(1)}}\left(a_2 + \frac{1}{\omega^2} D\cos\Theta\sin\Theta\right) =$$

$$= \frac{\mu^{(2)}}{\rho^{(2)}}\left(a_2 + \frac{1}{\omega^2} D\cos\Theta\sin\Theta\right)$$

so that

$$\frac{\mu^{(1)}}{\rho^{(1)}} a_2 - \frac{\mu^{(2)}}{\rho^{(2)}} a_2 = \frac{D\cos\Theta\sin\Theta}{\omega^2}\left(\frac{\mu^{(2)}}{\rho^{(2)}} - \frac{\mu^{(1)}}{\rho^{(1)}}\right)$$

$$a_2 = -\frac{\mu^{(1)}\rho^{(2)} - \mu^{(2)}\rho^{(1)}}{\mu^{(1)}\rho^{(2)} - \mu^{(2)}\rho^{(1)}}\;\frac{D\cos\Theta\sin\Theta}{\omega^2}$$

$$a_2 = -\frac{D\cos\theta\sin\theta}{\omega^2}$$

reinserting the value of a_2 into the solution for u_r , we obtain

$\phi = 0^\circ - 180^\circ$:

$$U_r^{(1)} = \frac{\mu^{(1)} D\cos\theta\sin\theta}{\rho^{(1)}(36\sin^2\theta + 1)}\left(\frac{-1-\cos\theta}{\omega^2}\right)\left(6\sin^3\theta\sin\phi - \sin^2\theta\cos\phi\right)$$

$\phi = 180^\circ - 360^\circ$: (26)

$$U_r^{(2)} = \frac{\mu^{(2)} D\cos\theta\sin\theta}{\rho^{(2)}(36\sin^2\theta + 1)}\left(\frac{-1-\cos\theta}{\omega^2}\right)\left(6\sin^3\theta\sin\phi - \sin^2\theta\cos\phi\right)$$

Equations (26) thus give the diurnal tidal displacements in both media.

(2). Semidiurnal tides:

Starting with

$$\frac{1}{\sin^2\theta}\frac{\partial^2 U_r}{\partial\phi^2} + \frac{6}{\sin\theta}\frac{\partial U_r}{\partial\phi} - \frac{\rho}{\mu}\frac{\partial^2 U_r}{\partial t^2} = D\cos^2\theta\cos 2\phi$$

and by the employment of analogous procedures we arrive at the result

$$\frac{\mu D\cos^2\theta}{\rho(36\sin^2\theta + 2)}\left(\frac{1-\cos 2\phi}{4\omega^2}\right)\left(\sin^2\theta\cos 2\phi + 3\sin^3\theta\sin 2\phi\right) \quad (27)$$

remembering the proper superscripts for the two different regions.

8.4.4. Integration of the deformation potential equation.- The equation derived in 8.3.3 was

$$\frac{\partial^2 V}{\partial \phi^2} = B U_r \sum_{n=1,3\ldots}^{p} \cos n\phi$$

where we have written $B = -3.458 \times 10^{-7} \sin\theta$.

Substituting $p = \frac{\partial V}{\partial \phi}$

$$\frac{dp}{d\phi} = B U_r \sum_n \cos n\phi$$

$$p = \frac{1}{n} B U_r \sum_n \sin n\phi + c_1$$

$$V = -\frac{1}{n^2} B U_r \sum_n \cos n\phi + c_1 \phi + c_2$$

To eliminate the constants, we first use the boundary condition that V must be identical at $\phi = 0^\circ$ and at $\phi = 360^\circ$;

therefore, automatically $c_1 = 0$ and hence

$$V = -\frac{1}{n^2} B U_r \sum_n \cos n\phi + c_2 \qquad (28)$$

We shall now digress somewhat from orthodoxy in order to evaluate the constant c_2. We note from (28) that at $\phi = 90^\circ$, the value will be

$$V_{90} = c_2 \qquad (29)$$

We have seen that by definition $V = kW2$ and therefore at $\phi = 90^\circ$ we have, by using the astronomical value $k = 4/15$

$$V_{90} = c_2 = \frac{4}{15} A(\cos\theta \sin\theta \cos 90^\circ + \cos^2\theta \cos 180^\circ)$$

therefore $c_2 = -(4/15)\ A \cos^2\theta$ (30)

where as before we are using

$A = f\ m\ /\ r^3$ (r = Earth-Moon radius vector).

Substituting equation (30) into (28) the result is

$$V = -\frac{1}{n^2} B u_r \sum_{n=1,3...}^{p} \cos n\phi - \frac{4}{15} A \cos^2\theta \qquad (31)$$

and we shall consider, as already mentioned, p =11 or 13 as providing sufficient accuracy.

8.4.5. Tabulation of the results.- Tables 8 and 9 show results for an overall Earth density of $(\rho^{(1)} + \rho^{(2)})/2$ and of 5.5 respectively. In model calculations the density considerations enter explicitly in the evaluation of the u_r's. A density term, moreover, enters in the evaluation of the gravitational acceleration (g) in the determination of the Love number (h); in this latter case two "density laws" were assumed in the calculation of our model, one being the average value $\rho^{(1)}$ and $\rho^{(2)}$ and the other and average Earth density of 5.5 corresponding to the true case. The results given in the two tables will show with extreme severity the importance of obtaining a reasonable density law, should an

Table:8 Resultant parameters of the lateral facies change Earth model.

θ	ϕ	Displacement $\times 10^{-4}$ cms	h	k	delta	$\delta_D - \delta_S$ difference	S-semid D-dium
80° N	$0-180^\circ$	.000739242	.06823	.22670	.72818	.56819	D
		.000184565	.04430	.58954	.15999		S
	$180-360^\circ$	.001273096	.11750	.22670	.77745	.58547	D
		.000317851	.07629	.58954	.19198		S
50° N	$0-180^\circ$	.003832925	.37392	.14502	1.15640	.44298	D
		.001035769	.24860	.35679	.71342		S
	$180-360^\circ$	.006600920	.64396	.14502	1.42643	.53347	D
		.001783762	.42814	.35679	.89296		S
20° N	$0-180^\circ$	.002590108	.62167	.07120	1.51488	.54427	D
		.000322510	.07741	.07120	.97061		S
	$180-360^\circ$	.004460588	1.07062	.07120	1.96383	.93731	D
		.000555415	.13331	.07120	1.02652		

"DENSITY LAW": $\rho = \dfrac{\rho^{(1)} + \rho^{(2)}}{2}$

Table: 9 Resultant parameters of the lateral facies change Earth model.

θ	φ	Displacement x 10^{-4} cms	h	k	delta	$\delta_D - \delta_S$ difference	S-semid D-dium
80^0 N	0 - 180^0	.000739242	1.15822	.22670	1.81816	.95048	D
		.000184565	.75199	.58954	.86768		S
	180 - 360^0	.001273096	1.99464	.22670	2.65458	1.41074	D
		.000317851	1.29505	.58954	1.41074		S
50^0 N	0 - 180^0	.003832925	6.34746	.14502	7.12994	2.44500	D
		.001035769	4.22012	.35679	4.68494		S
	180 - 360^0	.006600920	10.93136	.14502	11.71384	3.98129	D
		.001783762	7.26773	.35679	7.73255		S
20^0 N	0 - 180^0	.002590108	10.55310	.07120	11.44630	9.23906	D
		.000322510	1.31403	.07120	2.20724		S
	180 - 360^0	.004460588	18.17415	.07120	19.06736	15.91118	D
		.000555415	2.26297	.07120	3.15618		S

"DENSITY LAW": $\rho = 5.5$

attempt once be made to make as accurate an estimation of the tidal parameters as possible. In the case of $\rho = 5.5$, the values literally went berserk. Yet, the Earth's density is not an average of the densities of its crustal components.

8.4.6. Discussion of results.- It is necessary to state emphatically here that the model was not designed to provide accurate theoretical Love number predictions and indeed does not do so. It has, however, served its purpose of establishing some relationships of tidal parameters under conditions of differing crustal compositions:

(1). The results conform to all known models inasmuch as δ is always greater for the diurnal tides.

(2). The difference $\delta_D - \delta_S$ is greater in the region of denser rocks and smaller in those of lighter ones, but a crossover and indeed, a close approach, is never attained.

(3). It is of interest to note that the crossovers reported always occur in continental regions, i.e. in areas where the crust is less dense. This is in conformity with our model, or at least not in conflict with it. It is this author's "hunch" that under conditions actually occuring in nature some additional factors might be present, which reduce the difference in δ_D and δ_S or even produce the crossover. Such additional factors might be the occurence of strongly anisotropic rocks such as dynamometamorphosed rocks or a thick pile of sediments (especially shales), which frequently occur in continental areas. The elastic response of anisotropic material necessitates more than two Lamé constants to define its equations of motion. Results of a model worked on this basis would be very interesting, but the differential equations extremely difficult to solve.

(4). In conclusion we note that all parameters determined are always higher in the denser rocks; the deformational potential remains constant, as it was hoped it would and therefore the principle of conservation of energy is honored. And finally, the model appears to give the best values in the mid-latitudes, as was also expected.

9.0 CONCLUSIONS.

It was considered desirable to arrange the conclusions obtained from this work in an itemized manner in this final chapter.

9.1. Noise must be removed prior to analysis and it was found that any noise residue in excess of 10 μgals of amplitude (in the frequency domain) might affect unfavorably the determination of δ . It was found that the drift characteristics of the LaCoste - Romberg gravimeter used were so favorable, that a linear correction was generally sufficient for noise removal to the specified standards for records up to one month in length. Filtering will generally be necessary for longer records and such filters may have to be tailor made for each individual series. This author has never seen any mention of residual noise by other workers, and he firmly holds the belief that this should be done, in order to provide a measure of the reliability of the determined values. It also should be noted that on theoretical grounds alone such noise removal is not necessary, since the bandwidths are distinct and separate.

9.2. The results of this work would indicate that one month's record is of sufficient length to determine δ for the diurnal and semidiurnal tide respectively (by spectral analysis) as primary numbers by weighted averages, with δ for the total tide being obtained therefrom, since all significant tides are contained in the diurnal and semidiurnal waves. This author is of the opinion that these three determinations will give the most relevant tidal data. It was also found that for one month's record the semidiurnal tide δ value is well stabilized, whereas the same value for the diurnal tide undergoes greater oscillations. The most important information obtained from this analysis is, in the view of this writer, the numerical value of the δ 's with respect to each other (see section 9.7).

9.3. Should, nevertheless, values for individual tidal components be required, this author believes that interpolation in the frequency domain is a simpler and more accurate method for this task than filtering. Although the length of the tidal series was insuffiecient for a proper componental analysis, the individual values within each of the two frequency bands are closer together in magnitude than are the values obtained by filtering; such a distribution of values is more commendable from the point of view of physical reality within the system. This writer would also like to see the habit of listing with each δ value the value of the amplitude of the corresponding rigid tide, as a measure of numerical significance.

9.4. No significant effort was chanelled into phase work, but the phase spectra values obtained from the records are listed. The writer is highly reluctant to accept their significance without a great deal of further work.

9.5. We consider of great importance a small discussion of the two δ values for the diurnal and semidiurnal tidal waves. It is our opinion that the determination of these two values is of prime importance in tidal analysis, all the other mentioned δ values being merely of secondary interest (apart from the δ value for the total tide). We know from Earth models with a vertical variation that the diurnal should exceed in numerical value the semidiurnal δ . Our investigation proves that the opposite obtains in the Pittsburgh - Monroeville (Nike site) area. Similar results are obtained from a very significant number of stations from other parts of the world, especially central Asia. (NAKAGAWA (1966)). It is our opinion that this result cannot be overlooked but in any interpretation of this behavior we are severely hampered in arriving at any conclusions by a virtually complete lack of information as to what the behavior of δ should be in respect to the geological foundation.

9.6. The author has established a new tidal Earth model whose express purpose is to show such a relationship by varying the nature of the geological substrate. Two rocks from the opposite ends of the petrological spectrum (granite and dunite) were selected in order to maximize the possible contrast thus obtained. Calling $\Delta\delta$ the quantity $\delta_D - \delta_S$ (where D refers to diurnal and S to semidiurnal), we find that in our model, as well as in all other known models, this quantity is always positive, but its value is smaller over regions whose substrate is composed of lighter rocks. The tidal method can therefore be potentially of value in outlining lateral facies changes in the crust. The suspicion offers itself that the negative value of $\Delta\delta$ might be due to frequently occuring anisotropic rocks in large masses within continental areas, such as dynamometamorphosed rocks or prominently layered sediments, particularly shales. It would be of considerable interest to make some tidal determinations in areas with large light but isotropic or near-isotropic rocks, such as large granitic batholiths.

9.7. WYCKOFF (oral communication) has found the following interesting behavior of δ (for the total tidal wave):

From astronomical considerations the "world wide" values of the Love numbers are approximately

$$h = 0.624$$

$$k = 0.285,$$

so that, as a world average, $\delta = 1 + h - 3k/2 = 1.1965$ and the average for Pittsburgh is 1.19600. The overall value for Pittsburgh is thus remarkably close to the world average, the negative value of $\Delta\delta$ notwithstanding. This agreement would suggest that at this location here, the overall δ value is not seriously disturbed by the effects of oceanic tides.

9. 8. It might be of interest to laymen to obtain ideas about actual tidal ground motion. Such an estimation can be made by an approximation using the free air and Bouguer corrections in the following manner:

The observed maximum tidal change in the gravitational acceleration (g) may be taken as 300 μgal (double amplitude). The term "double amplitude" might be deserving of some further explanation. It is introduced as a simplification, because of the assymetry across the zero line. The estimated ground motion is therefore taken from saddle to trough and not from an (inferred) zero line.

Using the value $\delta = 1.2$, we obtain the difference due to yielding of the Earth

$= 300\ \mu\text{gal} - 300/1.2\ \mu\text{gal} = 50\ \mu\text{gal} = .050$ mgal;

The <u>free air gradient</u> (at sea level) = -0.003086 mgal/cm;

The <u>Bouguer correction</u> (infinite slab)

$\Delta g = 2\pi Gdh$, where d = density;

$$G = 6.67 \times 10^{-8} \text{ c.g.s. (gravitational constant)}$$

and thus for 1 cm, $\Delta g = 6.67 \times 10^{-5} \times 6.283\ d$ mgal/cm.

The latest values for the densities are given by WOOLARD (1969) as follows:

d (above Moho) = 2.93 gms/cm^3
d (below Moho) = 3.27 gms/cm^3.

(i). d = 2.93

Bouguer correction = 122.789 x 10^{-5} mgal/cm = .0012279 mgal/cm

therefore net decrease = free air - Bouguer =

= 0.003086 - 0.0012279 = 0.001858 mgal/cm

therefore .050 mgal corresponds to 26.91 cms.

(ii). d = 3.27

Bouguer correction = 137.03788 x 10^{-5} mgal/cm = .00137038 mgal/cm

therefore net decrease = .001716 mgal/cm
and .050 mgal corresponds to 29.14 cms.

We thus may say that the maximum surface tidal wave in the vicinity of Pittsburgh amounts to approximately 30 centimeters or about 1 foot with a period of 1 day.

This survey concludes our work. Having left behind here what hopefully might prove to be some lasting values, we consider it only essential decency to leave behind our totem as well.

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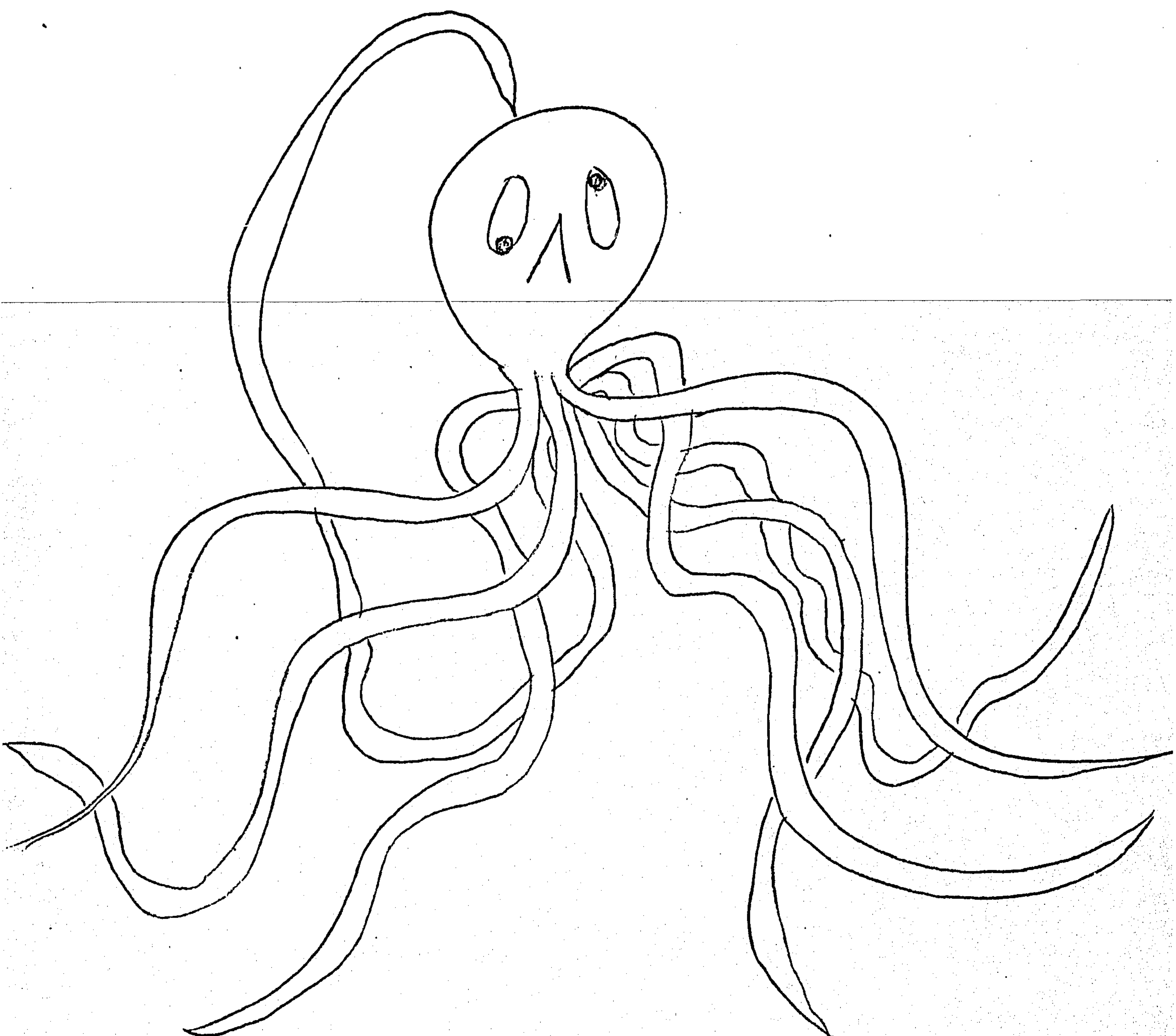


APPENDIX I.

THE OPERATION OF THE "SINC FUNCTION" FOR INTERPOLATION.

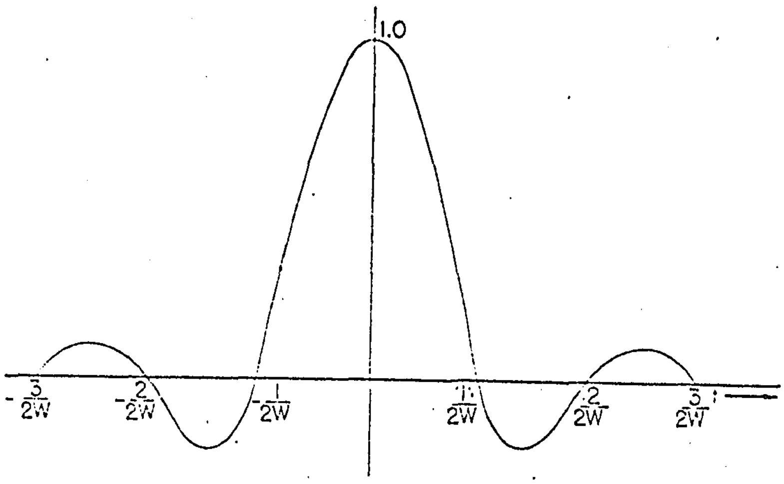


Fig. 16 - The sampling function

$$Sa(2\pi Wt) = \frac{\sin(2\pi Wt)}{2\pi Wt}$$

(after GOLDMAN (1953)).

In this Appendix, unless otherwise stated, T = length of the time series.

We shall illustrate the operation of the "sinc function" in the frequency domain, since the function is employed in this manner in our work. The same arguments can be applied to the operation in the time domain.

Let $F(\omega)$ be a function at any arbitrary frequency interpolated for;

Let $F(\frac{2\pi n}{T})$ be the value of this function at fundamental frequency increment stations. For the interpolation we have

$$F(\omega) = \sum_{n=-\infty}^{\infty} F\left(\frac{2\pi n}{T}\right) \frac{\sin\left(\frac{\omega T}{2} - n\pi\right)}{\left(\frac{\omega T}{2} - n\pi\right)} \tag{1}$$

where the first part under the summation sign is a weighting function and the second part is the "sinc function" proper (in the sequel this part will be abbreviated to SF). For the derivation of the above equation see Appendix II.

Whenever the argument of the sine equals zero, the value of the SF is equal to 1. (The proof of this is somewhat unusual and unexpected, see PETERSEN and GRAESSER, p. 13, (1961)).

Suppose this happens, i.e.

$$\frac{\omega T}{2} - n\pi = 0 \quad ;$$

also let

$$\omega_0 = \frac{2\pi}{T} \quad = \text{fundamental frequency}$$

and for any arbitrary frequency ω, we have

$$\omega = 2\pi f$$

using this in the preceding, we obtain

$$\frac{\cancel{2}\pi f T}{\cancel{2}} - n\pi = 0$$

i.e.

$$\pi f T - n\pi = 0$$

(1). At stations which are integer multiples of the fundamental frequency we have, by definition, the condition

$$f = n f_0 = \frac{n}{T}$$

Hence, inserting into preceding results

$\pi n f_0 T - n\pi = 0$, whence

$\dfrac{\pi n T}{T} - \pi n = 0$ is immediately evident.

We see, therefore, that the value of the SF vanishes at these stations.

(2). At an arbitrary frequency station, such as we might be interpolating for, the total value as seen from Eq. (1) will come from a summation of all the SF's in the system, which will not be zero, and multiplied by the corresponding weight values $F(\frac{2\pi n}{T})$; the contribution, however, from all except those in the immediate vicinity will be miniscule. This is due to the rapid dwindling of the value of the SF away from the maximum value.

The reader is here beseached to note another important fact: In the preceding rationale, as well as in the derivation of the interpolation equation (1) proper, no assumption of any kind has been made. The whole reasoning is purely analytical. It will be noted that the universe of Eq. (1) stretches from $-\infty$ to $+\infty$. In actual practice this is unattainable and a truncation has to be made, but because of the rapid dwindling of the SF, the error in consequence of this is negligible. It follows therefore, that this interpolation gives a virtually analytical result (cf. filters).

In passing, we also wish to point out that, because of the summation involved, the value obtained at an interpolated arbitrary station can exceed the value of the spectral peak attained at a fundamental frequency station.

To illustrate the action of the interpolation function further at the sample stations, let there for any fixed n be a sliding parameter

$$j = 0,\ 1,\ \ldots\ k$$

such that

$$\frac{\pi n T}{T} - (n+j)\pi = 0 \qquad ,\ j = 0$$

$$\pi n - (n+j)\pi = -j\pi \qquad \text{otherwise}$$

and j is any integer number; from the curve it is immediately seen that at such points

SF = 1, $j = 0$

SF = 0, $j \neq 0$

so that a digitized point is completely determined by its own data.

It is unfortunate that the discussion of a SF interpolation operation is frequently neglected in the literature.

An excellent account can be found in GOLDMAN (1954).

For a discussion of the relation of the SF interpolation formula, the degrees of freedom and the position of the frequency band (with respect to the zero frequency axis), return to the main text. These considerations can at times acquire a crucial importance, although no difficulty was encountered in our work due to the inherent statistical overdefinition.

APPENDIX II.

THE SAMPLING FUNCTION IN THE TIME DOMAIN.

We have seen in the body of the text (Chapter 5.0) that if a function G(t) has as its highest frequency the value W cycles per unit time, then a complete definition in the time domain occurs when the values are spaced 1/2W (unit time) apart. Any closer spacing will produce statistical overdetermination, whereas stations wider apart will be insufficient to define such a function.

(1). To show this, we have

$$G(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Omega(\omega)\, e^{j\omega t}\, d\omega \qquad (1)$$

$$= \frac{1}{2\pi}\int_{-2\pi W}^{2\pi W} \Omega(\omega)\, e^{j\omega t}\, d\omega$$

with the limits on the integral since no frequency above W occurs.

Substituting $t = \frac{n}{2W}$ (i.e. digitization)

$$G\left(\frac{n}{2W}\right) = \frac{1}{2\pi}\int_{-2\pi W}^{2\pi W} \Omega(\omega)\, e^{j\omega \frac{n}{2W}}\, d\omega \qquad (2)$$

because of definite limits on the integral, the Fourier series concept is applicable (in the range of the limits). Thus the right hand side of eq.(2) will then attain the value $2WC_{-n}$, where C_{-n} is the (-nth) complex Fourier series coefficient. Since $\Omega(\omega)$ is zero outside this range of expansion, G(t) is completely defined and the introductory statement at the head of the Appendix is thus proven.

(2). To derive the sampling function, we can expand $\Omega(\omega)$ in Fourier series because of the limitations on the bounds of the integral,

$$\Omega(\omega) = \sum_{n=-\infty}^{\infty} \bar{C}_n e^{-j\frac{2\pi n\omega}{4\pi W}} = \sum_{n=-\infty}^{\infty} \bar{C}_n e^{-j\omega\frac{n}{2W}}$$

where $\bar{C}_n = C_{-n}$ and we had before

$$\bar{C}_n = \frac{1}{2W} G\left(\frac{n}{2W}\right)$$

Hence

$$\Omega(\omega) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} G\left(\frac{n}{2W}\right) e^{-j\omega\frac{n}{2W}} \qquad (3)$$

From (1) and (3)

$$G(t) = \frac{1}{2\pi}\int_{-2\pi W}^{2\pi W} \frac{1}{2W}\left[\left(\sum_{n=-\infty}^{\infty} G\left(\frac{n}{2W}\right) e^{-j\omega\frac{n}{2W}}\right)\right] e^{j\omega t}\, d\omega$$

$$= \frac{1}{4\pi W}\sum_{n=-\infty}^{\infty} G\left(\frac{n}{2W}\right)\int_{-2\pi W}^{2\pi W} e^{j\omega\left(t-\frac{n}{2W}\right)}\, d\omega$$

~~assuming interchangeability of the integral and the sum-~~ mation.

Taking the part of the R.H.S. of the above under the integral, i.e.

$$\int_{-2\pi W}^{2\pi W} e^{j\omega\left(t-\frac{n}{2W}\right)}\, d\omega = \left[\frac{e^{j\omega\left(t-\frac{n}{2W}\right)}}{j\left(t-\frac{n}{2W}\right)}\right]_{-2\pi W}^{2\pi W}$$

$$= \frac{e^{j2\pi W\left(t-\frac{n}{2W}\right)} - e^{-j2\pi W\left(t-\frac{n}{2W}\right)}}{j\left(t-\frac{n}{2W}\right)}$$

$$= \frac{2 \sin (2\pi \bar{W} t - n\pi)}{t - \frac{n}{2W}}$$

and hence

$$G(t) = \frac{2}{4\pi W} \sum_{n=-\infty}^{\infty} G\left(\frac{n}{2W}\right) \frac{\sin(2\pi Wt - n\pi)}{\left(t - \frac{n}{2W}\right)}$$

$$= \sum_{n=-\infty}^{\infty} G\left(\frac{n}{2W}\right) \frac{\sin(2\pi Wt - n\pi)}{2\pi Wt - n\pi} \qquad (4)$$

where eq. (4) will be recognized as the sampling function. A corresponding equation can be derived for the frequency domain without introducing any innovations on the preceding discussion. The result shows that G(t) is defined everywhere, provided that the number of digitized stations of the function $G(\frac{n}{2W})$ is <u>not less than the number of degrees of freedom of the system.</u>

The number of degrees of freedom of the system of bandwidth W (with the bandwidth starting at zero frequency) and of signal duration $T = T_2 - T_1$, is <u>2TW.</u>

This latest statement is in need of some explanation:

In theory, if the function under consideration is limited in extent in one domain, then its inverse domain will be infinite. However, all signals known in nature are such that their "quadivas" (i.e. quadratic contents) are limited in both domains. Hence the above is justified.

The important conclusions of this Appendix are:

(i) The sampling function can describe the entire function, provided only that the number of digitized stations is sufficient for statistical determinancy.

(ii) This statistical determinancy is 2TW, the path being from either domain into the other one.

APPENDIX III.

THE SAMPLING FUNCTION (IN THE TIME DOMAIN) WHEN THE FREQUENCY BANDWIDTH DOES NOT START AT ZERO FREQUENCY.

The essential complications here, as compared to the subject matter in the preceding Appendix, are as follows:

(i) Since we do not start at $\omega_1 = 0$, but have now a bandwidth $W = \omega_2 - \omega_1$, where

$$\omega_2 = 2\pi f_2$$
$$\omega_1 = 2\pi f_1$$

we cannot obtain a continuous frequency range in the Fourier series expansion.

(ii) This limitation is generally bypassed by limiting our interval either to the range ω_1 to ω_2 or $-\omega_2$ to $-\omega_1$ which is done by <u>defining a complex function of time g(t)</u> such that

$$g(t) = G(t) - jG_1(t)$$

where G(t) is the same as in the preceding Appendix.

The resulting algebraic equations are essentially the same as in the preceding Appendix, but involving much more horrendous dogwork, and these will therefore not be carried out here. The interested reader is referred to Sec. 2.3 of GOLDMAN (1953).

The result is that, while the same amount of information must still be fed into the system (because of $G(t)$ and $G_1(t)$), the density of the resulting stations will be exactly one half as compared to the case where the bandwidth was started at zero. The net effect is therefore loss of resolution (by one half).

From our point of view the discussion in these two Appendices is irrelevant, since the statistical definition of our tidal series is more than ample. However, for a system with only marginal statistical definition, the content of these two Appendices represents very important information which hereby is submitted to any interested reader.

The essence of these two Appendices, with some modifications to suit our needs, was taken from GOLDMAN (1953).

APPENDIX IV.

THE FOURIER INTEGRAL.

This Appendix serves as an amplification of sections 5.3.1 and 5.3.2 to show the operation of the Fourier transform more intimately. It is not intended as a rigirous mathematical treatment and will thus of necessity be largely heuristic.

In physical terms a Fourier series can express a signal of any form in a limited range of time, e.g. in a period from -T/2 to +T/2 as a sum of components of frequencies 1/T and its harmonics 2/T, 3/T, ... If we now increase $T \to \infty$, the frequency spacing becomes smaller and smaller. In mathematical form, we could expect a Fourier series

$$G(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nt + b_n \sin nt \right) \qquad (1)$$

to pass into an equation of the type

$$G(t) = \int_0^{\infty} a(\omega) \cos \omega t \, d\omega + \int_0^{\infty} b(\omega) \sin \omega t \, d\omega \qquad (2)$$

and if t is a time denomination, then equation (2) would give us the frequency composition of the function G(t). If certain conditions are obeyed, it can be shown that the coefficients $a(\omega)$ and $b(\omega)$ can be calculated by formulae analogous to Fourier series coefficients, i.e.

$$a(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} G(t)\cos\omega t\,dt$$
$$b(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} G(t)\sin\omega t\,dt \tag{3}$$

Combining equations (3) and (2) we obtain

$$G(t) = \frac{1}{\pi}\int_{0}^{\infty}\left[\int_{-\infty}^{\infty} G(t)\cos\omega t\,dt\right]\cos\omega t\,d\omega + \frac{1}{\pi}\int_{0}^{\infty}\left[\int_{-\infty}^{\infty} G(t)\sin\omega t\,dt\right]\sin\omega t\,d\omega \tag{4}$$

which equation is known as the Fourier integral formula.

It can be simplified into

$$G(t) = \frac{1}{\pi}\int_{0}^{\infty} S(\omega)\cos\left[\omega t + \phi(\omega)\right]d\omega \tag{5}$$

where

$$S(\omega) = \sqrt{\left[\int_{-\infty}^{\infty} G(t)\cos\omega t\,dt\right]^2 + \left[\int_{-\infty}^{\infty} G(t)\sin\omega t\,dt\right]^2} \tag{6}$$

$S(\omega)$ is known as the spectrum (function) and its plot against ω will give the frequency (spectrum) composition of the signal $G(t)$. The importance of this development is that it shows the Fourier integral - as long as certain conditions for its existence are present - can describe any function, even a non-periodic one, in terms of its frequency composition. Equation (6) is used in program FOURIER.FILTR to calculate the frequency composition of the tidal signals.

Similarly $\phi(\omega)$, the phase spectrum, in eq. (5) is defined by

$$\tan \phi(\omega) = \frac{-\int_{-\infty}^{\infty} G(t) \sin \omega t \, dt}{\int_{-\infty}^{\infty} G(t) \cos \omega t \, dt} \tag{7}$$

which is likewise used in the computer program FOURIER. .FILTR. The interested reader will find an exhaustive discussion of $\phi(\omega)$ and especially the difference $k(\omega)$ together with their physical significance and implications in Chapter 7.0.

A simple example will be illustrative. We have a signal

$$G(t) = \begin{cases} \cos \omega_0 t, & t_2 - t_1 \\ 0 & \text{elsewhere,} \end{cases}$$

then, using preceding equations,

$$\int_{-\infty}^{\infty} G(t) \cos \omega t \, dt = \int_{-\infty}^{t_1} 0 \cos \omega t \, dt + \int_{t_1}^{t_2} \cos \omega_0 t \cos \omega t \, dt$$

$$+ \int_{t_2}^{\infty} 0 \cos \omega t \, dt$$

$$= \int_{t_1}^{t_2} \cos \omega_0 t \cos \omega t \, dt$$

$$= \left[\frac{1}{2(\omega+\omega_0)} \sin(\omega+\omega_0)t + \frac{1}{2(\omega-\omega_0)} \sin(\omega-\omega_0)t \right]_{t_1}^{t_2} \qquad (8)$$

In a likewise manner, the sine part will give

$$\int_{t_1}^{t_2} \cos\omega_0 t \sin\omega t \, d\omega =$$

$$= \left[\frac{-1}{2(\omega+\omega_0)} \cos(\omega+\omega_0)t + \frac{-1}{2(\omega-\omega_0)} \cos(\omega-\omega_0)t \right]_{t_1}^{t_2} \qquad (9)$$

These formulae will enable us to evaluate S(ω) for all values ω , except $\omega = \omega_0$. In the latter case

$$\int_{t_1}^{t_2} \cos\omega_0 t \cos\omega_0 t \, dt = \int_{t_1}^{t_2} \cos^2\omega_0 t \, dt$$

$$= \int_{t_1}^{t_2} \frac{1-\cos 2\omega_0 t}{2} dt = \left[\frac{t}{2} - \frac{\sin 2\omega_0 t}{4\omega_0} \right]_{t_1}^{t_2}$$

$$= \frac{t_2 - t_1}{2} - \frac{1}{4\omega_0} (\sin 2\omega_0 t_2 - \sin 2\omega_0 t_1) \qquad (10)$$

and similarly

$$\int_{t_1}^{t_2} \cos\omega_0 t \sin\omega_0 \, dt = \int_{t_1}^{t_2} \frac{\sin 2\omega_0 t}{2} dt$$

$$= -\frac{1}{4\omega_0} (\cos 2\omega_0 t_2 - \cos 2\omega_0 t_1) \qquad (11)$$

Equations (8) through (11) show the frequency distribution of a wave train of <u>any length</u>. The reader's attention is focused on the following points in particular:

(i) the more cycles there are in the wave train, the more peaked is the frequency distribution;

(ii) as the wave train becomes infinitely long, the case of a Fourier series component is approached;

(iii) if case (ii) holds, then $S(\omega) = 0$ everywhere, except at $\omega = \omega_o$, when $G(t) = A \cos \omega_o t$;

(iv) if $G(t)$ is an infinitely long <u>periodic</u> wave train, of period $2\pi / \omega_o$ and is not a pure cosine wave, then $S(\omega) = 0$ for all values of ω except ω_o and the harmonics $n\omega_o$, where $n = 1, 2 \ldots$

To complete the discussion, the Fourier integral formula (equation (4)) can be expressed in complex form and the <u>Fourier transform pairs</u> arrived at. Many standard textbooks will show this, but since no items of immediate relevance to our subject matter will be encountered, the development of this discussion to a further extent would be an unprofitable venture.

The final stages of this Appendix might benefit from some numerical work, in particular the estimation of the influence of the length of the tidal time series on the resolution of tidal components. We shall tacidly assume that the components are of equal amplitude.

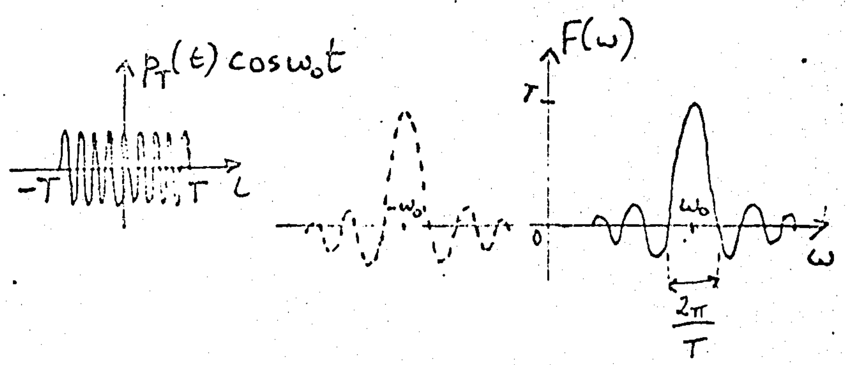
To do this, we shall postulate a clipped function $p_T(t)\cos\omega_o t$, where $p_T(t)$ is the clipping function, which we shall take as having a constant amplitude. Also $\omega_o = 2\pi f_o$, where f_o = 12 hours.

Thus

$$f(t) = p_T(t)\cos\omega_o t \quad \longleftrightarrow$$

$$\longleftrightarrow \quad F(\omega) = \frac{\sin(\omega-\omega_o)T}{(\omega-\omega_o)} + \frac{\sin(\omega+\omega_o)T}{(\omega+\omega_o)}$$

and the width of the peak will be $2\pi/T$ as shown in the sketch.



This width will be 1/2T in terms of non-angular frequencies and for consistency with our preceding numerical work.

We shall only consider time series containing at least two cycles.

Then

T (hrs)	cycles/hour	$1/2T = \frac{1}{2}f_s$
24	2.0	0.021
100	8.33	0.005
700	58.33	0.0007
4000	333.33	0.0001

At $2f_s$ apart the peaks will be at the point of complete separation. Assuming equal amplitude of the components, the following will be the relation between the length of time series and complete resolution:

T(hours)	f_s (cycles/hour)
24	0.042
100	0.010
700	0.0014
4000	0.0002

Comparing these values with the tabulation of the tidal components used in the Prependix, page v, the reader might conclude that sufficient resolution is obtainable with a series length of only a few cycles. We should like to point out here that the assumption was made that all tidal components have the same amplitude, which is contrary to actual practice, these variations being very substantial. The smaller amplitudes could easily be overlooked, unless they are given a chance to increase their peaks, as well as the sharpness of these peaks, by increasing the length of the time series.

APPENDIX V.

(PARTIAL) OUTPUT OF PROGRAM FOURIER.FILTR.

This Appendix lists the computer output of the amplitude spectrum of the tidal record Batch #1 which is 887 hours long. It is supplementary to the plot of the amplitude spectrum in Chapter 5.0 in order to supplement this information in more accurate numerical detail. It is shown here extended somewhat beyond the semidiurnal bandwidth, but not all the way to the Nyquist frequency.

The meaning of the individual columns is as follows:

FREQ = frequency, i.e. $f = \omega/2\pi$ (in cycles/hour);

SPEC1 = amplitude spectrum of the rigid tides (microgals);

SPEC2 = amplitude spectrum of the observed tides (microgals);

PERIOD = 1/FREQ, in hours/cycle;

GAMMA = SPEC2/SPEC1 = gravitational magnification factor (called δ in the body of the text).

FREQUENCY ADVANCING RELATED TO RECORD LENGTH

TABLE 10

Tidal time series without linear correction.

FOURIER AMPLITUDE SPECTRUM ...
(AMPLITUDES IN MICROGALS)

FREQ	SPEC1	SPEC2	PERIOD	GAMMA
FREQUENCY INCREMENTING UNINTERRUPTED				
.0000000	.000039	.0010602	NON-PER.	27.4889
.0011274	1.643146	24.7427180	887.000	15.0581
.0022548	1.296723	12.9687350	443.500	10.0012
.0033822	1.894333	7.2055057	295.667	3.8037
.0045096	1.094452	7.3314577	221.750	6.6987
.0056370	.549090	6.4842689	177.400	11.8091
.0067644	.545609	3.9284987	147.833	7.2002
.0078918	.696265	3.7887817	126.714	5.4416
.0090192	.602662	3.5716135	110.875	5.9264
.0101466	.477694	3.2083829	98.556	6.7164
.0112740	.577844	2.9067017	88.700	5.0302
.0124014	.622274	2.6694435	80.636	4.2898
.0135287	.604658	2.9136211	73.917	4.8186
.0146561	.676995	1.9307711	68.231	2.652[illegible]
.0157835	.706570	2.4374551	63.357	3.4497
.0169109	.654152	2.1995636	59.133	3.3625
.0180383	.787068	1.9927922	55.438	2.5319
.0191657	.782070	2.1654809	52.176	2.7689
.02029[illegible]	.885574	2.1163230	49.278	2.3898
.0214205	.832182	2.0493243	46.684	2.4626
.0225479	.913344	1.8145022	44.350	1.9867
.0[illegible]36753	.939614	2.1020445	42.238	2.2371
.0248027	1.000332	1.9511118	40.318	[illegible].9505
.0259301	.953076	2.1875693	38.565	2.2953
.0270575	1.169297	2.2522171	36.958	1.9261

.0281849	1.420480	2.2831524		35.480	1.6073
.0293123	1.503497	2.3371727		34.115	1.5545
.0304397	1.608079	2.5949410		32.852	1.6137
.0315671	1.868787	2.9078500		31.679	1.5560
.0326945	2.288755	3.2220443		30.586	1.4078
.0338219	2.659400	4.0050043		29.567	1.5060
.0349493	3.582354	4.7831312		28.613	1.3352
.0360767	5.895672	7.5292106		27.719	1.2771
.0372041	14.569522	17.6359713		26.879	1.2105
.0383315	28.719791	34.6627650	diurnal bandwidth.	26.088	1.2069
.0394588	14.461805	17.1926355		25.343	1.1888
.0405862	3.662504	4.4174486		24.639	1.2061
.0417136	32.392515	38.6213322		23.973	1.1923
.0428410	5.365339	6.2293422		23.342	1.1610
.0439684	4.611814	4.8478662		22.744	1.0512
.0450958	5.460306	6.2795522		22.175	1.1500
.0462232	3.354218	3.5664788		21.634	1.0633
.0473506	2.670572	2.6766785		21.119	1.0023
.0484780	2.381120	2.4408697		20.628	1.0251
.0496054	2.049138	1.9440893		20.159	.9487
.0507328	1.878009	1.8988883		19.711	1.0111
.0518602	1.780920	1.5461968		19.283	.8682
.0529876	1.655981	1.6416629		18.872	.9914
.0541150	1.648638	1.2654377		18.479	.7676
.0552424	1.626102	1.3795130		18.102	.8484
.0563698	1.501315	1.3152178		17.740	.8760
.0574972	1.259510	1.2240215		17.392	.9718
.0586246	1.403530	1.3213517		17.058	.9414
.0597520	1.451541	1.2453389		16.736	.8579
.0608794	1.436298	1.2963576		16.426	.9026

.0620068	1.442879	1.2554809		16.127	.8701
.0631342	1.518760	1.3584746		15.839	.8945
.0642616	1.458300	1.2677334		15.561	.8693
.0653890	1.547942	1.5094721		15.293	.9751
.0665163	1.639066	1.4317619		15.034	.8735
.0676437	1.694439	1.6020826		14.783	.9455
.0687711	1.787008	1.6745152		14.541	.9370
.0698985	1.922079	1.8429598		14.306	.9588
.0710259	2.056058	2.1451616		14.079	1.0433
.0721533	2.179910	2.2159046		13.859	1.0165
.0732807	2.509087	2.7500271		13.646	1.0960
.0744081	2.884171	2.8950826		13.439	1.0038
.0755355	3.462306	3.7932765		13.239	1.0956
.0766629	4.592033	4.9234546		13.044	1.0722
.0777903	8.011829	9.3981131		12.855	1.1730
.0789177	17.715405	21.0653563	semidiurnal bandwidth.	12.671	1.1891
.0800451	31.038204	37.1307540		12.493	1.1963
.0811725	21.989229	26.7220969		12.319	1.2152
.0822999	6.699616	8.4917443		12.151	1.2675
.0834273	30.997304	37.5448313		11.986	1.2112
.0845547	4.311881	5.6741763		11.827	1.3159
.0856821	2.941707	3.8369153		11.671	1.3043
.0868095	2.254366	3.2911377		11.519	1.4599
.0879369	1.975575	2.6818970		11.372	1.3575
.0890643	1.641288	2.4154692		11.228	1.4717
.0901917	1.387141	2.1206960		11.088	1.5288
.0913191	1.220842	1.8881141		10.951	1.5466
.0924464	1.181169	1.7792674		10.817	1.5064
.0935738	1.015529	1.4975244		10.687	1.4746
.0947012	.966736	1.6250907		10.560	1.6810
.0958286	.862820	1.2587327		10.435	1.4589

FREQUENCY ADVANCING RELATED TO RECORD LENGTH

TABLE 11

Tidal time series with linear correction.

FOURIER AMPLITUDE SPECTRUM ... (AMPLITUDES IN MICROGALS)

FREQ	SPEC1	SPEC2	PERIOD	GAMMA
FREQUENCY INCREMENTING UNINTERRUPTED				
.0000000	.072671	.3058600	NON-PER.	4.2088
.0011274	1.643146	1.1312184	887.000	.6884
.0022548	1.296723	.4368036	443.500	.3369
.0033822	1.894332	1.6439631	295.667	.8678
.0045096	1.094452	1.1124766	221.750	1.0165
.0056370	.549090	1.6840442	177.400	3.0670
.0067644	.545609	.9638223	147.833	1.7665
.0078918	.696265	.3788428	126.714	.5441
.0090192	.602662	1.0016949	110.875	1.6621
.0101466	.477694	.7384037	98.556	1.5458
.0112740	.577844	1.0957107	88.700	1.8962
[illegible]	.622274	1.0402828	80.636	1.6717
.0135287	.604658	1.3450263	73.917	2.2244
.0146561	.676995	.6274883	68.231	.9269
.0157835	.706570	.8950749	63.357	1.2668
.0169109	.654152	1.2231400	59.133	1.8698
.0180383	.787068	.7415302	55.438	.9421
.0191657	.782070	1.0702308	52.176	1.3685
.0202931	.885574	.9881110	49.278	1.1158
.0214205	.832182	1.0534134	46.684	1.2658
[illegible]	.913344	.9794524	44.350	1.0724
.0236753	.939614	1.2866427	42.238	1.3693
.0248027	1.000332	1.2092575	40.318	1.2089
.0259301	.953076	1.3707286	38.565	1.4382
.0270575	1.169297	1.5338053	36.958	1.3117

.0281849	1.420480	1.5740814		35.480	1.1081
.0293123	1.503496	1.7476837		34.115	1.1624
.0304397	1.608079	1.9822183		32.852	1.2327
.0315671	1.868786	2.3361663		31.679	1.2501
.0326945	2.288755	2.6560970		30.586	1.1605
.0338219	2.659400	3.4723344		29.567	1.3057
.0349493	3.582354	4.3197168		28.613	1.2058
.0360767	5.895672	7.2532086		27.719	1.2303
.0372041	14.569521	17.5013950	diurnal bandwidth	26.879	1.2012
.0383315	28.719792	34.3563728		26.088	1.1963
.0394588	14.461805	17.4187255		25.343	1.2045
.0405862	3.662504	3.7329463		24.639	1.0192
.0417136	32.392516	37.9980636		23.973	1.1731
.0428410	5.365341	6.5025796		23.342	1.2120
.0439684	4.611814	5.3210946		22.744	1.1538
.0450958	5.460307	6.8538120		22.175	1.2552
.0462232	3.354218	4.0989054		21.634	1.2220
.0473506	2.670572	3.1388185		21.119	1.1753
.0484780	2.381120	2.8489663		20.628	1.1965
.0496054	2.049138	2.3719766		20.159	1.1575
.0507328	1.878009	2.3045341		19.711	1.2271
.0518602	1.780920	1.9725132		19.283	1.1076
.0529876	1.655981	2.0470949		18.872	1.2362
.0541150	1.648638	1.6810940		18.479	1.0197
.0552424	1.626102	1.8018983		18.102	1.1081
.0563698	1.501315	1.7143809		17.740	1.1419
.0574972	1.259510	1.6372206		17.392	1.2999
.0586246	1.403530	1.7202107		17.058	1.2256
.0597520	1.451541	1.6575966		16.736	1.1420

.0620068	1.442879	1.6638759		16.127	1.1532
.0631342	1.518760	1.7622503		15.839	1.1603
.0642616	1.458300	1.6616865		15.561	1.1395
.0653890	1.547942	1.9171162		15.293	1.2385
.0665163	1.639066	1.8188464		15.034	1.1097
.0676437	1.694440	1.9999211		14.783	1.1803
.0687711	1.787008	2.0557596		14.541	1.1504
.0698985	1.922079	2.2333944		14.306	1.1620
.0710259	2.056058	2.5219125		14.079	1.2266
.0721533	2.179910	2.5978788		13.859	1.1917
.0732807	2.509087	3.1284452		13.646	1.2468
.0744081	2.884171	3.2720696		13.439	1.1345
.0755355	3.462306	4.1661970		13.239	1.2033
.0766629	4.592033	5.2875358		13.044	1.1515
.0777903	8.011829	9.7417998		12.855	1.2159
.0789177	17.715405	21.3959827	semidiurnal bandwidth.	12.671	1.2078
.0800451	31.038207	37.4833961		12.493	1.2077
.0811725	21.989230	26.3724144		12.319	1.1993
.0822999	6.699615	8.1546198		12.151	1.2172
.0834273	30.997305	37.2057233		11.986	1.2003
.0845547	4.311881	5.3372999		11.827	1.2378
.0856821	2.941707	3.5112391		11.671	1.1936
.0868095	2.254367	2.9684694		11.519	1.3168
.0879369	1.975575	2.3611444		11.372	1.1952
.0890643	1.641288	2.0942392		11.228	1.2760
.0901917	1.387141	1.8053143		11.088	1.3015
.0913191	1.220842	1.5744051		10.951	1.2896
.0924464	1.181168	1.4716747		10.817	1.2459
.0935738	1.015529	1.1911694		10.687	1.1730
.0947012	.966736	1.3224710		10.560	1.3680
.0958286	.862820	.9595454		10.435	1.1121

APPENDIX VI.

SOME RELATIONSHIPS BETWEEN DENSITY, YOUNG'S MODULUS, RIGIDITY MODULUS AND POISSON'S RATIO IN IMPORTANT IGNEOUS ROCKS.

ρ = density;

E = Young's modulus;

μ = rigidity modulus;

σ = Poisson's ratio.

It was seen in the Chapter developing the Earth tide model, that the solution of the elastic equation of motion is greatly simplified if one assumes that $\mu = \lambda$. This Appendix will show that in the realm of igneous rocks, this simplification is not far fetched and is, indeed, perfectly justified.

The data are taken from the "Handbook of Physical Constants", G.S.A. Memoir 97, 1966.

If one assumes $\sigma = .25$ for all important materials composing the Earth, we have

$$\sigma = \frac{\lambda}{2(\lambda+\mu)} \qquad \text{and putting} \quad \mu = \lambda$$

$$\sigma = \frac{\mu}{4\mu} = .25,$$

i.e. putting $\sigma = .25$ is assuming $\mu = \lambda$ as done above. Then, since

$$E = 2\mu(1+\sigma)$$

for our particular assumption we obtain

$$E = 2.5\mu$$

If there is any grain in the above assumption, then the true μ (obtained from $\mu = E/2(1+\sigma)$) should be similar in value to the "calculated μ" from $\mu = \frac{E}{2.5}$.

The last two columns in Table 12 show that this indeed is the case.

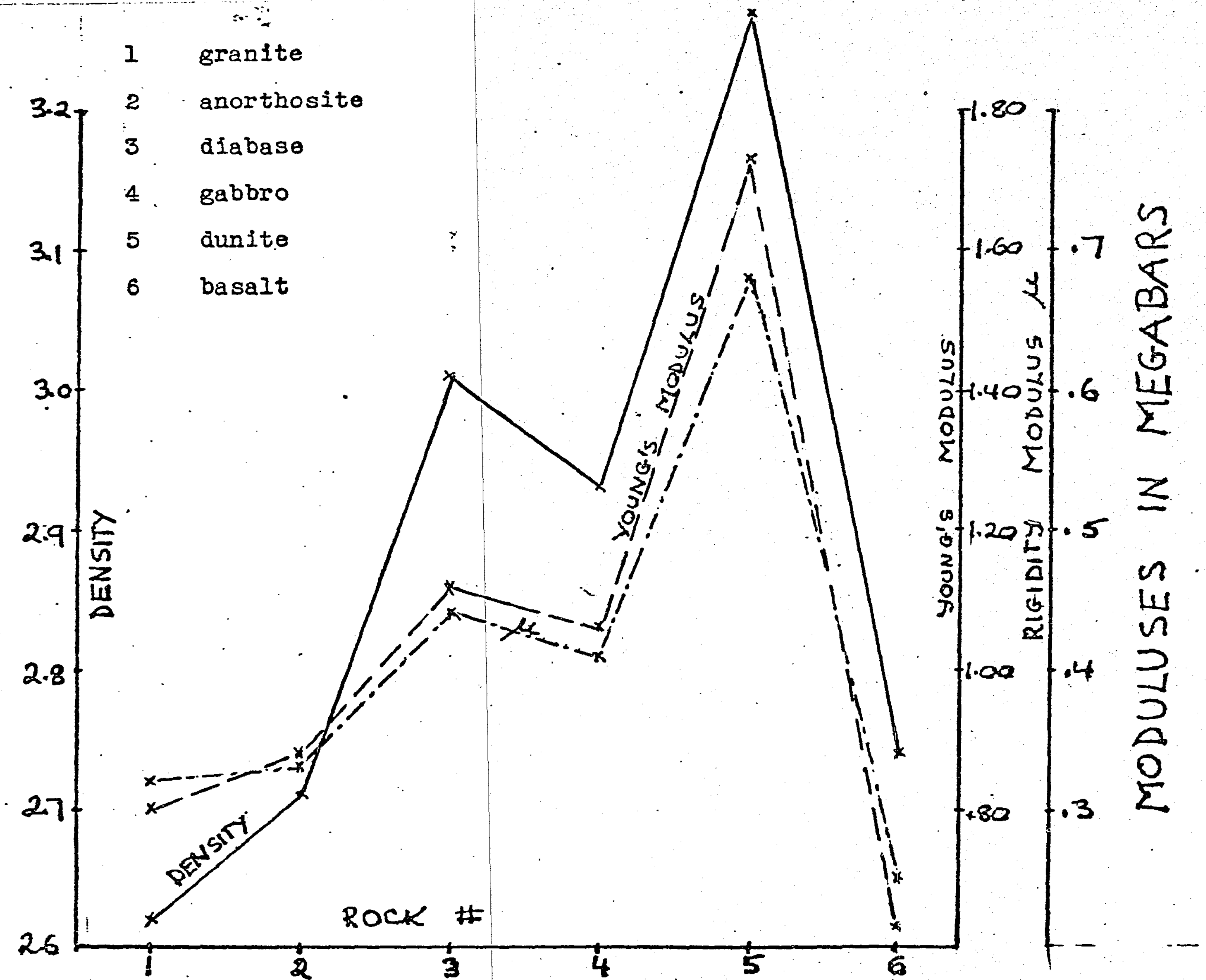
Another interesting fact emerges from Figure 17 . It will be seen that the variation in the moduluses and in the densities for important Earth-building rocks is completely sympathetic. There is, of course, no immediate reason for this behavior, but it goes a far way in introducing physical reality into the simplification of the solution of the equation of motion.

The moduluses are tabulated in megabars (1 megabar= $= 10^{12}$ dynes cm^{-2}).

Table 12: Elastic parameters of some important igneous rocks.

rock:	density:	σ:	E :	rigidity modulus: actual:	rigidity modulus: calculated:
granite	2.619	.26	.80	.317	.320
anortho-site	2.708	.32	.88	.333	.352
diabase	3.012	.27	1.12	.441	.448
gabbro	2.931	.31	1.06	.405	.424
dunite	3.267	.27	1.73	.681	.692
basalt	2.74	.25	.63	.252	.252
			..........megabars...............		

Fig. 17. Relation between density, Young's modulus and rigidity modulus.



All values from -Handbook of Physical Constants- G.S.A. 1966.

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