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A LOGICAL BASIS FOR NOUNS, ADJECTIVES, AND VERBS

by

Michael F. Stewart

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of the requirements for the degree of
Doctor of Philosophy
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Doctoral Committee:

Assistant Professor Peter Fodale, Chairman
Associate Professor Kenneth C. Hill
Professor Harold V. King
Assistant Professor Michael H. O'Malley

PREFACE

My interest in the logical connections between nouns, adjectives, and verbs began at about the time when, as an undergraduate student of English literature, I first encountered the poetry of Gerard Manley Hopkins. At first, the startling anthimerias of Hopkins persuaded me that verbs might as well be adjectives, that both of these might as well be nouns, and that all three of them might as well be something else. A closer look convinced me that Hopkins knew exactly what he was doing when he wrapped his meaning up in a verb rather than an adjective or a noun. In short, I became convinced that these grammatical distinctions were semantically important. My suspicions along these lines remained rather vague, however, until I read a work of an entirely different sort, Bertrand Russell's *Principles of Mathematics*.

It is unfortunate that linguists are largely ignorant of this work, and of the astute observations it contains on the nature of syntactic categories. Those readers who are familiar with it will recognize that this entire dissertation is an attempt to work out in detail the principles which Russell set down, even though the logical system which I have employed departs from Russell's in a number of ways.

My indebtedness to people I have known is no less than my indebtedness to people I have only read. I am indebted to all of the members of my committee: to Professor Peter Fodale, my chairman and my first teacher of syntax, who managed to communicate an excitement over the subject with which I have been stuck ever since; to Professors Kenneth Hill and Michael O'Malley, who will understand what I mean when I say thanks for what must have seemed to them at times like interminable conversations; and to Professor Harold King, who graciously agreed to replace one of my original committee members, Professor Joyce Friedman, on rather short notice. I am also grateful to Professor Friedman, who offered much valuable criticism of earlier versions of this work, and insisted that I learn something about logic before completing it.

To my good friend and colleague Professor James Rose I owe a special debt of gratitude. We were exposed to Gerard Manley Hopkins at the same time, and our discussions of him and of much else besides have affected my thinking in ways that are impossible to unravel. If there are any insights into the nature of language in this work, I can no longer tell which of them are his and which of them are my own. My sincere gratitude is also extended to Yves Morin, who took time away from his own work, probably much more valuable than my own, to teach me some French and to help me over some logical difficulties in Chapter II.

I also wish to thank the following erstwhile denizens of the Frieze Building basement: Professors William Ritchie, David Michaels, Charles Elliot, and Donald Smith. All of these people have contributed immeasurably to any education that I can lay claim to.

And finally, my deepest appreciation to my wife, Barbara, who knows very little of the substance of this work, yets pays me the supreme compliment of believing it worthwhile in spite of that.

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I. Introduction

Recent studies in the area which has come to be known as generative semantics have seriously challenged traditional assumptions about the status of the major lexical categories noun, adjective, and verb. In the traditional view, which is by and large preserved in the standard or 'lexicalist' formulation of transformational grammar due to Chomsky (Chomsky, 1965; Chomsky, 1969), these categories are formally distinct and must be counted among the irreducible primitives of any theory of syntactic description. In their search for progressively deeper 'deep structures', the adherents of generative semantics have departed from this view and have posited increasingly abstract representations in which lexical categories have either ceased to be independent or have disappeared altogether. Lakoff, for example, has proposed (Lakoff, 1965; Lakoff, 1966) that adjectives and verbs should be represented in terms of a single category at one level of analysis, and Ross has proposed (Ross, 1967) that adjectives should be represented in terms of noun phrases at a still deeper level. The ultimate reduction was suggested by Bach (Bach, 1968), who proposes the term contentive as the name for a supercategory including nouns, adjectives, and verbs alike. It is widely accepted that representations at a hypothetically deepest or 'semantic' level will closely approximate those of the predicate calculus, and in the typical applications of the latter, the distinction between lexical categories has been dispensed with.

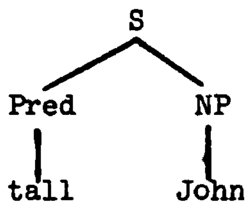
Lexical categories cannot be dispensed with on all levels of description, of course. The operation of various syntactic transformations depends crucially upon the lexical category membership of the items composing the strings which fall under their domain. Consider, for example, the following simple case. In a generative semantics framework, sentences such as

(1) John is tall

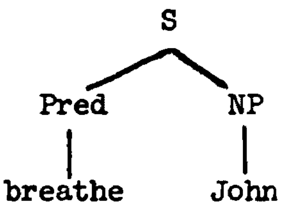
and (2) John breathes

have usually been assigned underlying representations like the following:

(3)



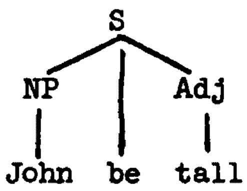
(4)



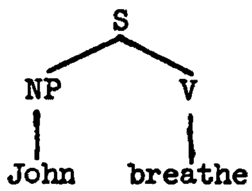
A transformation which reverses the order of argument and predicate applies to both (3) and (4). Then a transformation which inserts the copula applies to (3), converting it to a surface structure like that of (1). The latter transformation cannot apply to (4), however, since the copula does not occur before verbs. To assure the correct application of this transformation, then, we must somehow indicate that *tall* is an adjective and *breathe* is a verb. This can be done simply by marking them in the lexicon with an arbitrary syntactic feature, say ± Adj. If *tall* is marked +Adj and *breathe* is marked -Adj, we can achieve the desired result by stipulating that the transformation in question applies only in the environment of the feature +Adj. (Cf. Lakoff, 1965, pp. A5-A7).

In a lexicalist framework, the deep structures assigned to (1) and (2) would correspond much more closely to their surface structures. In particular, the adjective *tall* would be dominated by a node *Adjective* and the verb *breathe* would be dominated by a node *Verb*. Thus, in a somewhat simplified form, the representations would be as follows:

(5)



(6)



The fact that *be* occurs in (5) and not in (3) is the least of their differences. In fact, it would be perfectly possible to introduce *be* transformationally into (5) in the same way in which it was introduced into (3). From a lexicalist point of view, however, there is little justification for such a transformation, since the difference in structure between (5) and (6) allows us to account for the presence of be in the former and its absence in the latter directly by means of phrase structure rules.

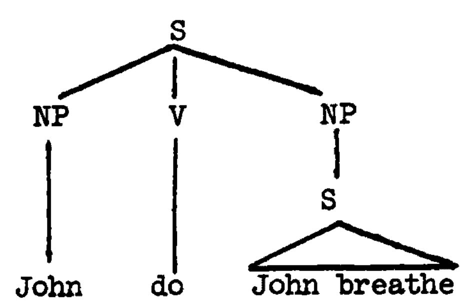
The essential distinction between (3) and (4), on the one hand, and (5) and (6) on the other, is the following. In (3) and (4), which are supposed to be semantic representations, there is an implicit claim that both adjectives and verbs bear a common semantic relationship to their subjects. Thus, any possible semantic relevance of differences in lexical category membership is dismissed. The lexicalist approach, as represented by (5) and (6), embodies the opposite claim--that adjectives and verbs bear a different relationship to their subjects, and that information on lexical category membership is available to the rules of semantic interpretation to which structures like (5) and (6) constitute the input.

Let us examine in some detail the theoretical implications of the above distinction. The lexicalist hypothesis is completely open to the possibility that semantic representations can be constructed in a system similar to the predicate calculus. The chief point of disagreement between lexicalism and generative semantics is the manner in which semantic representations are to be related to levels explicitly containing lexical categories. In a generative semantics framework, the former are related to the latter by a single sequence of transformational rules, including lexical insertion transformations. Since lexical categories are introduced transformationally, information pertaining to them will not be available at the semantically interpreted level. It follows, then, that distinctions between lexical categories must be considered wholly meaningless from a generative semantics point of view. On the other hand, lexicalism supposes that semantic representations are related by rules of interpretation to a level of syntactic deep structure in which lexical category membership is specified. Thus, on a lexicalist view, it is theoretically possible to relate differences in lexical category membership to differences in meaning.

Of the two theories, generative semantics has been the one associated with the general view that syntactic behavior is predictable on semantic grounds. Yet it would appear that a semantically motivated explanation of the existence of distinct lexical categories can be provided only on the basis of a theory such as lexicalism. If the usual analysis provided by generative semantics is correct, then the fact that *tall* is an adjective in (1) and *breathe* is a verb in (2) has nothing to do with any difference in the semantic content of these sentences. For example, it has nothing to do with the fact that *tallness* is a property of John while *breathing* is an activity which he performs. The grammatical distinction between (1) and (2) is written off as accidental--a matter of arbitrary syntactic features.

More recent developments in generative semantics have revised the estimate that (1) and (2) have parallel representations. While (1) continues to be represented by something like (3), it has been argued (Cf. Cantrall, 1969) that (2) must have a representation something like the following:

(6)

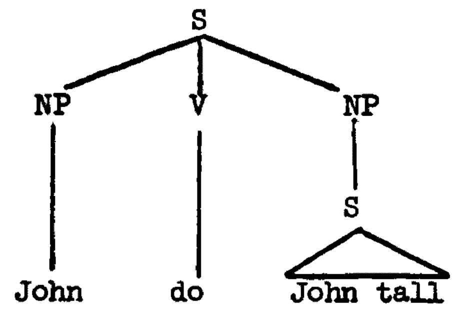


This alternative is an improvement over (4), since indirectly, at least, it seems to capture the fact that breathing is an activity. It is still unsatisfactory, however, for it simply postpones to a lower sentence the problem of analyzing the exact nature of the relation between John and this activity, a relation which is again not the same as that between John and tallness. Thus, one could reasonably ask why we might not have a sentence

(7) *John talls

derived from a structure parallel to (6):

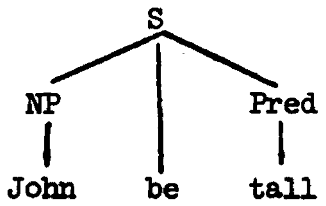
(8)



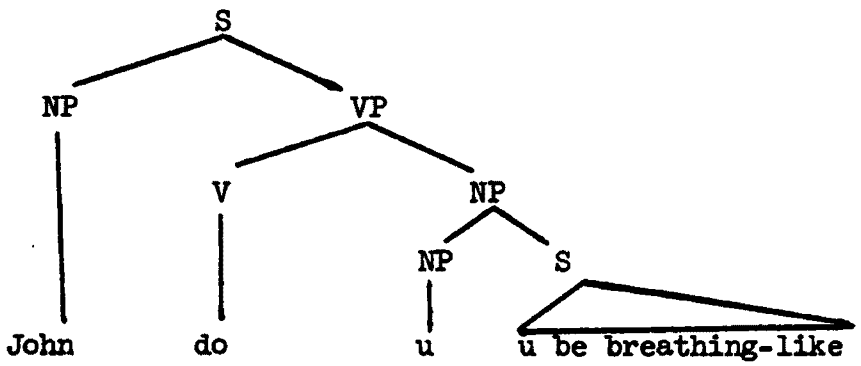
Presumably, the answer would be that only sentences which denote activities can appear as complements of the verb do, and that John breathe is such a sentence while John tall is not. But (6) and (8) do not present any such distinction. The embedded sentences John breathe and John tall are assigned exactly the same semantic structure. Thus, there would appear to be no semantic reason why they should not occur in exactly the same environments.

In Chapter III of this work, semantic representations of (1) and (2) will be proposed which circumvent the above difficulties. These representations will be presented in a system of notation like that of the predicate calculus. For our present purposes, however, they can be translated roughly into tree structures like the following:

(9)



(10)



Under this analysis, the lexical item tall belongs to a class of semantically primitive items which we will call predicates. On the other hand, the lexical item breathe will correspond to a semantically complex structure represented by the verb phrase in (10). The variable u occurring in (10) is a specialized variable which ranges over activities. Thus, (10) says that John performs an activity which is distinct from other activities by virtue of being an activity of breathing. The difficulties associated with the previous analysis will not arise here. In (9), tallness is predicated directly of John. However, (10) predicates breathing of an activity which John performs rather than of John himself. The deviance of a sentence such as *John talls* can now be explicated quite naturally by noting that the adjective *tall* is predicable of individuals, but not of activities.

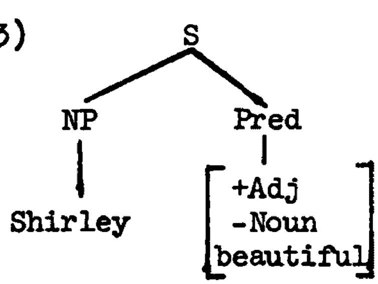
Now let us consider an example having to do with the distinction between adjectives and nouns. The sentences

(11) Shirley is beautiful

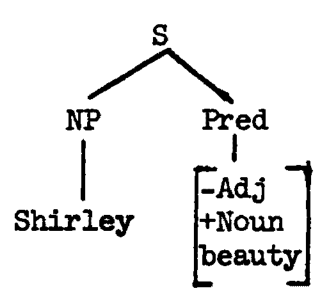
and (12) Shirley has beauty

are clearly logically equivalent. That is to say, they will have the same truth value regardless of the identity of Shirley. Generative semantics would appear to be committed to assigning them a common semantic representation. How, then, can it account for their difference in form? By analogy with the previous case, we might proceed as follows. Suppose that the lexical items *beauty* and *beautiful* are differentiated by the arbitrary syntactic features ±Adjective and ±Noun. Then (11) and (12) could be represented as

(13)



(14)



The transformation which applied earlier to (3) could now apply to (13), producing a surface structure like that of (11). In addition, we could suppose that *have* is introduced transformationally in (14), producing a surface structure corresponding to (12).

This analysis has a certain plausibility. But it is objectionable for a number of reasons:

(1) As before, this analysis assumes that the distinction between two lexical categories is a piece of syntactic arbitrariness--and furthermore, that the same proposition just happens to underlie two distinct surface structures. This is difficult to believe, however, in view of the regularity of the relation between pairs like <u>beauty</u> and <u>beautiful</u>. There are countless other pairs like this one in English in which the noun appears to denote the property which something must have in order to be correctly described by the adjective. Witness:

(a)	vain	vanity	(g)	just	justness
(b)	sane	sanity	(h)	red	redness
(c)	true	truth	(i)	incongruous	incongruity
(d)	wise	wisdom	(j)	happy	happiness
(e)	intelligent	intelligence	(k)	affable	affability
(f)	heavy	heaviness	(l)	polite	politeness

On the above analysis, we would have to suppose not only that <u>beauty</u> and <u>beautiful</u> were arbitrary syntactic variants, but also that there were many other pairs of such variants, and that it was simply an accident that all of these pairs happened to be related in the same way.

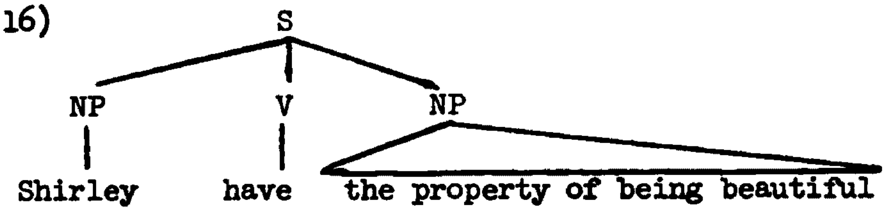
(2) All that we are really in a position to say about (11) and (12) is that they are logically equivalent--i.e., that they have the same truth value for all possible situations. The transformational approach, however, demands more than logical equivalence--it demands nothing less than complete cognitive synonymy. That is, it claims not merely that (11) and (12) state equivalent propositions, but that they state the <u>same</u> proposition. In view of the difference in form of (11) and (12), especially in view of the fact that (12) seems to refer explicitly to a property while (11) does not, it is difficult to see how this claim could ever be justified.

(3) The above analysis succeeds in relating (11) and (12), but it does not go far enough. To (11) and (12) we can add the sentence

(15) Shirley has the property of being beautiful

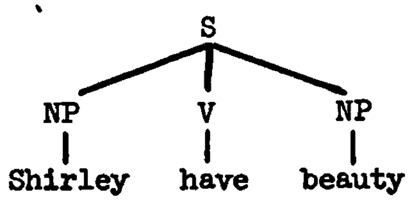
Now it appears that it would be perfectly possible to relate (12) and (15) in a generative semantics framework. Let us assume that (15) has the following structure:

(16)



Let us assume, further, that the underlying structure of (12) is not given by (14), but instead by (17) below:

(17)



Now it will be possible to derive (17) from (16) on the assumption that there is a lexical insertion transformation which replaces the noun phrase in (16) with the lexical noun <u>beauty</u>, i.e., a rule to the effect:

(18) the property of being beautiful $\rightarrow$ beauty

In this way, (12) and (15) can be assigned a common representation, and the difference between them would depend only upon whether the above transformation had applied. However, if we adopt this approach, it will no longer be possible to relate (11) and (12) as we did before, since they will now have completely different underlying representations. On the other hand, if we choose to derive (11) and (12) from a common structure, then (15) will be recalcitrant. Either way, it does not appear feasible to derive (11), (12), and (15) from a common representation.

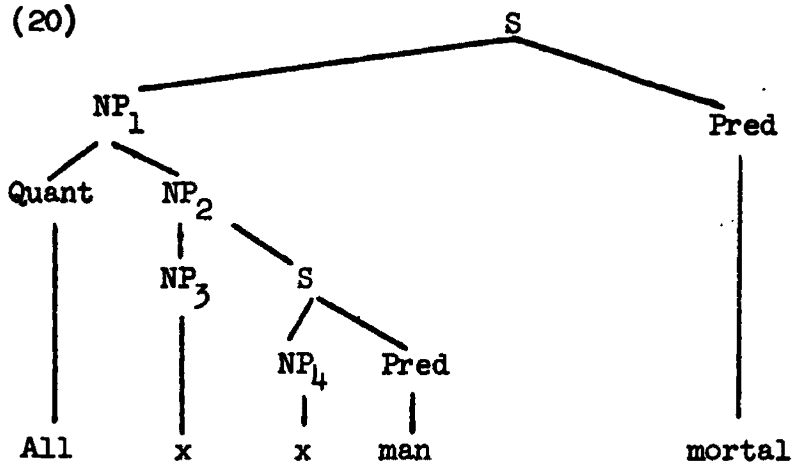
The lexicalist approach to the above problem is to assume that (11), (12), and (15) have distinct underlying representations, and that their meanings are related by rules of semantic interpretation. Granted that such rules can be formulated at all, they offer a definite theoretical advantage over the transformational approach. Instead of assuming that noun and adjective pairs like <u>beauty</u> and <u>beautiful</u> are simply arbitrarily distinct, we can, by virtue of such rules, demonstrate a regular semantic relation between them. In Chapter II of this work, it will be shown that a rule of interpretation relating sentences like (11) and (12) can indeed be formulated, as a theorem in a deductive system. Furthermore, it will be shown that (15) can be related to (12), hence also to (11), by a rule similar in effect to (18), in which the noun *beauty* is defined in terms of the noun phrase the *property of being beautiful*.

In his paper *Nouns and Noun Phrases* (Bach, 1968), Bach makes the suggestion that all English noun phrases correspond to logical structures of the form "(Qx)(F(x))", where "(Qx)" is some sort of quantifier or an operator, and F is some predicate, or in Bach's terminology, a contentive. To a sentence such as

(19) All men are mortal

Bach would assign a representation something like the following:

(20)



Under this analysis, the noun phrase *all men* corresponds to the string "All x (such that) x man", which is dominated by the node NP_1 and has the general shape "(Qx)(F(x))".

Now let us inquire, what, if anything, in the above representation corresponds to the superficial noun, *men*. The most deeply embedded predicate, *man*, is, in Bach's terminology, a contentive. That is to say, like the adjective *mortal* it is simply a term which describes something. There is no more reason to identify this term with the noun *men* than there is to identify it with an adjective such as *man-like*. In fact, there is good reason for saying that the function of the predicate *man* in (20) is just like that of the adjective *mortal*. The immediate structure in which the predicate man appears has the logical form

(21) F(x)

The adjective mortal also appears in such structures. The sentence

(22) Socrates is mortal

has a schematic representation like (21), in which the variable x is replaced by an individual constant. Moreover, it is implicit in (20) that the adjective mortal is truly predicable of all of those individuals for which the propositional function "man(x)" is true.

Unlike the predicates man and mortal, the noun men fulfills a dual semantic function. It not only describes a given sort of thing, but it also refers to a given sort of thing. Now the noun phrase NP_2 in (20) includes both a variable and a predicate. It is therefore a structure which incorporates both the function of description and reference. It seems natural, then, to identify the noun men with this noun phrase.

In the previous cases, we saw that the adjective tall corresponded directly to a predicate, and that the verb breathe corresponded to a verb phrase. Furthermore, it was suggested that the abstract noun beauty was intimately related to the noun phrase the property of being beautiful. In the present case, it appears again that a noun corresponds to a noun phrase, that is, to a semantic structure which includes both a variable and a predicate. Thus, the fact that it is a noun, men, which occurs after the quantifier in (19), and not an adjective or a verb, is predictable from the structure of the representation of (19), which we can assume will be similar to (20).

In all of the cases which we have examined so far, we have concluded that distinct lexical categories have distinct semantic correlates. The central thesis of this work is that this will be true in every case--that the existence of the lexical categories noun, adjective, and verb is not a fortuitous syntactic fact, as generative semantics would seem to suggest, but a fact which can be explicated on semantic grounds.

The thesis that semantic relationships are relevant to the facts of lexical category membership does not involve any claim as to the priority of semantics over syntax, or vice versa. The question to which this study is addressed is simply the following: granted a system of abstract representations which adequately reflects the meaning of sentences, what is the correspondence between the primitive elements of that system and the lexical categories noun, adjective, and verb.

Our present knowledge of semantics does not justify the attempt at constructing anything like a wholly comprehensive system of semantic representations--that is, a system which would allow us to represent unambiguously the meaning of every possible sentence in a given language. Nevertheless, we may profitably examine the logic of lexical categories in a system which is considerably less than complete. Accordingly, in Chapter II I will present a formal system of semantic representations which is restricted to a fairly manageable subset of English sentences. It is hoped that this system will succeed, at least, in establishing a set of theoretical minima for any system which will be able to deal adequately with meaning and with lexical categories at the same time.

I will be concerned only in a general way with the relation between the proposed semantic representations and syntactic representations. Hence, I will not propose any explicit set of rules for mapping semantic representations into surface structure or into deep structure in Chomsky's sense. The reader is cautioned at this point not to misinterpret expressions which he may find in the text to the effect that such-and-such a syntactic structure is derivable from such-and-such a postulated semantic representation, or that such-and-such a semantic representation underlies a given sentence. These expressions should be taken to mean only that the stated correspondence exists, and they are not meant to imply anything about direction of generation.

The formal system of semantic representations which will be presented in Chapter II is a modified version of standard quantificational logic, without set theory. The modifications have been made for the sake of capturing significant aspects of English sentences which symbolic logic was not designed to handle. It must be remembered that the developers of symbolic logic as a rule prided themselves on its ability to ignore many features of natural language, features which logicians have traditionally looked upon as irrelevant or too unsystematic to bother with. As we have already noted above, and as we will note again more carefully in Chapter III, the relation between a predicate adjective and its subject is different from the relation between an intransitive verb and its subject. For the purpose of analyzing purely syllogistic arguments, this difference can be safely overlooked. But it is obvious that no comprehensive theory of semantic structure can afford to overlook it.

The proposal of Bach's referred to above, that all English noun phrases correspond to logical structures of the form "$(Qx)(F(x))$", is perfectly commensurate with the view espoused in this work--i.e., the view that distinct syntactic structures have distinct semantic correlates. However, there is considerable difficulty in applying Bach's proposal in detail. Consider, for example, the question of how to represent semantically a sentence such as

(22) Aristotle was a Greek

On many accounts, a sentence like (22) expresses a relation of set membership between an individual and a set. It could therefore be represented in set theoretic notation by

(23) $A\epsilon(\imath x)(y)(y\epsilon x\equiv g(y))$

If we identified the symbol 'ϵ' here with the verb be and the expression '$(\imath x)(y)(y\epsilon x\equiv g(y))$' with the noun phrase 'a Greek', then (23) would conform to Bach's proposal. However, in Chapter II a number of arguments will be presented which show that sentences such as (22) cannot be adequately represented in terms of notations like (23).

Consider also the question of how to represent sentences like the following:

(24) Water is necessary for the origin of life

(25) Dinosaurs are extinct

In these sentences, the subject noun phrases are simple nouns which occur without quantifiers. It is not obvious, therefore, how they are to be represented in terms of expressions of the form "(Qx)(F(x))".

In Chapter II, an alternative to set theoretic notation will be presented which will allow us to represent indefinite singular noun phrases in the manner in which Bach suggests. This alternative will be possible only after we have supplemented standard quantificational logic in very special ways. In Chapter IV, the logical apparatus developed in Chapter II will be applied to the representation of generic noun phrases, and it will be shown that these, too, can be represented as Bach suggests, without the necessity of assuming, however, as Bach did, that there is a special 'generic quantifier' in English (Cf. Bach, 1968, p. 106). Finally, in Chapter V it will be shown that mass nouns, as they appear in sentences like (24), also correspond to semantic structures of the form "(Qx)(F(x))".

II. A Formal System of Semantic Representations

2.0. Introduction

In this chapter I will present a formal system of semantic representations. Since a number of sentence types and constituent structures have been deliberately excluded from it, the system is restricted to a small subset of English sentences. Nevertheless, it includes a great many more English sentences than do the standard systems of symbolic logic, and the set of sentences which it includes is particularly significant for an examination of the logic of syntactic categories. Except as otherwise noted, the system will include all of the basic notions of quantificational logic, as developed, for example, in Principia Mathematica. It will not require, however, any set-theoretic notions. The most significant deviation from standard logics is the addition of a primitive relation denoted by "*" and the addition of a number of axioms involving this relation. The system will be referred to, therefore, in this and in subsequent chapters, as *S.

2.1. Syntactic Categories of *S.

The primitive syntactic categories of *S are as follows: (1) denotators; (2) predicates; (3) relations; (4) truth-functional connectives; and (5) quantifiers. These will be discussed below in the order listed.

2.1.1. Denotators

The class of denotators includes an infinity of variables of several kinds: (a) individual variables, i, j, k, i_1, j_1, k_1, i_2, j_2, k_2 ...; (b) activity variables, u, v, u_1, v_1, u_2, v_2 ...; (c) property variables, p, q, p_1, q_1, p_2, q_2 ...; (d) sentential variables, s, s_1, s_2 ...; and (e) unrestricted variables, w, x, y, z, w_1, x_1, y_1, z_1, w_2, x_2, y_2, z_2 .... By interpretation, the range of variables in class (e) will include that of the variables in classes (a)-(d), while the latter will be mutually exclusive in range.

To each of the classes of variables (a), (b), and (c) there will correspond a finite set of constants. Hence, there will be individual constants, e.g., a, b, c, d, I, J; activity constants, e.g., U, V, U_1, V_1; and property constants, e.g., P, Q, P_1, Q_1. Since the range of class (e) is exhausted by classes (a)-(d), there will be no constants corresponding to class (e). By interpretation, the variables in class (d) range over the set of well-formed formulas of *S. The rules of formation presented below will specify how a formula of *S can be substituted for a sentential variable.

2.1.2. Predicates

The system *S contains a finite set of predicate constants, e.g., A, B, C, E, F, G, H, Fl, Fl, Gr, Tu. It also contains an infinite set of predicate variables, f, g, h, f_1, g_1, h_1, f_2, g_2, h_2 ..., which range over the set of predicate constants.

The predicate constants of *S are subcategorized with respect to the number and type of arguments they can have. The manner in which this subcategorization is to be expressed will be explained below.

2.1.3. Relations

The class of predicates and the class of denotators together make up the class of terms of *S. Atomic propositions are formed when terms are joined in one of the following ways by means of relations.

The relation "BE" asserts the relation of predication, i.e., the relation which holds between a denotator or a group of denotators and a predicate. This relation will be expressed in *S merely by juxtaposition of a predicate and an argument or arguments.

The relation "DO" asserts the relation which holds between an individual and some activity which that individual performs or undergoes. Atomic formulas in which this relation occurs will be represented by expressions of the form "iDu".

The relation "HAVE" asserts (a) a relation between something and a property, or (b) between two individuals. It will occur, therefore, in expressions of the form "xHp" and in expressions of the form "iHi".

Finally, the relation represented by "*" asserts that two denotata are not countably distinct, i.e., that they are not wholly separate entities. Schematically, the formulas in which this relation will occur have the form "x*y", where x and y are always variables of the same type.

In most systems, the relations "HAVE", "DO", and "*" would be assimilated to the class of dyadic predicates. The relation "BE", of course, is not reducible to a predicate, since it has the function of joining predicates to arguments. Below, we will see that the relations "HAVE", "DO", and "*", like the relation "BE", have a very special status in the system *S. In the first place, unlike ordinary predicates, they occur essentially in the basic axioms of the system. Furthermore, one or the other of these four basic relations occurs in every atomic proposition of *S. It seems warranted, therefore, to grant them a somewhat unusual syntactic status, if only for the sake of convenience of explication.

2.1.4. Truth-Functional Connectives

The primitive truth-functional connectives of *S are negation and conjunction, defined by standard two-valued truth tables. By means of these two primitives, the additional truth functions alternation, material implication, and material equivalence can be defined, as follows:

Df. (1) $s_1 \vee s_2 \equiv \text{df.} \; -(-s_1 \,\&\, -s_2)$

Df. (2) $s_1 \supset s_2 \equiv \text{df.} \; -s_1 \vee s_2$

Df. (3) $s_1 \equiv s_2 \equiv \text{df.} \; (s_1 \supset s_2) \,\&\, (s_2 \supset s_1)$

It should perhaps be pointed out here that, from a natural language standpoint, it appears rather supererogatory, at least to the present author, to attempt a further reduction of primitives by way of introducing Scheffer's stroke or some such device.

2.1.5. Quantifiers

Universal quantification is assumed as a primitive notion in *S. It will be expressed by prefixing a formula with a parenthsized variable. Quantification over predicates is permitted; hence, if s is any well formed formula of *S, (i)s, (u)s, (p)s, (s)s, (x)s and (f)s are all well-formed formulas.

So-called existential quantification is defined in terms of universal quantification in a familiar manner. Where v is any variable of *S

$$\text{Df. (4) } (\exists v)s \equiv \text{df. } -(v)-s$$

The interpretation of the existential quantifier in *S (hence also that of the universal quantifier) differs from that of standard systems. This difference in interpretation, which in no way affects the formal rules of the system, will be discussed below.

2.2. Rules of Formation

Subcategorization of predicate constants will be accomplished in the following manner. The class of all predicates which take a given number of arguments of a given type will be designated by expressions of the form

$$P_i;\ P_{i,j};\ P_{u,s};\ P_x;\ P_{x,y,z};\ P_{i,x,y,s}$$

and so on, where the subscripts indicate both the number and the type of arguments which can occur in the specified order with predicates of the given class. Thus, the expression

$$F\epsilon P_i$$

will mean that the predicate constant F denotes a monadic predicate which takes an individual variable or constant as argument, while

$$G\epsilon P_{x_1,x_2\ldots x_n}$$

will mean that G is an n-ary predicate allowing variables or constants of any type in any position.

In the following rules, the variables x, x_1, $x_2 \ldots x_n$ are to be regarded as meta-variables--i.e., they represent ambiguously variables of all of the classes (a)-(e) above, as well as constants correlated with classes (a)-(c). F represents any predicate constant, while f represents any predicate variable.

(1) $f(x)$; $f(x_1;x_2)$; $f(x_1;x_2;x_3)$; $f(x_1;x_2\ldots x_n)$ are wffs

(2) If $F\epsilon P_x$, then $F(x)$ is a wff

If $F\epsilon P_{x_1,x_2}$, then $F(x_1;x_2)$ is a wff

$\vdots \qquad \vdots \qquad \vdots$

If $F\epsilon P_{x_1,x_2\ldots x_n}$, then $F(x_1;x_2\ldots x_n)$ is a wff

(3) xHp is a wff

(4) iHi is a wff

(5) iDu is a wff

(6) If x and y are denotators of the same class, then x*y is a wff.

(7) If s is a wff, then -s is a wff.

(8) If s_1 and s_2 are wffs, then s_1&s_2 is a wff.

(9) If s is a wff, and if x is not a constant, then (f)s and (x)s are wffs.

(10) If s_1 is a wff, and if s_1 is substituted for a variable s occurring in any wff, the result is a wff.

(11) Nothing else is a wff.

The above rules specify recursively an infinite set of well-formed formulas. In addition, they exclude certain possible combinations of symbols as ill-formed. Thus, while such strings as

(a) pHp; i*j;(u)(i)(iDu);B(J;(i)(B(i;C(J))

are included in the set of well-formed formulas, those in (b) are not, and must be considered meaningless in *S:

(b) xHy; xDu; i*u; u*s; F(g); (f)(f(G))

It should be noted that the rules (1)-(11) permit multiple embeddings of formulas. The last formula in (a) exemplifies a case of triple embedding. If we take B=believe, J=John, and C=crazy, then this formula corresponds to the English sentence "John believes that everyone believes he is crazy." Actually, this correspondence is only a first approximation. Detailed analysis will show that the English verb "believe" cannot be regarded as a simple predicate. For our present purpose, however, the above example suffices to illustrate how the process of embedding functions in *S.

The manner in which (1)-(11) are stated will sometimes allow a quantifier to occur vacuously. For example, these rules will generate the formulas

(c) (x)(F(a)) and (f)(G(x))

The quantifiers (x) and (f) occur vacuously here, since no variable is available to be bound by them. Strictly speaking, such strings are meaningless. However, the rules of inference of *S, to be presented in the next section, allow us to regard such vacuously quantified formulas as equivalent to the formulas which result when the quantifiers are dropped. Thus, the above will be equivalent to

(d) F(a) and G(x)

We are therefore justified in stipulating that formulas like those in (c) will be interpreted in terms of a corresponding formula like those in (d).

2.3. Rules of Inference

In the following rules, the symbol "$\vdash$" can be read "may be asserted." The variables v_1 and v_2 are meta-variables ranging over the terms of *S, i.e., over denotators and predicates, and α and β represent any well-formed formulas. For any rules which have the effect of substituting v_2 for v_1 in a formula, the following general condition holds: if v_1 belongs to class (e), v_2 may belong to any class of denotators; otherwise, v_1 and v_2 must belong to the same class.

R1 If α is truth-functionally tautologous, $\vdash\alpha$

R2 If β is like α except that v_2 occurs free in β wherever v_1 occurs free in α, and if $\vdash(v_1)\alpha$, then $\vdash\beta$

R3 If $\vdash(\exists v_1)\alpha$, and if β is like α except that some arbitrary constant not occurring previously in a proof occurs free in β wherever v_1 occurs free in α, then $\vdash\beta$

R4 If $\vdash\alpha$, and if α does not contain a constant introduced by R3 into a formula containing a free variable, then if V_1 is not a constant, $\vdash(V_1)\alpha$.

R5 If $\vdash v_1=v_2$, and if $\vdash\alpha$, and if β is like α except that v_2 occurs in some position in β where v_1 occurs in α, then $\vdash\beta$

R6 If $\vdash(\alpha\supset\beta)$ and $\vdash\alpha$, then $\vdash\beta$

R1 gives as theorems of *S all formulas which are tautologous according to truth-table analysis. It specifies, therefore, an infinity of theorems, which includes the classical logical laws, e.g.,

(a) $--\alpha \equiv \alpha$ Double Negation

(b) $-(\alpha \& -\alpha)$ Contradiction

(c) $\alpha \vee -\alpha$ Excluded Middle

(d) $(\alpha \supset \beta \& -\beta) \supset -\alpha$ Reductio ad absurdum

R2 represents the process of universal instantiation, which corresponds more or less to the intuitive inference from the general case to the specific case. Since R2 is a rule which has the effect of substituting one term for another, the restriction noted in the first paragraph of this section applies. In the absence of this restriction, it would be possible to construct obviously invalid proofs like the following. The proposition that all properties are abstract entities can be written as

(1) (p)(A(p))

If we were alloed to substitute any variable for p in (1), we could derive

(2) A(x)

and applying R4 to (2) we would obtain

(3) (x)(A(x))

which, retranslating to English, would be "Everything is an abstract entity."

Although it excludes the above inference, R2 as formulated permits the valid inference from the premise "Everything is an abstract entity" to the conclusion "All properties are abstract entities", or in other words, the inference from (3) to (1). In general, R2 allows us to infer from the premise (x)(f(x)) any of the conclusions (i)(f(i)), (p)(f(p)), (u)(f(u)), and (s)(f(s)). However, the converses of these inferences will not hold.

Applications of R2 in subsequent proofs will be cited by means of the abbreviation UI.

R3 represents existential instantiation, which allows us to assert of some arbitrarily selected individual a propositional function which we know to be true in at least one instance. The general restriction explained above also applies here. Thus, in instantiating the formulas $(\exists i)(f(i))$, $(\exists u)(f(u))$, and $(\exists p)(f(p))$, we must select an individual constant, an activity constant, and a property constant, respectively. Instantiation of the formula $(\exists x)(f(x))$, however, allows us to select a constant of any of these types.

In addition to this general restriction, R3 carries the special restriction that the constant selected be one which has not appeared previously in the proof. The reason for the latter is obvious from the following consideration. Suppose that as premises of an argument we have the statements

(4) Some men are stupid

(5) Some women are stupid.

Suppose, further, that these premises are represented as

(6) $(\exists i)(M(i)\&S(i))$

(7) $(\exists i)(W(i)\&S(i))$

Application of R3 to (6) yields

(8) $M(a)\&S(a)$

In the absence of the indicated restriction, we could now apply the rule to (7) to give

(9) $W(a)\&S(a)$

Judicious application of R_1 and R_6 will now allow us to conclude from (8) and (9) that

(10) $W(a)\&M(a)$

which is to say that our arbitrarily selected individual 'a' is both a man and a woman. This highly unnatural consequence can be avoided if, on the second application of R3, we are careful to substitute a different individual constant from the one we substituted on the first.

In subsequent proofs, applications of R3 will be cited by means of the abbreviation EI.

R4, which we will call universal generalization (abbreviated UG), is the inverse of R2. The equivalence of the two sentences

(11) All dogs like bones

(12) Any dog likes bones

may be said to derive from the applicability of UI to (11) and, conversely, of UG to (12).

It might be supposed that R4 is in error, since it allows us to infer universal statements from particular ones. Thus, from "Socrates is Greek", which has the form "G(a)", R4 allows us to infer "(x)(G(a)". The latter does not claim, however, that all men are Greek. As noted earlier, formulas with vacuously applied quantifiers are equivalent to formulas derived by dropping the quantifier. This equivalence is itself justified by R2 and R4. In the present case, then, the formula "(x)G(a)" embodies the same claim as the formula from which it is derived. Hence, by interpretation, it means no more or no less than the original sentence "Socrates is Greek."

Universal generalization is not valid unless the variable generalized upon is perfectly arbitrary. If this condition is violated, then we can construct an erroneous deduction like the following. Let us represent the premise "Everyone has a mother" as

(13) $(x)(\exists y)(M(y;x))$

Applying UI and EI to (13), we obtain

(14) $M(a;x)$

Now, if universal generalization is applied to (14), we derive

(15) $(x)(M(a;x))$

which states, in effect, that there is an individual, a, who is the mother of everyone. This erroneous conclusion results for the following reason. By introducing the individual constant, a, we have imposed a condition on the free variable, x, and it is therefore no longer perfectly arbitrary. That is, it comes to represent the individual whose mother is a, and not <u>any</u> individual, as required. Errors of this sort may be avoided if we observe the restriction stated above, i.e., if R4 is not applied to certain formulas containing an arbitrary constant introduced by EI.

The additional restriction on R4 insures that no formula in which a constant appears in the position of a quantifier will be inferable in *S. This is necessary because no such formula is well-formed according to the rules of formation of *S.

R5 (SUBS) justifies substituting a variable or a constant for another at any place in a formula, as long as the two are identical. Intuitively, the principle of substitutivity embodied in R5 should apply to any two coreferential noun phrases. The work of Quine, however, has familiarized us with the fact that substitutivity in the intuitive sense fails in certain contexts--

namely, those contexts which Quine calls referentially opaque (Quine, 1960, Ch. 4; Quine, 1964, pp. 139-59). In section 2.7, below, we will see that the failure of substitutivity in such contexts can be accounted for without sacrificing the generality of R5 as it is stated above.

The last primitive rule of inference of *S, R6, is the principle Modus Ponens. Applications of Modus Ponens in subsequent proofs will be cited by means of the abbreviation MP.

In addition to the primitive rules R1-R6, there are three secondary rules which can be derived from these which are of sufficient importance to be introduced here. The first of these is the rule of inference known as Reductio ad Absurdum, which can be stated as follows:

RED If $\vdash (\alpha \supset \beta \,\&\, -\beta)$, then $\vdash -\alpha$

Proof:

$(\alpha \supset \beta \,\&\, -\beta) \supset -\alpha$ is assertable by R1. Hence, by MP, whenever $\alpha \supset \beta \,\&\, -\beta$ is assertable, $-\alpha$ is assertable

The next rule of inference is known as the Deduction Theorem. Formal proofs of this theorem, or rather, of its analogues in various systems, are somewhat involved, and none will be presented here. The reader can easily satisfy himself, however, that a formal proof of a deduction theorem for *S can be constructed along the lines of Church's proof for the system F^1 (Church, 1956, pp. 196-98). Following Church, we introduce the following definition:

A deduction of β from the hypotheses $\alpha_1, \alpha_2 \ldots \alpha_n$ is a finite sequence of wffs, $\beta_1, \beta_2 \ldots \beta_m$, such that $\beta = \beta_m$ and for each β_i, either

(1) β_i is an axiom

or (2) β_i is one of $\alpha_1, \alpha_2 \ldots \alpha_n$

or (3) β_i is truth-functionally tautologous

or (4) β_i is inferred according to R2 or R3 from a previous wff

or (5) β_i is inferred from a previous wff according to R4, with the provision that no variable generalized upon occurs free in any of $\alpha_1, \alpha_2 \ldots \alpha_n$

or (6) β_i is inferred from two previous wffs according to R5, with the provision that the denotator substituted for does not occur free in any of $\alpha_1, \alpha_2 \ldots \alpha_n$

or (7) β_i is inferred from two previous wffs according to R6

If there is a deduction of β from the hypotheses $\alpha_1, \alpha_2 \ldots \alpha_n$, we will say that β is <u>deducible</u> from $\alpha_1, \alpha_2 \ldots \alpha_n$. The deduction theorem can therefore be stated as follows:

If β is deducible from $\alpha_1, \alpha_2 \ldots \alpha_n$, then $\alpha_n \supset \beta$ is deducible from $\alpha_1, \alpha_2 \ldots \alpha_{n-1}$

In most of the application of the above, we will require only the special case in which there is only one hypothesis, i.e., where n=1. For this special case, we have the corollary:

If β is deducible from α, then $\vdash \alpha \supset \beta$

Applications of the Deduction Theorem and of the above corollary will be cited in subsequent proofs by means of the abbreviation DED.

The rule of inference known as existential generalization (EG) can be stated as follows:

EG If β is like α except that β contains free occurrences of v_2 wherever α contains free occurrences of v_1, and if $\vdash \alpha$, then $\vdash (\exists v_2)\beta$

Proof:

(1)	$(v_2)\text{-}\beta$	Hyp
(2)	$(v_2)\text{-}\beta \supset \text{-}\alpha$	UI, DED
(3)	$\alpha \supset \text{-}(v_2)\text{-}\beta$	R1, MP
(4)	$\alpha \supset (\exists v_2)\beta$	Df.4

From (4), it follows by MP that whenever α is assertable, $(\exists v_2)\beta$ is assertable. Since EG involves implicitly an application of UI, the general restriction on UI carries over. That is, an inference from "$f(v_1)$" to "$(\exists v_2)(f(v_2))$" will not be justified unless v_2 belongs to class (e) or v_1 and v_2 belong to the same class. Thus, if P is a property constant, the assertion $f(P)$ will entail that $(\exists x)(f(x))$ and $(\exists p)(f(p))$, but not $(\exists i)(f(i))$, $(\exists u)(f(u))$, or $(\exists s)(f(s))$.

2.4. <u>Axioms</u> <u>of</u> <u>*S</u>

Among the well-formed formulas specified by the rules of formation presented in 2.2 are those which are to count as axioms of *S. These will be presented in this section. The presentation of the axioms will also serve as an occasion for a number of observations on questions of interpretation.

Before the first axiom is introduced, it will be necessary to give some additional definitions. The first of these defines the relation of <u>inclusion</u>, analogous to set inclusion. In this definition, and in all subsequent ones, the variables w, x, y, z etc. are to be understood as representing any variable from classes (a)-(e).

Df. 5 $x \leq y$ for $(z)(z{*}x \supset z{*}y)$

This relation is transitive and totally reflexive, viz.

T1 $\vdash (x \leq y \,\&\, y \leq z) \supset (x \leq z)$

and T2 $\vdash (x \leq x)$

The proof of these theorems is straightforward and will be omitted here.

The next definition introduces the relation of identity:

Df. 7 $x=y$ for $(z)(z{*}x \equiv z{*}y)$

Identity is of course transitive, symmetric, and totally reflexive:

T3 $\vdash((x=y \,\&\, y=z) \supset x=z)$

T4 $\vdash x=y \equiv y=x$

T5 $\vdash x=x$

The relation of proper inclusion can now be defined in terms of inclusion and identity:

Df. 8 $x<y$ for $(x<y) \,\&\, -(x=y)$

Proper inclusion is transitive, irreflexive, and asymmetric:

T6 $\vdash (x<y \,\&\, y<z) \supset (x<z)$

T7 $\vdash -x<x$

T8 $\vdash x<y \supset -y<x$

The first axiom of *S is as follows:

A1 $\vdash x{*}y \equiv (\exists z)(z \leq x \,\&\, z \leq y)$

From A_1 it follows that the relation "*" is reflexive and symmetric:

T9 $\vdash x{*}x$

T10 $\vdash x{*}y \equiv y{*}x$

A demonstration of these two theorems is given below. First, we will establish the lemma

T11 $\vdash (\exists y)(y=x)$

Proof:

(1)	$(y)-(y=x)$	Hyp.
(2)	$-(x=x)$	UI
(3)	$x=x$	T5
(4)	$x=x\ \&\ -x=x$	R1, MP
(5)	$(y)-(y=x)\supset(x=x\ \&\ -x=x)$	DED
(6)	$-(y)-(y=x)$	RED
(7)	$(\exists y)(y=x)$	Df. 4

A direct proof of T9 can now proceed.

(1)	$x\lneqq x$	T2
(2)	$x\lneqq x\ \&\ x\lneqq x$	R1, MP
(3)	$a=x$	T11, EI
(4)	$a\lneqq x\ \&\ a\lneqq x$	SUBS
(5)	$(\exists z)(z\lneqq x\ \&\ z\lneqq x)$	EG
(6)	$x*x\equiv(\exists z)(z\lneqq x\ \&\ z\lneqq x)$	A1, UG, UI
(7)	$x*x$	MP

The proof of T10 is much simpler than that of T9:

(1)	$x*x\equiv(\exists z)(z\lneqq x\ \&\ z\lneqq x)$	A1, UG, UI
(2)	$y*x\equiv(\exists z)(z\lneqq y\ \&\ z\lneqq x)$	UG, UI
(3)	$(\exists z)(z\lneqq x\ \&\ z\lneqq y)\equiv(\exists z)(z\lneqq y\ \&\ z\lneqq x)$	R1, EG
(4)	$x*y\equiv y*x$	A1, R1, MP

In step (4) of the above proof, repeated applications of R1 and MP are required. In subsequent proofs, the procedure of citing such repeated rules only once will continue to be followed.

The importance of A1 in the system *S is that it secures the intended interpretation of the relation "*". As noted earlier, this relation holds between denotata which are not countably distinct from each other. Clearly, then, it must be symmetric and relexive. This result could be achieved, of course, merely by assuming T9 and T10, but the choice of A1 is more economical from a theoretical point of view.

The next axiom of *S asserts that every monadic predicate of the system determines a property:

A2 $\vdash(\exists p)(y)(yHp \equiv f(y))$

Several detailed observations on A2 are in order. First, it is analogous to an axiom frequently assumed in set theory, viz.

$\vdash(\exists x)(y)(y \epsilon x \equiv \alpha)$ where α is a wff which does not contain x. (Cf. Quine, 1964, p. 89)

It is well known that unless this axiom is restricted in some fashion, it gives rise to a contradiction. Suppose that α is taken as $-(y \epsilon y)$. Then we get

$(\exists x)(y)(y \epsilon x \equiv -y \epsilon y)$

and taking y as x we get

$(\exists x)(x \epsilon x \equiv -x \epsilon x)$

To avoid this contradiction, which is Russell's paradox, it is necessary somehow to rule out formulas such as $x \epsilon x$ and $-y \epsilon y$. Russell's theory of types is one method of accomplishing this, and there are various others.

Now it might be thought that a similar paradox would result in the case of A2. It appears that it makes sense in English to say that there are some properties which possess themselves and others which do not. The property of being old, for example, would qualify as old, whereas the property of being fat is not fat. Let us call properties which possess themselves <u>predicable</u> and those which do not <u>impredicable</u>.

This classification is mutually exclusive and exhaustive. Thus, it seems to make sense to inquire whether or not the property of impredicability possess itself. If <u>impredicable</u> means $-pHp$, this question could be answered formally, it appears, by substituting $-yHy$ for $f(y)$ in A2, deriving

$(\exists p)(y)(yHp \equiv -yHy)$

and then taking y as p. But from the latter step we get

$(\exists p)(pHp \equiv -pHp)$

This paradox cannot arise in *S for the simple reason that $-yHy$ is not a well-formed formula of the system, and hence is not a legitimate substitution for f(y) in A2. To this extent, *S embodies something of a type theory. On the other hand, it is not nearly as radical. Although the formulas yHy and -yHy are ruled out, the formulas pHp and -pHp are perfectly respectable. Thus, it is possible to define the property of impredicability in *S, and to do so without paradox. By instantiating A2 we can derive

$$(\exists p)(qHq \equiv f(q))$$

Now suppose $f(q) \equiv -qHq$

then $(\exists p)(qHp \equiv -qHq)$

which is the desired result.

The necessity of avoiding contradiction in a formal system makes it imperative to distinguish certain possible strings of symbols as meaningless. In *S, this is accomplished in a very natural way by introducing distinct styles of variables. If we did not distinguish property variables from the others, we could not allow a formula like -pHp and at the same time rule out a formula like -yHy. Some of the other distinctions among variables in *S are not as strongly motivated as the above. For example, it is not obvious that any outright contradiction would result if individual variables were classed together with activity variables. On the other hand, it is clear that such a procedure would lead to the formation of meaningless strings. As we will see below, the operation of addition, in an ordinary language sense, is definable in *S. In the ordinary language sense, only denotata which belong to some common class can constitute a sum. Since there appears to be no general term in English which is applicable to both individuals and events, it is impossible to return an answer to questions like "What is the sum of two wombats and two raids on a blind pig?" The ill-formedness of such questions can be accounted for in *S in terms of the ill-formedness of the string i*u.

The set of predicate constants in *S is finite. As a consequence of this, and of A2, the set of properties definable in *S is also finite. In this respect, *S differs radically from standard systems. In most systems which accomodate them at all, properties are defined for any propositional function, by means of the device known as intensional abstraction. In general, the expression

$$(\lambda x)(\ldots x\ldots)$$

denotes the property which something has if it meets the condition (...x...). (Cf Carnap, 1956, p.3; Rescher, 1968, p. 139). If applied to the full range of propositional functions available in natural languages, this device would allow us to define such entities as the property of having voted for Spiro T. Agnew or the property of being about to eat an omelet for breakfast. It is clear that neither of these is a property in English. In general, properties are specified in English only by intransitive adjectives. With few exceptions, they never correspond to transitive adjectives or to verbs, both of which, as we will see, always express some sort of relation. If we suppose, then, that the set of predicates in *S corresponds to the set of underlying intransitive adjectives in English, A2 will define properties in the appropriate places.

One final observation about A2. Given a predicate of the proper sort, it defines some property. It does not exclude the possibility, however, that there are properties which are undefined. It is perfectly conceivable that two entities might be described by exactly the same predicates, yet be distinct in some property for which a given language has no available predicate. This hypothetical situation can be expressed in *S in the following way:

$$(\exists x)(\exists y)(\exists p)(xHp\&-(\exists f)(f(x)\&-f(y)))$$

Obviously, it would be impossible to express this logically possible state of affairs in a system in which there is no concept of properties as independent of the predicates which determine them

The third axiom of *S states that two properties are in fact the same property if, and only if, whatever has one must have the other:

A3 $\vdash p=q \equiv (x)(xHp \equiv xHq)$

By means of A2 and A3, it can now be shown that every monadic predicate of *S determines a unique property. This proposition can be written as

T12 $\vdash (\exists p)(y)\{(yHp \equiv f(y)) \& (q)(y)[(yHq \equiv f(y)) \supset p=q]\}$

The proof of T12 is as follows:

(1)	$(f)(\exists p)(y)(yHp \equiv f(y))$	A2, UG
(2)	$yHP \equiv f(y)$	UI, EI, UI
(3)	$yHq \equiv f(y)$	Hyp
(4)	$yHP \equiv yHq$	R1, MP
(5)	$P=q$	A3
(6)	$(yHq \equiv f(y)) \supset P=q$	DED
(7)	$(q)(y)[(yHq \equiv f(y)) \supset P=q]$	UG
(8)	$(yHP \equiv p(y)) \& (q)(y)[(yHq \equiv f(y)) \supset P=q]$	R1
(9)	T12	UG, EG

At this point it will be convenient to introduce a notation for expressions of the type "the unique x such that x satisfies the condition ...x...". Such an expression is known as a <u>definite description</u>. These can occur in any of the atomic formulas specified by the rules of formation in 2.2. Letting "C(x)" represent the appearance of x in any of these contexts, we write:

Df.9 $C(\iota x)(\ldots x\ldots)$ for $(\exists x)(\ldots x\ldots \& (y)(\ldots y\ldots \supset y=x) \& C(x))$

We now wish to demonstrate the theorem

T13 $\vdash (f(y) \equiv yH(\iota p)(y)(yHp \equiv f(y))$

Proof:

(1)	(8) of preceding proof	
(2)	$yHP \equiv f(y)$	R1, MP
(3)	$f(y)$	Hyp
(4)	(8)&yHP	R1, MP
(5)	$yH(\iota p)(y)(yHp \equiv f(y))$	EG, Df.9
(6)	$f(y) \supset yH(\iota p)(y)(yHp \equiv f(y))$	DED
(7)	$yH(\iota p)(y)(yHp \equiv f(y))$	Hyp
(8)	$f(y)$	Df.9, EI, UI, R1, MP
(9)	$yH(\iota p)(y)(yHp \equiv f(y)) \supset f(y)$	DED
(10)	T13	R1, MP

T13 expresses a very regular equivalence in English--namely the equivalence between sentences of the form NP_1+BE +Adj and NP+HAVE+NP_2, where NP_2 denotes the property which something must have in order to be truly described by the corresponding adjective. This equivalence is evident in the following sentences:

(1a) John is intelligent
(2a) Sheila is beautiful
(3a) Max is wise

(1b) John has the property of being intelligent
(2b) Sheila has the property of being beautiful
(3b) Max has the property of being wise

The (a) sentences all have the schematic representation "F(J)", while the (b) sentences have the form "$JH(\imath p)(y)(yHp \equiv F(y))$." If "F" corresponds to the English adjective <u>wise</u>, then the latter can be rendered in English as "The individual J has the property which something has if, and only if, it is wise."

The correspondence of the (a) and (b) sentences above can be carried a step further. The abstract nouns <u>intelligence</u>, <u>beauty</u>, and <u>wisdom</u> may be substituted for the predicate noun phrases in (1b)-(3b), yielding

(1c) John has intelligence
(2c) Sheila has beauty
(3c) Max has wisdom

The equivalence of the (b) and (c) sentences can be accounted for on the assumption that the predicate nouns in (c) are property constants corresponding in reference to the definite descriptions in (b). To express this, we could write the following rules:

(d) $Int =df. \ (\imath p)(y)(yHp \equiv I(y))$

(e) $Bea =df. \ (\imath p)(y)(yHp \equiv B(y))$

(f) $Wis =df. \ (\imath p)(y)(yHp \equiv W(y))$

Rules such as (d)-(f) are analogous to lexical insertion rules of the type

$$(cause(become(not\ alive))) \rightarrow kill$$

Unlike the latter, however, they do not literally convert one structure to another. Instead, they allow us to conclude that two strings which are independently generated are logically equivalent. Thus, (2b) and (2c) have the logical structures

$$\text{(g)}\quad SH(\neg p)(y)(yHp \equiv B(y))$$

and

$$\text{(h)}\quad SHBea$$

which are demonstrably equivalent by virtue of (e).

The fourth axiom of *S asserts that there is no more than one performer for each numerically distinct act:

$$\text{A4}\quad \vdash((iDu \& jDu) \supset i=j)$$

A4 is a truism. but it may occur to some readers that it is violated by sentences such as

(i) John and Mary made love

Under the normal interpretation, however, (i) is understood to assert the performance of two separate acts--namely, John's making love to Mary and reciprocally. These reciprocal acts are of the same genus, but they are numerically distinct. Hence, (i) and other sentences like it are not exceptions to A4.

It might be objected that (i) asserts only that a single act took place, an act performed by John and Mary jointly. If this is the case, then A4 is still not violated, however, for it would be incorrect to assert of an act which John and Mary performed jointly that John performed it and Mary also performed it. In a sense, it is a composite individual consisting of but distinct from either John or Mary separately to which we need to attribute the performance of a single act. A means of doing this will be introduced below.

The final axiom of *S allows us to define what we will call the generic extension of a predicate, or simply a generic or type:

$$\text{A5}\quad \vdash(\exists x)(f(x) \& (y)(f(y) \supset y \leq x))$$

That there is in fact a unique generic for every predicate is expressed by the theorem

$$\text{T14}\quad \vdash(\exists x)\left\{\left[f(x) \& (y)(f(y) \supset y \leq x)\right] \& (z)\left[(f(z) \& (w)(f(w) \supset w \leq z)) \supset z = x\right]\right\}$$

Proof of T14:

(1)	$(\exists x)(f(x)\&(y)(f(y)\supset y\leq x))$	A4
(2)	$f(a)\&(y)(f(y)\supset y\leq a)$	EI
(3)	$f(z)\&(w)(f(w)\supset w\leq z)$	Hyp
(4)	$f(z)$	R1, MP
(5)	$f(z)\supset z\leq a$	R1, MP, UI
(6)	$z\leq a$	MP
(7)	$(w)(f(w)\supset w\leq z)$	R1, MP
(8)	$f(a)\supset a\leq z$	UI
(9)	$f(a)$	R1, MP
(10)	$a\leq z$	MP
(11)	$z\leq a\&a\leq z$	R1, MP
(12)	$a=z$	R1, Df.7
(13)	$f(z)\&(w)(f(w)\supset w\leq z)\supset a=z$	DED
(14)	$(z)(f(z)\&(w)(f(w)\supset w\leq z)\supset a=z)$	UG
(15)	(2)&(14)	R1, MP
(16)	T14	EG

T14 justifies us in introducing the following contextual definition of a generic, where C again represents any of the primitive contexts in which a denotator can appear:

Df.10 $C(\gamma f)$ for $C(\iota x)(f(x)\&(y)(f(y)\supset y\leq x))$

The generic extension of a predicate corresponds in significant ways to the English notion of a type of entity. Consider the following sentences:

(a) The dinosaur is extinct
(b) The automobile was invented in 1889
(c) I prefer the dandelion to all other weeds

The noun phrases in these sentences do not refer to individuals, but to types of individuals. The closest translation available for such sentences in standard logics is one which interprets them as statements about classes or sets. Yet it is certainly a deviant brand of English which would allow

(d) The class of dinosaurs is extinct
(e) The class of automobiles was invented in 1889
(f) I prefer the class of dandelions to all other weeds

Note that there is no difficulty in paraphrasing (a)-(c) in the following manner:

(g) The dinosaur is a type of animal which is extinct
(h) The automobile is a type of vehicle invented in 1889
(i) I prefer dandelions to all other types of weeds

The notion of class, particularly when it is treated informally, as is all too often the case in discussions of natural language, is easily confused with the notion of type. The formal distinction between these two is very clear-cut, however, as we will see in the following paragraphs.

Suppose that U is the predicate unicorn and E is the predicate exists. The proposition that nothing unicorn-like exists may then be represented as

$$\text{(j)} \quad -(\exists x)(U(x)\&E(x))$$

In spite of (j), however, the class of all unicorns still exists. It can be denoted by the definite description

$$\text{(k)} \quad (\iota x)(y)(y\epsilon x \equiv U(y)\&E(y))$$

This class is identical, of course, to the null class. Nevertheless, its intensional definition is unique, and distinct from that which specifies, say, the class of all griffins or the class of all round squares. In standard set theories, one is justified in asserting that the class of all unicorns is identical to the class of all griffins. In ordinary English, however, it would never be justified to say

(l) The unicorn is identical to the griffin.

A fundamental distinction between classes and types, then, is the following: any two memberless classes are necessarily identical, while two types which have distinct intensional definitions are not.

As we have just seen, the nonexistence of any members in a class does not imply the nonexistence of the class itself. In ordinary English, however, we would conclude that if no unicorns exist then the unicorn does not exist either. More generally, we would conclude that if there are no exponents of a given type, then the type does not exist. This proposition can be represented as follows:

$$\text{T15} \vdash -(\exists x)(f(x)\&E(x)) \supset -E(\gamma_f)$$

Proof of T15:

(1)	$E(\gamma_f)$	Hyp
(2)	$f(a)\&E(a)$	Df.10, EI, R1, MP
(3)	$(\exists x)(f(x)\&E(x))$	EG
(4)	$E(\gamma_f)\supset(\exists x)(f(x)\&E(x))$	DED
(5)	T15	R1, MP

As noted above in the discussion of A3, strings such as $y\epsilon y$ and $-y\epsilon y$ must be rejected as meaningless in standard set theory. This restriction may also be stated as follows: where f is the defining concept for some set, the statements

(m) $f(\gamma x)(y)(y\epsilon x\equiv f(y))$

and (n) $-f(\gamma x)(y)(y\epsilon x\equiv f(y))$

must be considered meaningless. If the referents of generic noun phrases like those in (a)-(c) are identified with classes, however, it will be impossible to maintain this, in view of the following.

In ordinary English, all of the following sentences are true:

(o) The lion is rapacious
(p) The lion is carnivorous
(q) The lion is tawny
(r) The lion is four-footed
(s) The lion is maned.

This list could obviously be extended to include all of the attributes which are criterial for lionhood. Whatever the referent of the noun phrase <u>the lion</u> is in these sentences, it is clearly something to which we can attribute all of the properties of a lion. Moreover, all of these attributes can be summed up in the single adjectival term <u>lionlike</u>, and it is clear that, in the intended sense of the term, the sentence <u>the lion is lionlike</u> is true, even tautologous.

Generally, we can conclude that the predicate which is critical for membership in a given type is predicable of the type itself. Symbolically, this is

$$\text{T16 } f(\gamma_f)$$

The theoremhood of T16 is evident from a simple inspection of A5. It expresses the logical truth of innumerable possible sentences in English, including:

(t) The lion is lionlike
(u) The tiger is tigerlike
(v) Beauty is beautiful
(w) Truth is true

and, since mass nouns are merely a special case of generics,

(x) Powder is powdery
(y) Bone is osseous
(z) Calcium carbonate is calcariferous

and so on.

The applicability of the above notion of generics to English generic noun phrases will considered in detail in Chapter IV, and its applicability to the analysis of mass nouns will be examined in Chapter V. We now turn our attention to the analysis of indefinite predicate nominals, or to expressions of the form "x is an f."

2.5. Indefinite Descriptions

In spite of the fact that our generic lion considered above has all of the properties of a lion, he does not count as an individual lion. That is, in ordinary English we would not say the lion is a lion, anymore than in set theory we would say that the set of all lions is a member of the set of all lions. This result is rather embarrasing, however, for logicians who have claimed that English sentences of the form "x is an f" correspond to assertions of set membership (Cf. Reichenbach, 1966, p. 192). In standard set theory, where f represents the common properties of the members of a given set, we have the following equivalence:

$$\text{(a)} \quad f(x) \equiv x \in (\imath y)(z)(z \in y \equiv f(z))$$

If a proposition of the form "x is a lion" is translated as

$$\text{(b)} \quad x \in (\imath y)(z)(z \in y \equiv f(z))$$

then it would be impossible, by (a), for the generic lion to have all the properties of a lion and still not be a lion.

There is an additional reason for supposing that the relation of set membership does not provide an adequate translation of English sentences of the form "x is an f". In set theory, it is possible to define classes which have a single member. In general, the class whose only member is x is given by

$$\text{(c)}\quad (\iota y)(z)(z\epsilon y\equiv z=y)$$

It is of course, logically true that x belongs to this set, i.e.,

$$\text{(d)}\quad x\epsilon(\iota y)(z)(z\epsilon y\equiv z=x)$$

Now suppose that we form the class whose only member is, say, the city of New York. Where N=New York, it will follow that

$$\text{(e)}\quad N\epsilon(\iota y)(z)(z\epsilon y\equiv z=N)$$

But if the relation "ϵ" corresponds to the English relation "is a", this would have to be read "New York is a New York."

What is wrong with this is evidently that English sentences of the form "x is an f" imply that there are at least two logically possible f's. As a further illustration of the same problem, consider the sentence

(f) The ruler of the universe is a ruler of the universe

All theological and ontological questions aside, (f) is a deviant sentence, since it embodies the contradictory claims that (a) there is one and only one ruler of the universe, and (b) that this being is one of such things. The putative representation of (f) in standard set theoretical notation, however, far from being contradictory, is logically true:

$$\text{(g)}\quad (\iota x)(R(U;x))\ \epsilon\ (\ y)(z)(z\epsilon y\equiv R(U;z))$$

In the system *S, it is possible to construct a representation of indefinite singular predicate nominals which circumvents the above difficulties. First, we introduce a notation which represents indefinite singular nominals in general:

Df.11 $C(ly)(\ldots y\ldots)$ for
$$(\exists y)\{\ldots y\ldots \& (\exists x)(\ldots x\ldots \& (z)((\ldots z\ldots \& z*y)\supset y\leq z))\& C(y)\}$$

An expression of the form "$(ly)(\ldots y\ldots)$" will be called an indefinite description. Like definite descriptions, these are incomplete symbols, i.e., they are not constituents of the strings in which they appear and they have a meaning only as part of some larger context.

The contexts in which we are immediately interested are those of the form "x is an f." If we take the copula to express identity in such expressions, then we can represent them as

$$\text{(h)}\quad x=(ly)(f(y))$$

By a previous convention, the C in Df.11 represents only the contexts in which a denotator can occur in an atomic formula. Hence, the expression "$(ly)(f(y))$" is not defined for a context such as (h), and Df.11 must be supplemented as follows:

$$w=(ly)(f(y)) \text{ for } (\exists x)\{f(x)\&(\exists y)(f(y)\&-y=x)\&(z)(f(z)\&z*x\supset x\leq z)\ \&w=x\}$$

Now let us examine in detail how the schema (h) applies to English sentences. Taking R=Rover and H=hound, the sentence

(j) Rover is a hound

will be assigned the representation

$$\text{(k)}\quad R=(ly)(H(y))$$

By Df.11, (k) becomes

$$\text{(l)}\quad (\exists y)\{H(y)\&(\exists x)(H(x)\&-x=y)\&(z)((f(z)\&z*y)\supset y\leq z)\&y=R\}$$

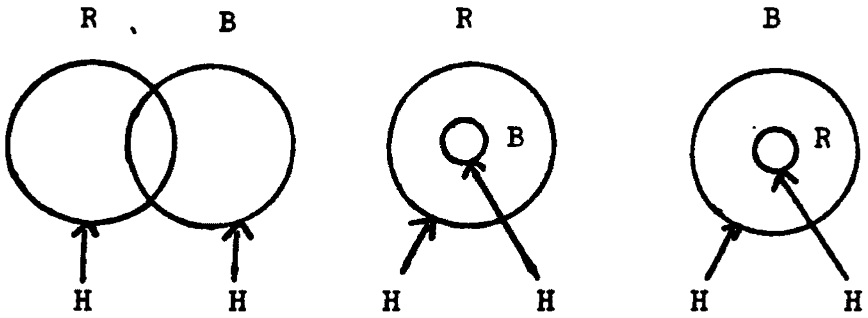
Existential instantiation and substitution of identities allows us to isolate the three statements

(m) $H(R)$

(n) $(\exists x)(H(x)\&-x=R)$

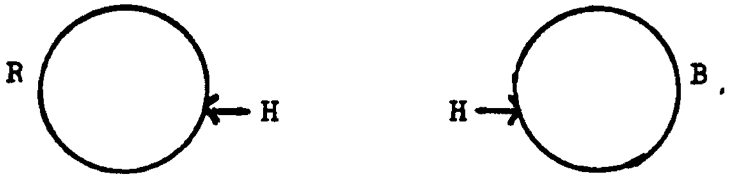
(o) $(z)((H(z)\&z*R)\supset R\leq z)$

According to (m), Rover has all of the properties which are requisite for hounds. Together, (m) and (n) state that there are at least two individuals which have them. Finally, (o) states that Rover constitutes a unit of houndhood which is countably distinct from all other such units. The significance of the latter can perhaps best be appreciated graphically. Suppose that the circles labelled R and B below represent Rover and Bowser, and that an arrow drawn between the symbol H and a given circle means that the predicate <u>hound</u> is applicable to that particular region of space. Then the following relations between Rover and Bowser are ruled out by condition (o):



(A) (B) (C)

The only relation between Rover and Bowser which is allowable by (o) is that represented in (D):



(D)

Which is of course exactly what one should expect.

Formally, the matter can be expressed as follows. What is true in all the cases (A)-(C) which is not true in case (D) is the assertion represented in *S as

(p) R*B

or, translated into English, Rover and Bowser are not countably distinct. If Rover and Bowser are both individual hounds, however, this cannot be the case. In general, any two individuals which have the same indefinite description must be countably distinct, i.e., T17 $\vdash(x=(\iota y)(f(y))\&w=(\iota z)(f(z))\&-x=w)\supset -x*w$

Proof:

(1)	$x=(\iota y)(f(y))\&w=(\iota z)(f(z))\&-x=w$	Hyp
(2)	$x=(\iota y)(f(y));w=(\iota z)(f(z));-x=w$	R1, MP
(3)	$f(x)\&(\exists z)(f(z)\&-z=x)\&(w_1)((f(w_1)\&w_1*x)\supset x\leq w_1)$	Df.11, EI, SUBS
(4)	$f(w)\&(\exists z)(f(z)\&-z=w)\&(w_2)((f(w_2)\&w_2*w)\supset w\leq w_2)$	Df.11, EI, SUBS
(5)	$f(x);f(w)$	R1, MP
(6)	$(w_1)((f(w_1)\&w_1*x)\supset x\leq w_1);(w_2)((f(w_2)\&w_2*w)\supset w\leq w_2)$	R1, MP
(7)	$(f(w)\&w*x)\supset x\leq w\ ;(f(x)\&x*w)\supset w\leq x$	UI
(8)	$f(x)\&x*w$	Hyp
(9)	$w\leq x$	MP
(10)	$x*w$	MP
(11)	$w*x$	T10, MP
(12)	$f(w)\&w*x$	R1, MP
(13)	$x\leq w$	MP
(14)	$x=w$	R1, Df.7, MP
(15)	$(f(x)\&x*w)\supset x=w$	DED
(16)	$(f(x)\&x*w)\supset x=w\&-x=w$	R1, MP
(17)	$-f(x)\&x*w$	RED
(18)	$-x*w$	R1, MP
(19)	T17	DED

In the introduction to this section it was noted that the sentence

(f) The ruler of the universe is a ruler of the universe was deviant by virtue of being self-contradictory. We are now in a position to explicate this self-contradictoriness. The subject noun phrase of (f) is a definite description and the predicate noun phrase is an indefinite description. If the correspondence between grammatical and logical structures outlined above is general, then (f) ought to have the form

(q) $(\eta x)(R(x;U))=(\iota y)(R(y;U))$

There is a minor difficulty, however, in presenting (q) as an appropriate translation of (f). As it stands, (q) contains two incomplete symbols, neither of which is defined in the context of the other. This problem is easily rectified, though, by introducing the following definition:

Df. 12 $(\text{℩}x)(\ldots x\ldots)=(\iota y)(\ldots y\ldots)$, where $(\ldots x\ldots)$ and $(\ldots y\ldots)$ are any two propositional functions, for

$$(\exists z)(z=(\text{℩}x)(\ldots x\ldots)\&z=(\iota y)(\ldots y\ldots))$$

Df. 12 permits us to represent (f) by means of (q). The contradictoriness of (q) can be readily shown:

T18 $\vdash -((\text{℩}x)(R(x;U))=(\iota y)(R(y;U))$

Proof:

(1)	(q)	Hyp
(2)	$(\exists z)(\exists x)(R(x;U)\&(y)(R(y;U)\supset y=x)\&z=x$ $\&(\exists y)(R(y;U)\&(\exists w)(R(w;U)\&-w=y)$ $\&(w_1)_\wedge(R(w_1;U)\&w_1 * y)\supset y\leq w_1)\&y=z)$	Df.12, Df.11, Df.10
(3)	$R(a;U)\&(y)(R(y;U)\supset y=a)$ $\&R(b;U)\&(\exists w)(R(w;U)\&-w=b)$	EI, SUBS
(4)	$(y)(R(y;U)\supset y=a)$	Rl, MP
(5)	$R(a;U)_\wedge R(b;U)$	Rl, MP
(6)	$a=b$	UI, MP
(7)	$R(c;U)\&-c=b$	Rl, MP
(8)	$R(c;U)$	Rl, MP
(9)	$c=a$	Ul, MP
(10)	$c=b\&-c=b$	T3, Rl, MP
(11)	$(q)\supset c=b\&-c=b$	DED
(12)	T18	RED

It should be noted that the above proof in no way depends on the choice of the particular propositional function $R(x;U)$. As long as $(\ldots x\ldots)$ and $(\ldots y\ldots)$ impose the same conditions on x and y, respectively, the following more general theorem could be proved in the same manner as T18:

T19 $\vdash -((\text{℩}x)(\ldots x\ldots)=(\iota y)(\ldots y\ldots))$

Let us return now to a consideration of our generic friend, the lion. As noted in the introduction to this section, this entity has all of the properties of a lion, yet fails to be one. In set theory, this would be a paradox, but it is in no way paradoxical in the system *S. The fact that the lion is not a lion, the tiger is not a tiger, truth is not a truth, and so on, is reflected in *S by the theorem:

T20 $\vdash -(\gamma_f=(\iota y)(f(y)))$

Proof:

(1)	$\gamma_f=(\iota y)(f(y))$	Hyp
(2)	$(\exists y)(f(y)\&(\exists x)(f(x)\&-x=y)\&(z)((f(z)\&z*y)\supset y\leq z)\&y=\gamma_f)$	Df.11
(3)	$(f(a)\&(\exists x)(f(x)\&-x=a)\&(z)((f(z)\&z*a)\supset a\leq z)\&a=\gamma_f)$	EI
(4)	$a=\gamma_f$	R1, MP
(5)	$f(\gamma_f)\&(\exists x)(f(x)\&-x=\gamma_f)\&(z)((f(z)\&z*\gamma_f)\supset\gamma_f\leq z)$	R1, MP, SUBS
(6)	$(z)((f(z)\&z*\gamma_f)\supset\gamma_f\leq z)$	R1, MP
(7)	$(\exists x)(f(x)\&-x=\gamma_f)$	R1, MP
(8)	$f(b)\&-b=\gamma_f$	EI
(9)	$f(b);-b=\gamma_f$	R1, MP
(10)	$(y)(f(y)\supset y\leq\gamma_f)$	A5, EI, R1, MP
(11)	$b\leq\gamma_f$	UI, MP
(12)	$b*\gamma_f$	Df.5, R1, MP
(13)	$(x\leq y\&-y=x)\supset -y\leq x$	Df.3, Df.7 R1, MP
(14)	$(b\leq\gamma_f\&-b=\gamma_f)\supset-\gamma_f\leq b$	UG, UI
(15)	$-\gamma_f\leq b$	MP
(16)	$f(b)\&b*\gamma_f\&-\gamma_f\leq b$	R1, MP
(17)	$((f(z)\&z*\gamma_f)\supset\gamma_f\leq z)\supset-(f(z)\&z*\gamma_f\&-\gamma_f\leq z)$	R1
(18)	$-(f(z)\&z*\gamma_f\&-\gamma_f\leq z)$	UI, MP
(19)	(18)$\supset$-(16)	UG, UI
(20)	(16)$\supset$-(18)	R1, MP
(21)	(18)&-(18)	MP, R1, MP
(22)	(1)$\supset$(18)&-(18)	DED
(23)	T20	RED

With the proof of T20 it has been demonstrated that the proposed analysis of indefinite singular predicate nominals overcomes the difficulties noted in the introduction to this section. The general outlines of this analysis are by no means new. That is, the interpretation of structures of the form "x is an f" as statements of an identity between two referring expressions is not new, having been proposed by numerous analysts, including such redoubtables as Russell and Quine (Cf. Russell, 1956, Ch. IV; Quine, 1960, p. 118). In spite of such support, however, this analysis has not been widely accepted by linguists.

Of the arguments which have been explicitly formulated to discredit the analysis in question, most have been altogether lacking in cogency. For example, one is frequently confronted with arguments like the following:

The sentence

(a) Bill is a logician

> cannot possibly be regarded as an identity statement, since it is clearly a statement about Bill and not a statement about Bill and a logician. (Cf. Karttunen, 1969, p. 4).

It must certainly be granted that (a) is not a statement about Bill _and_ a logician. But far from refuting the analysis of (a) as an identity, this fact follows from it. Conjoined noun phrases in English always have distinct referents. Thus, we may be perfectly sure that the law firm of Roedecker, Roedecker, Stein and Stein has four partners and not two or three. In the same way, we may be sure that a statement about Bill and a logician is a statement about two different people. The fact that (a) is not a statement about Bill _and_ a logician, then, is a direct consequence of the fact that it is an identity.

A much more reasonable (but in its consequences equally mistaken) argument runs as follows. Ordinarily, referring expressions can function as antecedents for relative pronouns. Thus, to an undeniably referential term such as the proper name _Bill_ we can adjoint a nonrestrictive relative clause, as in the following sentence:

(a) Bill, who I admire, is a logician

On the other hand, it is impossible to conjoin the relative clause in (a) to the noun phrase _a logician_. Hence, the nonexistence in English of

(b) Bill is a logician, who I admire

It is widely believed that examples like (a) and (b) show that indefinite singular predicate nominals are nonreferential. This is not the case, however, since the nonexistence of sentences like (b) is exactly what is to be expected in terms of an analysis employing indefinite descriptions.

It is generally accepted that (a) is derived from a structure like that of (c):

(c) Bill is a logician and I admire Bill

In general, the transformation which converts (c) to (a) functions by adjoining S_2 to some NP in S_1 which is coreferential with some NP in S_2. Accordingly, we must assume that (b) has an underlying structure like that of (d):

(d) Bill is a logician and I admire a logician

It is obvious, however, that (d) cannot be the underlying form of (b), since there is no guarantee that the two occurrences of the noun phrase a logician in (d) are coreferential, as required if relativization is to apply. There are actually two readings for (d), the distinction between them depending upon whether the second occurrence of the noun phrase a logician refers to some particular logician or to any logician. Coreference between the two occurrences of a logician fails in both of these cases. This situation is reflected exactly in the system *S. Assuming (for the moment, and incorrectly as well) that the verb admire corresponds to a simple two-place predicate, the representation of the first reading of (d) would be

(e) $B=(\iota y)(L(y))\&((x=(\iota y)(L(y)))\supset A(I;x))$

while the second reading would be represented by

(f) $B=(\iota y)(L(y))\&A(I;(\iota y)(L(y))$

When the indefinite descriptions in (f) are expanded according to Df.11, the variable y occurring in them will appear bound to two separate existential quantifiers. But this is tantamount to saying that y will represent two separate variables. In general, variables which are bound to separate quantifiers cannot be identified, even though they may be alphabetically the same. Very much simplified, the expanded form of (f) will be a proposition of the form

(g) $(\exists y)(C(y))\&(\exists y)(D(y))$

where C and D are propositional functions. A schema like (g) does not imply the schema

(h) $(\exists y)(C(y)\&D(y))$

In other words, it will not be possible to conclude from (f) that the logician named Bill and the logican I admire are one and the same person.

Essentially the same observations apply to (e), except that by devious means (e) will give rise to the proposition that I admire Bill, whereas (f) will not. In neither case should relativization be expected to occur, since coreferentiality between the relative pronoun and its presumptive antecedent would fail in both. However, (e) and (f) are the only plausible sources of (b) in the system *S. If it is assumed that the representations of *S correspond in some consistent way to syntactic representations, and if various transformations, including relativization, are to apply to the latter, then the fact that (b) is unattested in English is precisely what one would predict.

It is time now to discuss the relevance of indefinite descriptions to a theory of syntactic categories. In standard applications of symbolic logic to English, the grammatical distinction between predicate adjectives and predicate nominals is obliterated. Thus, the sentences

(1) Rover is carnivorous

and (2) Rover is a carnivore

are both analyzed as instances of the schema

(3) $F(a)$

If the standard predicate calculus is supplemented with set theory, then we can choose representations of (1) and (2) which reflect their difference in syntactic form. We may continue to represent (1) by means of (3), while (2) is assigned the representation

(4) $a\epsilon(\text{?}x)(y)(y\epsilon x \equiv F(y))$

This is preferable, since we may now consistently identify schema (4) with syntactic strings of the form NP+BE+Indefinite Singular Nominal, reserving schema (3) for strings of the form NP+BE+Adj.

But now a problem arises--namely, the problem of how to relate the predicate F in (3) to the set-description $(\text{?}x)(y)(y\epsilon x \equiv F(y))$ in (4), or, in other words, how to relate the adjective <u>carnivorous</u> to the noun <u>carnivore</u>. It is obvious that the latter can be defined in terms of the former. That is, a carnivore is simply a creature which is carnivorous.

If the range of our variables were restricted to individual creatures, then we could express this definition by means of the equivalence

(5) $F(z) \equiv z \epsilon (\imath x)(y)(y \epsilon x \equiv F(y))$

Unfortunately, the range of variables necessary for natural language is not restricted to individual creatures. It must include, among other things, such abstract entities as the carnivore. And while it is true that the carnivore is carnivorous, it is not true that the carnivore is a carnivore.

The only consistent way to handle this situation in set-theoretic terms would be to consider the adjective carnivorous and the noun carnivore as independently defined predicates. Thus, (1) and (2) could be represented by (6) and (7), respectively:

(6) $F_1(a)$

(7) $a \epsilon (\imath x)(y)(y \epsilon x \equiv F_2(y))$

In place of the equivalence in (5), one could adopt the following meaning postulate to express the fact that whatever is a carnivore is carnivorous:

(8) $a \epsilon (\imath x)(y)(y \epsilon x \equiv F_2(y)) \equiv F_1(a)$

No inconsistencies will arise from this approach, but we will have failed to explicate the obvious and systematic relation which exists between cognate nouns and adjectives like carnivore and carnivorous.

In the system *S, (1) and (2) are represented as

(9) C(R)

and (10) R=(1y)(C(y))

These formulas correspond term for term to the syntactic structures which they represent. In general, the schema F(a) will correspond to syntactic structures of the form NP+BE+Adj, while the schema a=(1y)(F(y)) will correspond to those of the form NP+BE+Indefinite Singular Nominal. To this extent, then, we will be able to maintain that syntactic categories are definable in terms of logical or semantic representations.

The meaning of the indefinite description (1y)(F(y)) is wholly determined once the predicate F is determined. Thus, in the system *S, the adjective carnivorous and the noun carnivore, when it appears as the head of an indefinite singular noun phrase, can both be specified in terms of a single underlying predicate. The same thing will be true of countless other pairs of adjectives and nouns.

The fact that whatever is a carnivore is carnivorous, which we tried to express in terms of (8) above, turns out to be a theorem in *S. In general terms, we have

$$\text{T21} \quad \vdash x=(\iota y)(f(y)) \supset f(y)$$

Proof:

(1)	$x=(\iota y)(f(y))$	Hyp
(2)	$(\exists y)\{f(y)\&(\exists x)(f(x)\&-x=y)\&(z)((f(z)\&z*y)\supset y\leq z)\&x=y\}$	Df.11
(3)	$f(a);x=a$	EI, R1, MP
(4)	$f(x)$	SUBS
(5)	T21	DED

In other systems, the work accomplished by T21 would have to be done by a very large number of meaning postulates, each one of them amounting to an extra assumption. Since T21 follows without any special assumptions, the system *S clearly has a great theoretical advantage.

2.6. Some Additional Definitions

In this section a number of definitions will be given for terms which will figure to some extent in the representations of sentences in subsequent chapters. The definitions will also serve to illustrate some of the resources of the system *S.

The first definition specifies the logical sum of two terms:

$$\text{Df.13} \quad xUy \quad \text{for} \quad (\iota z)(w)(w*z \equiv (w*x \vee w*y))$$

The notation "xUy", which can be read simply "x and y", has a very important application in translating certain kinds of phrasal conjunction in English. It has been pointed out frequently that sentences like

(1) John and Mary played tennis

are ambiguous, the two readings depending upon whether the two actors performed jointly or separately. Thus, (1) may be paraphrased by either of the following:

(2) John and Mary each played tennis with someone

(3) John and Mary played tennis with each other

In the first reading, (2), there are two activities of tennis playing which are asserted to have taken place. This can be represented in *S by

(4) $(\exists u)(\exists v)(JDu\&MDv\&P(u)\&P(v))$

In the second reading, (3), only one activity of tennis playing occurs. This reading can be represented by

(5) $(\exists u)(JUMDu\&P(u))$

It is important to realize that the term "JUM" represents a composite of two individuals and not the sum or union of two sets. It is not the set composed of John and Mary which is asserted to have played tennis in (1), rather the members of this set, that is, John and Mary themselves. There has been a good deal of confusion on this point, and various authors who have tried to explicate the ambiguity of sentences like (1) on the basis of set theory have wound up attributing all sorts of highly improbable activities to sets. McCawley seems to have been aiming at the solution arrived at here when he writes

> The 'set theory' here is one which ignores the difference between an individual and a one-member set, and thus allows individuals to be combined by the union operation:
>
> $x_1 U\ x_2 = \{x_1, x_2\}$ (Cf. McCawley, 1968, p. 146)

What McCawley suggests is, of course, literally impossible in set theory, for it is imperative that the distinction between a singularly set and its single member be maintained. It is quite possible, however, to define the notion of the union of two individuals independently of set theory, as we have just done here. It should be pointed out that the definition we have given is by no means new; it corresponds directly to one first provided by Goodman and Leonard in their paper *The Calculus of Individuals* (Goodman and Leonard, 1941).

A word must be added about what the notation "xUy" does *not* translate. It does not translate instances of phrasal conjunction such as

(6) John and Harry are similar

At this point we have no means of translating (6) directly into *S. However, it is obvious that (6) is equivalent to

(7) John is similar to Harry and Harry is similar to John

The latter can be assigned the representation

(8) S(J;H)&S(H;J)

Now it follows from the meaning of the predicate <u>similar</u> that whenever one of the conjuncts in (8) is true the other is also true. That is, we have the general rule

(9) S(x;y)⊃S(y;x)

If (9) is adopted as a meaning postulate, then (8) will be equivalent to either of the assertions S(J;H) and S(H;J). In other words, the order of the arguments will be truth-functionally irrelevant. This is true of a whole class of predicates which are called <u>symmetric</u>. A convenient notation for sentences involving such predicates can be introduced by way of the following definition:

Df.14 Where f is any symmetric predicate,

f(x,y) for f(x;y)&f(y;x)

We can now represent (6) directly by means of

(10) S(J,H)

Like the notation "JUH", the symbol "(J,H)" may be read "John and Harry." However, the latter represents an unordered couple and not a sum.

It is possible to construct a certain portion of arithmetic in *S. The operation of addition can be defined as follows:

Df.15 If -x*y, then

x+y for xUy

The above definition must be conditional, since x and y cannot be added if they have some common part. It should also be observed that addition is undefined for terms which belong to different classes. By Df.13, a string such as iUu would have to be defined as

(11) (x)(y)(y*x≡(y*i v y*u))

But (11) is well formed only if w is taken as an individual variable or an activity variable. That is, (11) must represent either

(12a) (℩z)(v)(v*z≡(v*u v v*u)) or (12b) (℩z)(j)(j*z≡(j*i v j*u))

Both of these strings are ill-formed. Accordingly, no meaning can be assigned by way of Df.15 to the string i+u. Thus, *S reflects the fact, noted earlier, that wombats and two raids on a blind pig do not constitute a sum in English.

The following theorems, which will be presented without proof, show that addition has the properties of commutativity and associativity. Where C is any propositional function,

T22 $C(x+y)\equiv C(y+x)$

T23 $C(x+(y+z))\equiv C((x+y)+z)$

The assertion "$x=(1y)(f(y))$" states that x is a single individual of the type f. Now that addition has been defined, we can go on to define a collection of two, three, four individuals, and so on:

Df.16 Where $x=(1y)(f(y))$ and $z=(1y)(f(y))$
$(2y)(f(y))$ for $x+z$

Df.17 Where $x=(1y)(f(y))$ and $z=(2y)(f(y))$
$(3y)(f(y))$ for $x+z$

etc.

This chain of definitions can be extended indefinitely to include a collection whose numerosity is any finite positive integer. Thus, the system *S provides a definition in context of all of the natural numbers, excluding zero.

The English sentence

(13) Two camels plus two camels equals four camels

can be represented in *S by the theorem

(14) $\vdash(x=(2y)(C(y))\&z=(2y)(C(y)))\supset x+z=(4y)(C(y))$

More generally, (14) is provable in *S when C represents any propositional function whatever. Since numbers are defined only contextually in *S, the proposition

(15) $2+2=4$

can be assigned a meaning in *S only if it is taken as an elliptical version of (14) in its general sense, i.e., only if it is interpreted as

(16) Two of something plus two of something equals
four of something

The usual translation of (15) in set theory is parallel to (14). Where "2" and "4" are the classes of all classes with two and four members, respectively, (15) may be expressed as:

(17) $(x\epsilon 2\&z\epsilon 2)\supset x+z\epsilon 4$

The essential distinction between (14) and (17) is that the antecedent of (14), "$x=(2y)(C(y))\&z=(qy)(C(y))$", imposes a common predicative condition in addition to mere two-foldness on x and z, while the antecedent of (17), "$x\epsilon 2\&z\epsilon 2$", does not. For applications to ordinary language addition, then, *S is clearly superior to set theory, since it reflects the fact that terms may be added in ordinary languages only if they belong to some common genus.

In view of the fact that the system *S dispenses with set theory, it would be of considerable philosophical interest to investigate the extent to which classical arithmetic is constructable in it. However, such an investigation would take us well beyond the scope of the present work. We will return, therefore, to a consideration of the applicability of *S to English.

A predication of the form "$f(ny)(f(y))$" attributes f to the sum represented by "$(ny)(f(y))$" and not to its component terms. When we say, for example,

(18) Three people is a crowd

we are asserting something of a sum of three people and not of three individuals. The translation of (18) into *S is as follows:

(19) $(x)\{x=(3y)(P(y))\supset x=(1y)(C(y))\}$

Number can be used in English in a distributive sense as well as in a collective sense, however, The sentence

(20) Four people are not allowed on the sidewalk in Selma, Alabama

is ambiguous. Treating <u>not allowed on the sidewalk in Selma, Alabama</u> as an unanalyzed predicate, it has the collective sense of

(21) $(x)\{x=(4y)(P(y))\supset F(x)\}$

It can also be interpreted, however, as referring to four definite people. This is its distributive sense, which is given by:

(22) $(\exists w)(\exists x)(\exists y)(\exists z)(F(w)\&F(x)\&F(y)\&F(z)\&-w=x\&-x=y\&-y=z)$

It will be convenient to express propositions like (22) more compactly. Accordingly, we introduce the following definitions:

Df.19 $(\exists_2 x)(\ldots x\ldots)$ for

$(\exists x)(\exists y)(\ldots x\ldots\&\ldots y\ldots\&-x=y)$

Df.20 $(\exists_{3} x)(\ldots x\ldots)$ for

$(\exists x)(\exists y)(\exists z)(\ldots x\ldots \& \ldots y\ldots \& \ldots z\ldots \& -x=y \& -y=z)$

Df.21 $(\exists_{4} x)(\ldots x\ldots)$ for

$(\exists w)(\exists x)(\exists y)(\exists z)(\ldots w\ldots \& \ldots x\ldots \& \ldots y\ldots \& \ldots z\ldots \& -w=x \& -x=y \& -y=z)$

etc.

Numerals which appear in quantifiers, as above, will have a distributive sense, while those that appear in operators, as in the foregoing examples (19) and (21), will have a collective sense.

According to Df.19, a formula such as "$(\exists_{2} x)(F(x))$" means that there are <u>at least</u> two things which satisfy the propositional function "F(x)". It does not exclude the possibility that there may be more than two things for which this function is true. It frequently happens, however, that we wish to assert that a given propositional function is true of <u>exactly</u> n things. Thus, the English sentence

(23) There are six men who have walked on the moon

means that exactly six men have walked on the moon, no more and no less. Assuming that <u>have walked on the moon</u> is represented by the unanalyzed predicate W, it would be incorrect to translate (23) as

(24) $(\exists_{6} x)(x=(\iota y)(M(y)) \& W(x))$

Taking into account the fact that both (23) and (24) contain an implicit reference to the present time, the assumption that in fact there were 76 men who had walked on the moon by now would falsify (23), but not (24).

The required sense of numerals is captured in the following definitions:

Df.22 $(\exists_{1.} x)(\ldots x\ldots)$ for

$(\exists x)(\ldots x\ldots \& (y)(\ldots y\ldots \supset y=x))$

Df.23 $(\exists_{2.} x)(\ldots x\ldots)$ for

$(\exists x)(\exists y)(\ldots x\ldots \& \ldots y\ldots \& -x=y \& (z)(\ldots z\ldots \supset (z=x \vee z=y)))$

etc.

Lakoff has observed that the sentence

(25) That archeologist discovered nine tablets

is ambiguous (cf. Lakoff, 1970; p. 12). It can mean that the archeologist in question made a single discovery of a group of nine tablets, or that he made a number of discoveries such that the total number of discovered tablets was nine. These distinct readings Lakoff calls the group reading and the quantifier reading. He does not suggest any formal way of representing the distinction. However, they can be readily differentiated in *S. Letting A=that archeologist, the group reading can be presented as

(26) $(\exists_{1.}u)(\exists i)(ADu\&i=(9j)(T(j))\&Di(u;i))$

and the quantifier reading as

(27) $(\exists_{9.}i)(\exists u)(ADu\&i=(1j)(T(j))\&Di(u;i))$

Lakoff's terms are therefore very appropriate. In the group reading, the numeral appears as part of the operator forming an indefinite description, and in the quantifier reading it is associated with the existential quantifier.

2.7. Substitutivity in *S

A referentially opaque context is one which does not necessarily permit substitution salva veritate of coreferential grammatical structures. As an example of this phenomenon, consider the following:

(1) Oedipus wanted to marry Jocasta

(2) Jocasta is Oedipus' mother

Substitution of the noun phrase Oedipus' mother into (1) on the basis of the identity in (2) is not permissible, since under the usual interpretation

(3) Oedipus wanted to marry his mother

differs in truth value from (1). There is in fact a reading of (3) on which it does follow from (1) and (2), but that it does not follow on all readings is enough to show that the interchangeability of coreferential noun phrases must be restricted in some way.

Tentatively, (1) and (2) may be assigned the following representations in *S:

(4) W(Oe;M(Oe;J))

(5) J=(℩x)(Mo(x;Oe))

The reading of (3) on which it follows from (1) and (2) may be called its referentially transparent reading. We will represent this as

(6) (∃y){y=(℩x)(Mo(x;Oe))&W(Oe;M(Oe;y))}

The normal reading of (3) is its referentially opaque reading. This can be represented as

(7) W(Oe;M(Oe;(℩x)(Mo(x;Oe))))

It will be observed that the definite description "(℩x)(Mo(x;Oe))" occurs inside of the predicate W (=want) in (7) and outside of it in (6). The importance of this distinction will emerge as we proceed. For the present, however, it should also be observed that (4) and (7) are identical in structure except that the individual constant J (=Jocasta) occurs in (4) where the definite description occurs in (7).

Intuitively, it would appear that (7) could be derived from (4) on the basis of the identity in (5). However, such a substitution would be illegitimate in *S. The rule of substitution presented in 2.3 allows us to interchange identical terms, but it does not allow us to interchange a term and a definite description. Thus, the failure of (3) to derive by substitution from (1) and (2) is reflected exactly in *S.

At this point, however, another question arises. It appears that there are many cases in which substitution of a definite description for a term does not have any untoward consequences, and ought therefore to be allowed. Consider, for example, the sentences

(8) Nixon is unscrupulous

(9) Nixon is the current President of the U.S.

(10) The current President of the U.S. is unscrupulous

It is intuitively obvious that (10) is a valid conclusion from (8) and (9), or for that matter, that (8) is a valid conclusion given (9) and (10). A formal semantic theory must certainly account for this. If the proper name <u>Nixon</u> and the definite description <u>the current President of the U.S.</u> were mutually substitutable, it would be an easy matter. But such an approach is not open in *S.

As it turns out, however, (8)-(10) represents a valid argument schema in *S in any case, and it is not necessary to revise the rule of substitution to account for it. Let us first provide a representation for the premises (8) and (9) and for the conclusion (10):

(11) $U(N)$

(12) $N=(\iota x)(CP(x))$

(13) $U(\iota x)(CP(x))$

That (13) follows from (11) and (12) will now be demonstrated:

(a)	$U(N)$	Hyp
(b)	$N=(\iota x)(CP(x))$	Hyp
(c)	$(\exists x)(CP(x)\&(y)(CP(y)\supset y=x)\&x=N)$	Df.9
(d)	$CP(a)\&(y)(CP(y)\supset y=a)\&a=N$	EI
(e)	$a=N;CP(a)\&(y)(CP(y)\supset y=a)$	R1, MP
(f)	$U(a)$	SUBS
(g)	$CP(a)\&(y)(CP(y)\supset y=a)\&U(a)$	R1, MP
(h)	$(\exists x)(CP(x)\&(y)(CP(y)\supset y=x)\&U(x))$	EG
(i)	$U(\iota x)(CP(x))$	Df.9

In a similar manner it can also be shown that (12) and (13) entail (11). Since this proof will be important for the purpose of comparison, it will also be presented in detail:

(a)	$U(\iota x)(CP(x))$	Hyp
(b)	$N=(\iota x)(CP(x))$	Hyp
(c)	$(\exists x)(CP(x)\&(y)(CP(y)\supset y=x)\&U(x))$	Df.9
(d)	$(\exists x)(CP(x)\&(y)(CP(y)\supset y=x)\&x=N)$	Df.9
(e)	$CP(a)\&(y)(CP(y)\supset y=a)\&U(a)$	EI
(f)	$CP(b)\&(y)(CP(y)\supset y=b)\&b=N$	EI
(g)	$(y)(CP(y)\supset y=a)$	R1, MP
(h)	$CP(b)\supset b=a$	UI
(i)	$CP(b);U(a)$	R1, MP
(j)	$b=a$	MP
(k)	$U(b)$	SUBS
(l)	$b=N$	R1, MP
(m)	$U(N)$	SUBS

The above results show that in certain cases terms and definite descriptions are effectively interchangeable after all. This is not a matter of direct substitution of one for the other, however. It remains to be seen now why even such indirect substitution fails in the case of (1)-(3).

Let us examine first whether (1) is entailed by (2) and (3) in the same way in which (12) and (13) entail (11). Assuming (5) and (7) as premises, we can proceed exactly as before to the conclusions

(e') $W(Oe;Mo(a;Oe)\&(y)(Mo(y;Oe)\supset y=a)\&Ma(Oe;a))$

(f') $Mo(b)\&(y)(Mo(y;Oe)\supset y=b)\&b=J$

It will not be possible to go on from here, however, to show that a=b, as we did in the previous proof. This would require us to produce the assertion

(g') $(y)(Mo(y;Oe)\supset y=a)$

But (g') does not follow from (e') in the way that (g) follows from (e). (e) has the general form $S_1\&S_2\&S_3$. It is clear that whenever we can assert such a conjunction, we can assert each of the conjuncts independently. By contrast, (e') has the form $F(i;S_1\&S_2\&S_3)$. In general, it is is not the case that an assertion of this form allows us automatically to assert one of the conjoined S's, or in fact, even to assert the simple conjunction of all of them. That is, it is not generally true that $F(i;S_1\&S_2\&S_3)$ entails S_1, or S_2, or S_3, or $S_1\&S_2\&S_3$. The appropriateness of this generality should be especially clear in the case of a verb like <u>want</u>, since it never has been true that a state of affairs which we happen to desire will automatically obtain.

We see, now, the significance of the fact that the definite description which purports to denote Oedipus' mother occurs inside the scope of the verb <u>want</u> in (7). The individual who meets this description has only a tenuous existence in the world of Oedipus own wishes. She cannot be identified with Jocasta, whose existence is an objective fact, independent of anything which Oedipus might think. The only connection between Jocasta and the mother of Oedipus' wishes is a verbal one. That is, under given circumstances they can both be described by the same definite description.

The last statement may appear to be paradoxical, in view of the fact that a definite description is by definition an expression which applies to one and only one individual. There is no paradox, however. Suppose that we present (7) in the alternative form

(14) $W(Oe;a=(\imath x)(Mo(x;Oe))\&Ma(Oe;a))$

In view of (5), and granted that the individual "a" cannot be identified with Jocasta, it would indeed be paradoxical if (14) were to give rise to the proposition

(15) $a=(\imath x)(Mo(x;Oe))$

However, (15) cannot follow from (14) for the very reasons which were discussed in the next to last paragraph. In other words, (7) and (14) do not assert that there is (or even that there could be) any such thing as a mother of Oedipus who is not identical to Jocasta. What they assert is only that the fulfillment of Oedipus' wishes would involve the existence of such a person, which is a case of hopeless wishing, to be sure, but not a contradiction.

Let us now examine the question of whether (3) is entailed by (1) and (2) in the manner in which (13) is entailed by (11). With (4) and (5) representing (1) and (2) and replacing (11) and (12) as premises, we can proceed exactly as in the former proof:

(a)	(4)	Hyp
(b)	(5)	Hyp
(c)	$(\exists x)(Mo(x;Oe)\&(y)(Mo(y;Oe)\supset y=x)\&x=J)$	Df.9
(d)	$Mo(a;Oe)\&(y)(Mo(y;Oe)\supset y=a)\&a=J$	EI
(e)	$a=J;Mo(a)\&(y)(Mo(y;Oe)\supset y=a)$	R1, MP
(f)	$W(Oe;Ma(Oe;a))$	SUBS
(g)	$Mo(a;Oe)\&(y)(Mo(y;Oe)\supset y=a)\&W(Oe;Ma(Oe;a))$	R1, MP
(h)	$(\exists x)\{Mo(x;Oe)\&(y)(Mo(y;Oe)\supset y=x)\&W(Oe;Ma(Oe;x))\}$	EG

As before, the next step should be that which introduces the definite description, in this case reducing (h) to (7). This step cannot be taken, however. It will be recalled that definite descriptions are defined only for the contexts in which denotators can appear in atomic formulas. Thus, the string "$W(a;M(a;(\imath x)(F(x))))$"

corresponds obligatorily to

(16) $W(a;(\exists x)(F(x)\&(y)(F(y)\supset y=x)\&Ma(a;x)))$

rather than to

(17) $(\exists x)(F(x)\&(y)(F(y)\supset y=x)\&W(a;Ma(a;x)))$

As it stands, there is no way in which (h) may be abbreviated by means of a definite description. Let us back up a step, however, to (g). Since we have the assertion a=J, we may derive by substitution

(18) $Mo(J;Oe)\&(y)(Mo(y;Oe)\supset y=J)\&W(Oe;Ma(Oe;J))$

Conjoining (18) with the assertion a=J and applying existential generalization we get

(19) $(\exists x)\{Mo(x;Oe)\&(y)(Mo(y;Oe)\supset y=x)\&x=a\ \&W(Oe;Ma(Oe;a))\}$

Definition 9 can apply to (19). As a consequence, we get

(20) $a=(\iota x)(Mo(x;Oe))\&W(Oe;Ma(Oe;a))$

Finally, applying existential generalization to (20) we obtain

(21) $(\exists y)\{y=(\iota x)(Mo(x;Oe))\&W(Oe;Ma(Oe;y))\}$

(21) is identical, of course, with (6), which represents the referentially transparent reading of (3). We started out to derive the opaque reading of (3) from (1) and (2), and discovered that this did not appear possible. Since the referentially transparent reading of (3) is that reading in which the noun phrase Oedipus' mother and the proper name Jocasta are supposed to be interchangeable in the first place, however, it should not be surprising that we could succeed in deriving the latter.

2.8. Existential Quantification in *S

One of the more obvious consequences of the fifth axiom of *S is the following theorem:

T24 $\vdash (\exists x)(f(x))$

The truthfulness of T24 may appear highly doubtful, however. Suppose that *S contains a predicate U, corresponding to the English unicorn. Then we would also have the theorem

T25 $\vdash (\exists x)(U(x))$

If existential quantification is rendered in the usual fashion as "there exists an x such that...", then T25 will certainly be false.

There is no compelling logical reason, however, for rendering existential quantification in this usual fashion. (Cf. Mates, 1952; p. 223). It is assumed that the universe of discourse over which the variables of *S range contains both existent and nonexistent things. Hence, existence must be asserted or denied of something by the use of a predicate. A formula will have what is commonly called "existential import" only if it explicitly predicates existence. If F is anything other than the predicate existent, the formula "$(\exists x)(F(x))$" will mean simply that at least one of the referents in the universe of discourse of *S is truly described by the predicate F, whether any such entity exists or not.

Under this interpretation, universal affirmative statements, i.e., statements of the form

(1) $(x)(A(x)\supset B(x))$

will come to have a stronger meaning than usual. Since the propositional function $A(x)$ will in general have nonexistent as well as existent things among its true substitution instances, (1) as a whole will be true only if it is true of all possible entities, not just of actual ones. If we wish to express the merely factual assertion that all existing A's are B's, then we must write

(2) $(x)(A(x)\&E(x)\supset B(x))$

As an illustration of the distinction between (1) and (2), consider the sentences

(3) All creatures with hearts are creatures with kidneys

(4) Triangles have three sides

As far as we can tell, there is no logical connection between having a heart and having kidneys, even though the co-existence of these organs may be necessitated on some biological or morphological basis. In any case, it is logically possible for there to be such a thing as a creature with a heart who has no kidneys, even if it biologically impossible. Thus, (3) must be interpreted as a purely factual assertion, and must be represented according to schema (2). On the other hand, there is more than a factual connection between being triangular and having three sides. It is inconceivable that the term triangle, granted that its meaning is preserved, could ever be applied to something which did not have three sides. Thus (4) expresses a logically necessary relationship between triangularity and three-sidedness, and it must be represented by means of schema (1).

Although the universe of discourse assumed for *S contains entities which are merely possible, such as unicorns, Santa Claus, Pegasus, and so on, it does not contain any logically impossible ones, such as two-sided triangles or round squares. All formulas of the form

(5) $-(\exists x)(F(x))$

are considered false, since it it assumed that all predicates of *S describe something. However, a formula of the form

(6) $-(\exists x)(F(x)\&G(x))$

can readily be true, if F and G are contradictory predicates, that is, if it is true that

(7) $(x)(F(x)\supset -G(x))$

Assuming that the predicates <u>round</u> and <u>square</u> are related as F and G in (7), then the assertion that nothing is both round and square would be represented according to (6).

Now consider the sentence

(8) There are no unicorns with two horns

Assuming that <u>with two horns</u> may be represented by the unanalyzed predicate W, we can represent (8) also in terms of the schema (6), more particularly, by

(9) $-(\exists i)(U(i)\&W(i))$

In view of the meaning of the terms involved, it is clear that (8) ought to be regarded as a non-factual assertion. That is to say, it ought not to possess any so-called "existential import." This is reflected by the absence of the predicate E in (9). On the other hand, the sentence

(10) There are no unicorns

clearly does possess existential import, of the negative variety. The representation of (10) in *S will therefore be

(11) $-(\exists i)(U(i)\&E(i))$

In standard systems, (8) and (10) would be assigned the representations

(12) $-(\exists x)(U(x)\&W(x))$

(13) $-(\exists x)(U(x))$

Now it is obvious that (12) is a logical consequence of (13). If

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(8) and (10) are assigned representations in this way, then, it is implied that the truth of (8) in its normal sense follows automatically from the truth of (10). But this is obviously not the case. In saying that there are no unicorns with two horns, one is making a claim about the use of the term unicorn, and the truth of this claim has nothing whatever to do with the existence of unicorns. Since (10) is a factual assertion and (8) is a non-factual one, there is no way in which the truth of one could depend upon that of the other. The representations assigned to (8) and (10) in *S assure that this is the case.

As a final example of the distinction between quantification in *S and in standard systems, consider the following. In standard systems, the sentence

(14) All unicorns have two horns on their heads

must count as true. This is the case even though the sentence

(15) There are no unicorns with two horns on their heads

must also be regarded as true. It is clear, however, that (14) and (15) will be regarded as contradictory in normal English. Let us examine therefore the relation of (14) and (15) in terms of their representations in *S.

We will assign to (14) and (15) the schematic representations

(16) $(i)(U(i)\supset H(i))$

and (17) $-(\exists i)(U(i)\&H(i))$

It is an easy matter to show that (16) entails the denial of (17):

(a)	$(i)(U(i)\supset H(i))$	Hyp
(b)	$(\exists x)(U(x))$	T25
(c)	$U(a)$	EI
(d)	$U(a)\supset H(a)$	UI
(e)	$H(a)$	MP
(f)	$U(a)\&H(a)$	R1, MP
(g)	$(\exists i)(U(i)\&H(i))$	EG

The contradictoriness of (14) and (15) corresponds, therefore, to the contradictoriness of their representations in *S. In this respect, *S provides a much more faithful translation of English sentences than standard systems do.

III. Adjectives and Verbs

3.0. Introduction

The majority of logicians and philosophers who have attempted to apply the predicate calculus to natural languages have ignored the distinction which English draws between adjectives and verbs. Thus, they treat both

(1) John is tall

and (2) John breathes

as instances of the single schema

(3) F(a)

This treatment is not universal. In his Principles of Mathematics (Russell, 1956, Ch. IV), Russell argues that (2) is to be taken as an instance of the schema

(4) aRb

According to this analysis, the verb breathe, unlike the adjective tall, does not appear as a constituent in semantic structure. Instead, it corresponds to a complex of semantic elements which performs a double duty--that of specifying some relatum and at the same time indicating the relation which it bears to the subject. Thus, (2) is viewed as having the same semantic structure as

(5) John performs the activity of breathing

In part, the thesis of this chapter is that Russell's analysis is preferable to the more traditional one. More generally, I will argue that verbs of all kinds--transitive and intransitive, active and stative--are distinct from adjectives in three respects: (1) unlike adjectives, verbs always incorporate the assertion of some relation; (2) unlike adjectives, verbs never correspond directly to semantically primitive predicates; and (3) unlike adjectives, all verbs except the semantically primitive verbs Have, Be, and Do always incorporate some variable.

Under the traditional analysis, both tall and breathes are considered to be irreducible terms on the semantic level, and to bear the same relation to their subjects--namely, the relation of predication. The fact that the copula occurs in (1) and not in (2) has been viewed as an idiosyncratic and totally insignificant fact about English syntax. With minor variations, usually only terminological, this doctrine has been propounded by generations of logicians. More recently, it seems to have been adopted by linguists in the generative semantics camp, who have of course introduced still further terminological modifications. Subject and predicate, argument and predicate, singular term and general term--all of these are expressions which have been used in one way or another to refer to the supposedly elementary constituents of sentences like (1) and (2) and of the schema (3). A fairly representative statement of the analysis under consideration is that of Geach, whose elementary constituents are called referring terms and predicables (Cf. Geach, 1968, Ch. 2).

The paradigmatic case of the referring term for Geach is the proper name. A name has the semantically primitive function of denoting that entity or entities which a proposition concerns. Predicables, on the other hand, do not refer to anything. Rather, says Geach, they are <u>true of</u> those things to which referring terms refer.

Now there is no doubt that we do sometimes speak loosely of predicates being true of their subjects. But if we were to be more scrupulous, we would note, as Strawson does, that the relation expressed by <u>true of</u> is more properly a relation which holds between a whole predication and its subject, rather than a relation which holds between a predicate and its subject. Thus, we would ordinarily say "it is true of Socrates that he is a philosopher" rather than "a philosopher is true of Socrates". (Cf. Strawson, 1963, pp. 155-56).

Even if we were inclined to accept the latter infelicitous usage, the fact would remain that <u>being true of something</u> is not characteristic of predicates alone, but applies equally well, or better, to propositions or sentences. Similar difficulties obtain if we attempt to replace the relation <u>true of</u> with something like <u>describes</u>, <u>characterizes</u>, <u>says something about</u>, etc. In fact, among linguistic constructions which are less than sentences, the first two of these expressions apply generally only to adjectives and adjective phrases, and not to verbs.

At this point one might well ask how the persistent invocation of the subject-predicate distinction is to be accounted for, if it is true that the notion of predicate itself is indefinable. As it turns out, the notion of predicate can be defined, but only in an essentially negative way. We do seem to have a firm notion of what constitutes a referring term. Granted that this is the case, we can arrive at predicates simply by cancelling out the referring terms of a sentence. Furthermore, it seems clear that sentences typically do two sorts of things--they refer to something and they say something about it. It seems natural, then, to say that what we have left of a sentence when we have gotten rid of the parts that refer to things is the part that says something about those things.

The genesis of the idea of grammatical predicates is probably very much like this. And the reasoning is not entirely incoherent. We may not know how to define the relation of predication in a non-circular fashion, but at least we can define it consistently: predication is that relation which holds between the predicating and non-predicating parts of a sentence.

It is legitimate enough, logically, to define things in this way. Moreover, such a definition even turns out to be useful (though not indispensible) in its own limited sphere, that is, when it comes to schematizing classical syllogistic arguments. Consider, for example, the following syllogisms:

(a) All people who are fat are overweight
John is fat
____
John is overweight

(b) All people who eat too much are overweight
John eats too much
____
John is overweight

If the distinction between the grammatical structure of the relative clauses who are fat and who eat too much is ignored, then both (a) and (b) can be justified in terms of the valid argument schema

$$(c)\quad \begin{array}{l} (x)(F(x)\&G(x)\supset H(x)) \\ F(a)\&G(a) \\ \hline H(a) \end{array}$$

One way of justifying (c) itself is to adopt a rule of inference which says that a predicate which is true of all individuals of such-and-such a description is true of any particular individual of that description. Formally, this can be expressed by saying that the inference from a statement of the form "(x)(F(x)&...)" to a statement of the form "(F(a)&...)" is always justified, regardless of the internal structure of the predicate "F".

A rule of inference formulated in the above fashion makes explicit reference to predicates. However, it is not necessary to formulate our rules of inference in this way. To justify the inferences in (a) and (b) above, we can adopt instead a rule which says that from a sentence of the form "$(x)\alpha$" it is always possible to infer a sentence "β", as long as the only difference between α and β is that x occurs free in α whenever some other referring term occurs free in β. When expressed in this way, our rule of inference appeals to the relation between a predication and a referring term, but it does not appeal to any relation between a predicate and a referring term. The reader will recall that the rule of R2 of *S is formulated in precisely the above fashion. Thus, as far as rules of inference are concerned, the notion of predicate in the above sense is dispensible in *S.

In this chapter, I will present several semantic arguments which tend to show that the notion of predicate which we have been considering is entirely too general to be very useful in semantic description, and that we must distinguish carefully between the relation which predicate adjectives bear to their subjects and the relation which verbs bear to theirs.

In the system *S, adjectives will correspond to primitive descriptive terms represented by predicate constants F, G, H, Fl, Bl, Flu, Tr, etc. These will appear only in structures of the form

(6) F(x); G(y); H(z); F(x;y); G(x;y), etc.

The relation which terms such as F, G, and H above bear to their subjects is expressed in English by the copula. For the sake of economy, and also to conform to standard practice, this relation is expressed in *S simply by juxtaposition. It is interesting to note that there are many natural languages which follow the same procedure. This fact has persuaded a number of linguists that the copula in English expresses no relation at all. But it by no means follows that this is true. Languages which have no overt copula express what English expresses in the same manner that the system *S does, by merely juxtaposing a predicate and its argument or arguments. In either case, the relation is there, regardless of the means chosen to express it.

It should be noted that predicate constants which take two or more arguments, while they are in a sense relational, do not by themselves assert relations. Adjectives like *fond of*, *aware of*, and so on, may be thought of as describing a relation, but no relation can be actually asserted until they are joined to various arguments by means of the verb *be*. Similarly, there are inherently relational nouns like *brother*, which characterizes a given relation, but does not by itself assert that this relation exists.

The job of asserting relations is characteristic of verbs. Active verbs, whether they are transitive or intransitive, assert the relation of an actor to the activity he or it performs or undergoes. In the system *S, therefore, sentences with active verbs will correspond to logical structures of the general form

(7) iDu&...u...

in which "(...u...)" is a condition specifying what kind of activity u is.

Stative verbs typically assert a relation between an individual and his affective states--his emotions, desires, thoughts, perceptions, and so on. It would probably be justified to introduce a special style of variable to refer to such states, but it will be tentatively assumed that they are denoted by individual variables. Thus, sentences with stative verbs will correspond to logical structures of the general form

(8) $iHj\&\ldots j\ldots$

where "$(\ldots j\ldots)$" specifies the nature of the affective state in question.

In sections 3.1 and 3.2 below, a detailed justification will be given for relating sentences with active verbs to semantic representations like (7) and sentences with stative verbs to representations like (8). For the present, let us consider the relation between representations such as (7) and (8) and more familiar syntactic representations.

Ignoring the problem of tense, the sentence

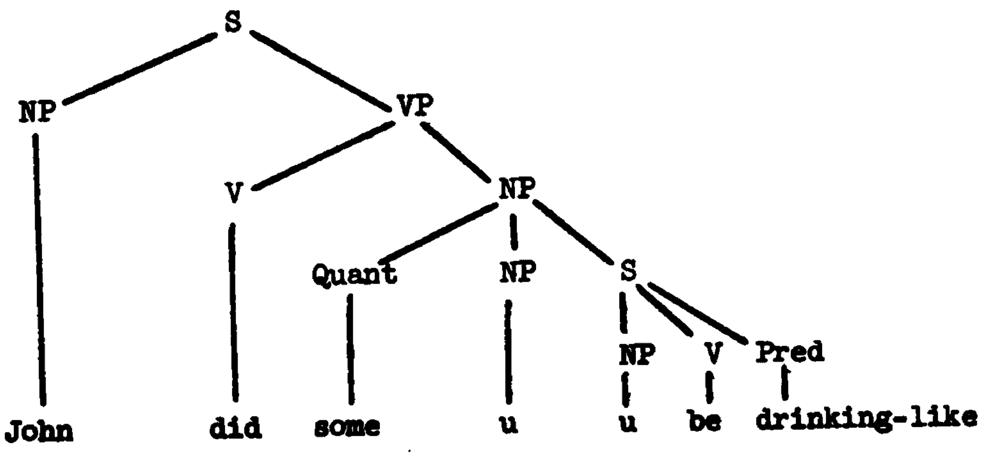
(9) John drank

would be assigned the following representation in *S:

(10) $(\exists u)(JDu\&Dr(u))$

The individual constant J in (10) corresponds directly to the proper noun <u>John</u> in (9). However, there is no constituent in (10) which corresponds to the verb <u>drank</u>. But it is possible to derive a branching diagram from (10) in which there is a constituent corresponding to the verb <u>drank</u>. Thus, (10) may be correlated in a regular way with the following syntactic representation:

(11)



The conversion of (10) to (11) is accomplished by picking out the string "Du&Dr(u)" as a verb phrase, by subordinating the sentence "Dr(u)" to the first occurrence of the activity variable "u" as a relative clause, and by attaching the existential quantifier to the noun phrase so formed. The verb drank corresponds directly to the constituent labeled VP in (11). Thus, it also corresponds to the string "(∃u)(...Du&Dr(u))" occurring in (10). That is to say, it corresponds to a semantic structure which incorporates the relation Do, a variable, and a descriptive predicate.

The predicate "Dr" occurring in (10) is, from a semantic point of view, an adjective, since its only function is to characterize a certain activity. Obviously, this predicate does not appear superficially in English as an adjective. In general, it will not be the case that every predicate of *S corresponds to a superficial English adjective. The converse of this will hold, however--that is, every adjective of English will correspond to a primitive predicate of *S.

The sentence "John drank" can be paraphrased by the sentence

(12) John did some drinking

Interestingly enough, (12) also has a direct correspondence with the representation (11), since the noun phrase some drinking corresponds to the NP complement of the verb Do. It appears, then, that (9) and (12) can be assigned the same underlying representation. The only difference between them is that in (9) the noun phrase some drinking and the relation Do are incorporated into a single lexical item. Thus, the semantic relation between the activity noun drinking and the verb drink can be explicated by noting that they incorporate the same descriptive predicate, and that they differ only insofar as the noun is used to denote an activity, while the verb is used to assert that it takes place. The same analysis will obviously apply to a great many other pairs of verbs and nouns in English.

The method whereby strings of the form "iDu&...u..." are related to syntactic structures of the form "i+active verb phrase" is indifferent to the analysis proposed here. In a generative semantics model, we could assume that there is a lexical insertion transformation which replaces a string like "...Du&...u..." with a lexical verb. Such a transformation would replace the VP in (11) with the verb drink. On the other hand, in a lexicalist model, we could assume that there is a rule of semantic interpretation which maps sentences like (9) and (12) into semantic representations like (10). In either case, there would still be a direct correspondence between active verbs and semantic structures of the form "...Du&...u..."

Consider now the sentence

(13) John hit George

This will be represented in *S by

(14) $(\exists u)(JDu\&H(u;G))$

In this case, the verb hit is to be identified with the string "$(\exists u)(\ldots Du\&H(u\ldots))$", with the term "G" left over as its complement. As before, the verb will correspond to a semantic structure which incorporates a relation, a variable, and a descriptive predicate. Again, the underlying predicate "H", which is adjectival in function, does not appear on the surface as an adjective. However, just as before, it can be incorporated into a nominal structure which does appear on the surface. Thus, we have the nominalization

(15) John's hitting of George was unforgivable

which can be represented in *S as follows:

(16) $Un(\imath u)(JDu\&H(u;G))$

Here, the nominal hitting of George corresponds to the structure

(17) $(\imath u)(\ldots u\&H(u;G))$

Thus, the difference between the verb phrase hit George and the noun phrase (the) hitting of George depends only on the fact that the former includes an assertion of the relation Do while the latter does not.

The analysis of stative verbs will be exactly parallel to the above, except that the relation which they incorporate will be Have instead of Do. It should be pointed out that the Have occurring in the analysis of stative verbs asserts a relation of inalienable possession, and is therefore not to be confused with the Have of sentences like the following, which asserts alienable possession:

(18) John has a dog

The sentence

(19) John has deep feelings

will be represented in *S by

(20) $(\exists_2 i)(JHi\&Fe(i)\&De(i))$

The Have+Complement structure in (20) gives rise to a stative verb, just as the Do+Complement structure in (10) gave rise to an active verb. Thus, (20) may be paraphrased by

(21) John feels deeply

If (18) were represented in a manner parallel to (20), i.e., by

(22) $(\exists i)(JHi\&i=(\iota j)(Do(j))$

then we could expect there to be a stative verb <u>dogs</u> such that (18) would be paraphrasable by

(23) *John dogs

The fact that there is no stative verb <u>dogs</u> in English indicates that the <u>Have</u> in (18) is very different from the <u>Have</u> in (19). In fact, the former <u>Have</u> is itself a derived stative verb, with the same sense as the stative verb <u>own</u>. The sentence

(24) John owns a dog

will be represented by

(25) $(\exists i)(JHi\&(\exists j)(j=(\iota k)(Do(k)))\&Ow(i;k))$

This corresponds also to the sentence

(26) John has ownership of a dog

Thus, (24), (25), and (26) are related in the same manner as (19), (20).and (21).

On the above analysis, the semantic distinction between various lexical verbs belonging to the same class will depend only upon the semantic content of the descriptive predicates which they incorporate. This formalizes an observation due to Weinreich that all active verbs have in common the assertion of some activity, and that their difference depends upon some "semantic residue" (Cf. Weinreich, 1966, p.423). The only verbs in *S which will not be factorable, so to speak, are the semantically primitive verbs

Have, Be, Do and *. Excluding the latter, which does not occur on surface in English anyway, these verbs form a natural class with respect to a large number of syntactic phenomena. It is highly interesting to note, therefore, that they also form a natural class in the system *S. A detailed examination of this point is beyond the scope of this study, however.

3.1. Active Verbs

In this section I will present several arguments which justify the procedure of deriving active verbs from structures of the general form

(1) iDu&...u...

The logical constant D corresponds to the superficial English verb Do. There are a number of rather compelling syntactic arguments for including this verb in the deep structure of English sentences with active verbs (Cf. Cantrall, 1969). These arguments, which I will not review here, offer additional support for the analysis I am suggesting. Since I am concerned primarily with the adequacy of semantic representations, however, and not that of syntactic representations, the arguments which I will present will be primarily semantic.

3.11.

It has been pointed out that sentences like

(1) John and Harry went to Cleveland

are ambiguous. This particular example is due to McCawley (McCawley, 1968, p. 152), who suggests that (1) is disambiguated by

(2) John and Harry each went to Cleveland

and (3) John and Harry went to Cleveland together.

These readings may be called the non-joint and joint readings, respectively. McCawley further suggests that these readings may be kept distinct by supplying the verb with indices. Thus

(4) $(\exists y)$(John go_y to Cleveland and Harry go_y to Cleveland)

expresses the joint reading of (1), while the non-joint reading is expressed by

(5) $(\exists x)(\exists y)$(John go_x to Cleveland and Harry go_y to Cleveland).

This procedure makes little sense, however, if we regard go as a predicate on a par with adjectives like red, fat, incorribible, etc. An index on an adjective would express, if anything, that it carried some special sense. Ambiguities like that of (1) also arise in sentences with predicate adjectives. Consider, for example, the sentence

(6) These two mountains are beautiful.

This can mean either that each of the two mountains in question is beautiful or that they are beautiful together. Denoting the two mountains as a and b, consider the absurdity of attempting to disambiguate (6) by way of

(7) $(\exists x)$(a is beautiful_x and b is beautiful_x)

and (8) $(\exists x)(\exists y)$(a is beautiful_x and b is beautiful_y)

If these are meaningful at all, they would have to mean that a and b are beautiful in the same sense or that they are beautiful in different senses. But this utterly fails to express the ambiguity of (6).

It is clear that the indices in (4) and (5) are not meant to express a difference in sense of the word go, but a difference in reference. The ambiguity of (1) depends upon whether going to Cleveland is taken to specify a single act or two independent acts.

This distinction can be directly expressed in the system *S. The non-joint reading of (1) is given by

(9) $(\exists u)(\exists v)(JDu\&HDv\&G(u)\&G(v)\&T(u;c)\&T(v;c))$

On this reading, the fact that two distinct acts of going to Cleveland are involved is expressed by representing the sentence with two separate activity variables. This is explicit, of course, only if we add to (9) the condition that $u \neq v$. However, this condition does not need to be written, since it is a consequence of the axiom that no single act can be performed by two distinct individuals.

The joint reading of (1) might be expressed, as a first approximation, by

(10) $(\exists u)(J\mathrm{D}u\&H\mathrm{D}u\&G(u)\&T(u;c))$

This corresponds directly to McCawley's analysis in (4). There are two reasons, however, for considering it unsatisfactory. First, as we have just noted, it does not appear to be sensible to say that two different individuals can separately perform a single act. But second, even if we did want to accept this possibility, the proposed solution would not be general enough, since it would fail to apply to cases like (6).

As an alternative to (10), therefore, consider the representation

(11) $(\exists u)((J\cup H)\mathrm{D}u\&G(u)\&T(u;c))$

As explained in chapter II, the notation 'J∪H' stands for the composite individual consisting of the two individuals 'J' and 'H'. (11) asserts that the act of going to Cleveland was performed by this composite individual. This representation is not open to the objections concerning (10). As required, (11) asserts the existence of only a single act, but it does not claim that this act was performed separately by two different individuals. Moreover, this kind of representation is general enough to apply to cases like (6). The joint reading of (6) can be given by

(12) $B(a\cup b)$

and the non-joint reading by

(13) $B(a)\&B(b)$

It is widely recognized that a grammar of English must distinguish two sources of conjoined NP. They may arise transformationally from conjoined sentences, or they may represent phrasal conjunction in the syntactic base (Lakoff and Peters, 1966). The separate readings of (1) and (6) correspond to these two possibilities. In the system *S, phrasal conjunction is represented as the union or the logical sum of two terms. On the other hand, sentential conjunction is represented in the usual manner by the truth-functional connective <u>and</u>. In surface structure, both kinds of conjunction are realized typically by *and*. Nevertheless, their logical functions are distinct, and this is pointed up in the system *S by the fact that they are represented by completely different symbols.

One further observation on the role of activity variables is in order here. As McCawley notes, (McCawley, 1968, p. 161), the nominalizations of the following sentence

(14) John and Harry departed for Cleveland

depend on whether it is taken in its joint or non-joint sense. In the former case, we can speak of

(15) John and Harry's departure for Cleveland

but in the latter case we must say something like

(16) John and Harry's departures for Cleveland.

These facts can be automatically accounted for on the hypothesis that the noun *departure* corresponds in reference to the activity variables in (9) and (11). In the non-joint sense, we get

(17) $(\iota u)(JDu\&D(u)\&T(u;c))$

and (18) $(\iota u)(HDV\&D(v)\&T(u;c))$

as expressions for John's departure and Harry's departure. In the joint sense, we get

(19) $(\iota u)(JUHDu\&D(u)\&T(u;c))$

The noun *departure* and the verb *depart* have a common semantic interpretation. What distinguishes them is not their meaning, but the way in which they enter into the structure of a sentence. In the system *S, the common meaning of *depart* and *departure*, and the common meaning of countless other pairs like it, is expressed by a single primitive descriptive term. When this term is taken together with a variable, it constitutes a noun, and when it is taken together with both a variable and a relation it constitutes a verb.

This way of accounting for things suggests that there is a potential noun or noun phrase lurking in the logical history of every verb. Such nouns and noun phrases can actually put in an an appearance in surface structure, and they do so with regularity. Consider the set of logical equivalences in (20a)-(24b):

(20a) Max rustled cattle in Nevada.
(21a) Homer never sang in his life.
(22a) On our trip, we fished and hunted a little.
(23a) We drank some, and then we just talked.
(24a) Sturdley and his wife argue constantly.

(20b) Max did some cattle rustling in Nevada.
(21b) Homer never did any singing in his life.
(22b) On our trip, we did a little fishing and hunting.
(23b) We did some drinking, and then we did nothing but talk.
(24b) Sturdley and his wife constantly engage in arguing.

The <u>somes</u>, <u>anys</u> and <u>a</u> <u>littles</u> which appear in (20b)-(24b) can be taken as reflexes of the existential quantifiers which also appear in the semantic representation of (20a)-(20b). The expression <u>engage</u> <u>in</u> in (24b) can be interpreted as a superficial variant of the logical constant D. It is not always possible to construct grammatically well-formed paraphrases in the manner of (20a)-(24b). As a general rule, however, structures of the form i + active verb always yield the conclusion that there is something which i does. This inference is validated by the ordinary rules of logic in the system *S, since from structures of the form $iDu\&F(u)$ it is always legitimate to infer $(\exists u)(iDu)$. If active verbs were assigned to the same syntactic class as adjectives, then this inference would no longer be justified.

There is a further complication in the representation of nominals like <u>John's</u> <u>departure</u> <u>for</u> <u>Cleveland</u>. In (17), (18) and (19) the fact that <u>departure</u> is a count noun was ignored. Taking this into account, they would have to be revised to

(25) $(\exists u)(JDu\&u=(\iota v)(D(v;c)))$

(26) $(\exists v)(HDv\&v=(\iota u)(D(v;c)))$

(27) $(\exists u)(JUHDu\&u=(\iota v)(D(v;c)))$

These revisions do not affect any of the arguments previously given in favor of representing active verbs by means of embedded activity variables. The joint and non-joint readings

(28) John and Harry departed for Cleveland

still arise, and it remains true that these readings cannot be diferentiated unless the verb <u>depart</u> is assumed to contain an activity variable. The joint and non-joint readings of (28) are represented by

(29) $(\exists u)(JUHDu\&j=(\imath v)(D(v;c)))$

and (30) $(\exists u)(JDu\&u=(\imath u_1)(D(u_1;c)))\&(\exists v)(HDv\&v=(\imath u_2)(D(u_2;c)))$

It also remains true that from (28), taken in either sense, we may infer that there is something which John and Harry did. Note that the latter also has its joint and non-joint readings, depending on whether it is a conclusion of (29) or (30).

In the case of the sentence

(20a) Max rustled cattle in Nevada

we were able to adduce the paraphrase

(20b) Max did some cattle rustling in Nevada.

On the other hand, we cannot paraphrase

(31) John departed for Cleveland

by (32) John did some departing for Cleveland.

The reason is that <u>cattle rustling</u> incorporates a mass-type predicate, while <u>depart</u> contains a count-type predicate, and the quantifier <u>some</u> appears only with the former. What does count as a legitimate paraphrase of (31) is

(33) John made a departure for Cleveland.

It is evident that <u>made</u> in sentence (33) performs the same function as <u>did</u> in sentences like (20b). That is, it merely expresses the relation between someone and an activity which he performs. We may therefore consider <u>make</u> as a stylistic variant of the abstract verb <u>do</u>, one that occurs idiosyncratically before activity nouns which are also count nouns.

3.12

There is a rather interesting distinction to be drawn in English between those activities which require a different object when they are repeated and those which do not. The grammatical and semantic relevance of this distinction is obvious in the following sentences:

(1) Lindbergh made three flights over a large body of water

(2) Moosberger committed three murders.

From (2), we can conclude that there were three people who Moosberger murdered. However, it does not necessarily follow from (1) that there were three large bodies of water which Lindbergh flew over. This conclusion is false if two or all of the flights in question were over the same body of water.

The same distinction appears in the following sentences:

(3) Nicklaus birdied 47 holes during the tournament

(4) Nicklaus ate 32 hot dogs during the tournament

Anyone who knows anything about golf knows that (3) does not imply that Nicklaus birdied 47 separate holes. If he knows something about arithmetic as well, he also knows that (3) entails that there was at least one hole which Nicklaus birdied three times. On the other hand, (4) would never imply that there was a hot dog which Nicklaus ate three times. The distinction obviously depends on the meaning of the verbs involved. If we birdie a hole, we do not in the process destroy it, and it stays around to be birdied again. On the other hand, although a cow can *ruminate* the same clump of grass several times, she can eat it only once.

Activities which destroy their objects obviously cannot be repeated again with respect to the same object. But the same thing goes for activities which *create* their objects. If John performed three acts of building a table, for example, then there are three tables which John built. In this respect, the verb *build* belongs in a class with the verbs *eat* and *murder*. The verbs of this class are opposed to those of another which includes *birdie* and *fly*.

Let us call the latter class *iterable* and the former class *non-iterable*. Since verbs differ only in terms of the predicates which they incorporate, a non-iterable verb is one which incorporates a non-iterable predicate. The notion of a non-iterable predicate can be formulated in *S as follows:

(5) f is non-iterable $=_{df.}$ $(x)(y)(u)(v)((f(u;x)\&f(u;y)\&{-}u{=}v)\supset{-}x{=}y)$

A predicate F is considered iterable if (5) does not hold of F.

If transitive verbs are treated as unanalyzable relations, as in the usual applications of the predicate calculus, then the distinction between iterable and non-iterable predicates cannot be formulated at all. Where f is a primitive two-place predicate, the analog of (5) would be

(6) f is non-iterable $\equiv$df. $(x)(y)(z)\{(f(z;x)\&f(z;y))\supset -x=y\}$

But (6), far from expressing the desired generality about non-iterable predicates, is demonstrably false. Holding x and z constant, (6) becomes

(7) f is non-iterable $\equiv$df. $(y)((f(a;b)\&f(a;y))\supset - b=y)$

Now suppose that y=b. The right-hand side of (7) becomes

(8) $(f(a;b)\&f(a;b))\supset - b=b$

which is the same thing as

(9) $f(a;b)\supset - b=b$

Since it is a logical truth that

(10) $(x)(x=x)$

(9) would be true only if there were no non-iterable predicates. Since there are in fact such things, (9) is false and so is (7). What comes to the same thing is that no such predicate as <u>non-iterable</u> is definable in the intended sense when verbs are treated as unanalyzable relations. The ability to define such a predicate depends upon the ability to define a verb as a composite of a relation, an activity variable and a descriptive predicate.

3.13.

Consider the ambiguity of the sentence

(1) John almost hit George

This can be disambiguated in the following way:

(2) John almost performed the act of hitting George

(3) John performed an act which almost amounted to hitting George.

If <u>almost</u> is taken as a sentential predicate, these two readings of (1) can be presented in *S as

(4) $A((\exists u)(JDu\&H(u;G)))$

and (5) $(\exists u)(JDu\&A(H(u;G)))$

Thus, the ambiguity of (1) depends upon a question of scope, its separate readings depending upon whether almost is construed as inside or outside the scope of the quantifier which binds the activity variable. If verbs like hit are taken as unanalyzed two-place predicates, then ambiguities of this sort cannot be explicated.

It should be pointed out that the readings (2) and (3) are not entirely independent. (2) has variable truth-conditions, one set of which are those expressed by (3) itself. If (4) and (5) are adequate semantic representations of (2) and (3), respectively, it must be the case that (5) logically implies (4). This result can be secured by adding a meaning postulate for the predicate almost, e.g.

(6) $iDu\&A(f(u))\supset A(iDu\&f(u))$

or some more general statement to this effect.

3.14.

In his Elements of Symbolic Logic, Reichenbach suggests that the sentence

(1) John drives slowly

is to be represented by

(2) $(\exists D_1)(D_1(J)\&D(D_1)\&S(D_1))$

The predicate D_1 is intended here to specify those properties which are characteristic of John-as-driver. These properties are in turn part of driving properties in general, which is indicated by the term $D(D_1)$. Finally, the properties which are characteristic of John's driving are asserted to be slow.

Note that it would not do to represent (1) simply by

(3) $D(J)\&S(D)$

since (3) would assert that all driving is slow, whereas we want to assert only that John's driving is. Reichenbach therefore rightly rejects (3) as a possible representation of (1).

There are a number of difficulties, however, with his alternative representation. (1) It is nonsense to consture driving as a property of John, or of anyone else. (2) It is even greater nonsense to construe slowness as a property of a property. (3) Existential quantification over the predicate D_1 makes it appear that the truth of (1) depends upon the existence of some predicate, but (1) could be true even if there were no such things as predicates. In this case, of course, we could not express the truth of (1), but it is no part of its meaning to assert the existence of some predicate. (4) What, if anything, is the difference in meaning between D and D_1, both of which are supposed to specify driving properties? Is D_1 identical in meaning with the D_2's and D_3's which we would need to specify Bill's driving and Fred's driving? If so, how can we tell on purely semantic grounds, that is, without knowing how Bill and Fred happen to drive? If not, then apparently we will need indefinitely many driving predicates, one for each possible driver.

All of these difficulties can be overcome in the system *S. The representation of (1) is

(4) $(\exists u)(JDu\&D(u)\&S(u))$

Under this analysis, slowness is attributed to an activity and not to another property. This activity is asserted to have a certain description, that which identifies it as an activity of driving. But the actor is not asserted to have any driving properties, or any kind of properties at all. Rather, he is asserted to have engaged in some sort of activity. Finally, the necessity of introducing an arbitrary predicate with a meaning arbitrarily different from that of driving is completely avoided.

3.2. Stative Verbs

The grammatical distinction between active and stative verbs is well documented. Active verbs occur freely in imperatives, but stative verbs do not. Active verbs undergo so-called do something replacement, whereas stative verbs do not. The latter distinction is illustrated in the sentences

(1) John hit George and I don't know why he did that.

(2) *Bill knew the answer, and I don't know why he did that.

According to the analysis of active verbs given here, the appearance of do in sentences like (1) could be explained as a matter of do-retention rather than do-replacement. The antecedent of that in (1) may be taken as the activity of hitting George. Thus, ignoring the logical structure of the clause I don't know why, (1) may be represented as follows:

(3) $(\exists u)(JDu\&H(u;G)\&$I don't know why $JDu)$

Under the general provision for deriving lexical verbs, the sequence $Du\&H(u;G)$ is replaced by the verb phrase "hit George." The final bound occurrence of the activity variable u, however, cannot give rise to a lexical verb, since it occurs without its descriptor. Instead, it is pronomialized to that and the relation do remains.

Since stative verbs like know do not behave in this fashion, it is clear that (2) could not be represented in the same manner as (1), that is, by

(4) $(\exists u)(BDu\&K(u;a)\&$ I don't know why $BDu)$

However, in this section I will present evidence that sentences with stative verbs must have semantic representations analogous to those required for sentences with active verbs. I will show that stative verbs as well as active verbs cannnot be regarded as unanalyzable predicates, and that they are essentially alike in that both incorporate a semantically primitive relation, a descriptive predicate, and a variable. The difference between them depends on two things: the variables which active verbs incorporate are activity variables, but those incorporated in stative verbs are individual variables; and while active verbs incorporate the primitive relation do, stative verbs incorporate the primitive relation have.

3.21.

All sentences with stative verbs will have, as a minimum, shared structures of the form

(1) $iHj\&\ldots j\ldots$

The arguments presented in the following sections will demonstrate only that sentences with stative verbs must have semantic structures of the form

(2) iRj&...j...

where the precise nature of the relation R is unspecified. However, that this relation is a relation of having is indicated by the fact that virtually every construction of the form NP + Stative Verb has a paraphrase of the form NP + Have + NP. Consider, for example, the following correspondences:

(3a)	John loves Shirley	(3b)	John has love for Shirley
(4a)	John admires Spiro	(4b)	John Has admiration for Spiro
(5a)	Max hates Elmira	(5b)	Max has a hatred for Elmira
(6a)	Frank likes franks	(6b)	Frank has a liking for franks
(7a)	Duke owns a dog	(7b)	Duke has ownership of a dog
(8a)	George needs a friend	(8b)	George has need of a friend
(9a)	That soup tastes funny	(9b)	That soup has a funny taste

The paraphrase relations in (3a)-(9b) can be accounted for automatically on the analysis suggested here. Note first that they are exactly parallel to those listed in section 3.11. for active verbs. In both cases, the noun phrases arise through the combination of a descriptive predicate and a variable. And in both cases, as a further step, verbs arise when these noun phrases are combined with relations.

The presence of the indefinite article in some of the above examples indicates that, as with active verbs, we must distinguish between those that are derived from count-type predicates and those that are derived from mass-type predicates. The latter occur in structures of the form

(10) iHj&F(j)

while the former are those which must occur in structures of the form

(11) iHj&j=(1k)(F(k))

3.22.

The distinction between iterable and non-iterable predicates noted in section 3.11. does not arise for stative verbs. This is to be expected, since it applies only to activities, and stative verbs do not express activites. However, there is a somewhat parallel distinction among stative verbs which is revealed in sentences like the following:

(1) John owned that house twice

(2) *John knew that fact twice

At first sight, it may appear that the distinction between the verbs <u>own</u> and <u>know</u> indicated in (1) and (2) is exactly the same as that between iterable and non-iterable activities. That is, it appears that ownership is a relation which can be repeated with respect to the same relatum, whereas knowledge is not. But this is not quite correct. It is possible, in fact, to know something on two or more separate occasions. Thus,

(3) John knew that fact on two separate occasions

is unexceptionable. It can mean that John knew a certain fact, forgot it, and then learned it again, or it can mean that John was capable of demonstrating knowledge of the given fact on two separate occasions, whether he happened to forget it in between or not being left out of the question. It is by no means clear how best to represent the latter sense of (3). Its former sense seems to be entirely straightforward, though, and it could be represented in a conventional notation by

$$(6) \quad (\exists t_1)(\exists t_2)(t_1 \neq t_2 \& K(J;a;t_1) \& K(J;a;t_2))$$

In this translation, <u>know</u> is treated as a three-place predicate, so that the expression $K(J;a;t_1)$ has the interpretation "John knows the fact a at time t_1." As a whole, then, (6) can be read "There are two separate times at which John knows the fact a."

Now it is evident that the quantifier <u>twice</u> in (2) cannot be interpreted in the same fashion as above, that is, as a quantifier over time variables. The deviance of (2) vis-a-vis the acceptability of (3) can be accounted for only if they have distinct semantic representations. Hence, we must find some alternative to (6).

Whatever alternative we choose, it must be one which makes available some variable for the quantifier <u>twice</u> to bind. But such a variable is provided automatically according to the analysis of stative verbs suggested above. Specifically, (2) can be represented by

$$(7) \quad (\exists_2 i)(JHi \& K(i;u))$$

and (1) can be represented in the same manner by

(8) $(\exists_2 i)(JHi \& O(i;b))$

We are now in a position to explain the difference in acceptability of (1) and (2). First, note that the sentence

(9) John owned that house

is represented by

(10) $(\exists i)(JHi \& O(i;b))$

But according to the pattern noted in the previous section, (10) is also a representation of

(11) John had ownership of that house

Consequently, (8) can also be interpreted as

(12) John had two ownerships of that house

This is perhaps only marginally acceptable, but the important thing is that it makes much better sense than

(13) *John had two knowledges of that fact.

It makes sense to talk about two different ownerships of the same piece of property, but it makes no sense to talk about two different knowledges of the same fact. Thus, we can say

(14) John's second ownership of that house was more propitious than his first

but we would not say

(15) John's first knowledge of that fact earned him more money than his second

The expression *first knowledge of something* does occur, for example, in sentences like

(16) My first knowledge of chemistry I acquired in high school

It is significant, however, that in such a context the expression means something like *the first bit of knowledge*; when we are speaking of knowledge of individual facts, which comes whole and not in bits and pieces, then we must refrain from speaking of first and second.

Why there should be such a distinction among verbs like *own* and *know* is not at all clear. Note that it does not depend upon the fact that *ownership* is a count noun and *knowledge* is a mass noun. The noun *belief* is a count noun, yet it behaves in this respect like *knowledge*:

(17) *John had two beliefs in that principle

There can be two different beliefs in the world, but there cannot be two different beliefs by the same person of the same thing. Belief of a given thing counts as the *same* belief whenever it occurs, even if it has been suspended and subsequently reaffirmed. The same is true of knowledge. On the other hand, when ownership has been lost and then reacquired, we speak of a new ownership.

The distinction can be captured in *S in the following way. Let us call relations like *know* and *believe* *determinable*. Then

(18) f is determinable $\equiv$df. $(i)(j)(z)\{(aHi\&aHj\&f(i;z)\&f(j;z))\supset i=j\}$

A predicate F is indeterminable just in case (18) does not hold of it. Now it will be readily seen that (2) is contradictory to the assumption that *know* is a determinable predicate as defined by (18). Hence, (2) can be approached in two different ways: (a) if *know* is taken in its normal sense, i.e., as determinable, then it makes a logically impossible claim; (b) if (2) makes a claim which is possibly true, then *know* must be interpreted in some abnormal way.

It was possible to specify in an intuitive way the difference between iterable and non-iterable verbs: the latter are those, namely, which either create or destroy their objects. A similar explanation of the essential difference between determinable and indeterminable relations does not appear possible. But a formal semantic theory ought, at least, to enable one to formulate the distinction. It is very difficult to see how this could be done if verbs like *know* and *own* are to be treated as unanalyzable n-ary predicates.

3.23.

An argument parallel to that of section 3.13. for active verbs can also be adduced for stative verbs. That is, there are occasional ambiguities in sentences with stative verbs which can be explicated only on the assumption that the verb contains at least two terms.

Compare the sentences

(1) Harry almost saw that

and (2) Harry almost owned that woman

The normal sense of (1) is that the relation of seeing between Harry and the object in question almost obtained. This could be represented without analyzing the verb <u>see</u> by means of

(3) $A(S(a;b))$

The normal sense of (2), on the other hand, cannot be captured in this way. (2) would normally be taken to mean that Harry's relationship to the woman in question almost amounted to ownership. This can be expressed only if the verb <u>own</u> is analyzed into two parts--an assertion of the existence of some relation and an assertion of the nature of that relation. In other words, it must be represented by something like the following:

(4) $(\exists i)(aHi\&A(O(i;b)))$

Now observe that (2) may also be interpreted in the manner of (1), that is, in such a way that <u>almost</u> includes the entire predication in its scope. When understood in this way, no relation between Harry and the woman is actually asserted to exist. This reading might arise, for example, if it were understood that the woman was a slave and Harry was a slaveowner, and that he would have purchased her if he had had the chance. This is represented by

(5) $A(\exists i)(aHi\&O(i;b))$

Similar ambiguities of scope arise with other sentential predicates. Consider, for example

(6) It is possible for Sam to recognize Sue

(7) It is possible that Sam recognizes Sue

These two sentences have virtually identical deep structures. The essential difference has been thought to reside in the aspect of the verb. But there is a further distinction which is made clear by a consideration of the circumstances under which each would be appropriate. (6) could be followed naturally by something like "if he would only look in her direction," whereas (7) could not. On the other hand, (7) could be followed by something like "That must be why he is staring at her," while (6) could not. This difference depends upon the fact that (7) presupposes the existence of some relationship between Sam and Sue--for example, that he is now looking at her--while (6) does not. Hence, (6) and (7) can be given the representations

(8) $P(\exists i)(SHi\&R(i;Su))$

and (9) $(\exists i)(SHi\&P(R(i;Su)))$

In (8), the existential quantifier is inside the scope of the modal <u>possible</u>. Hence, (8) does not assert the existence of any actual relationship. It merely asserts that the relationship of recognition between Sam and Sue, in that order, is possible, or that it could occur. In (9) the quantifier is outside the scope of <u>possible</u>. (9) therefore asserts that there is some actual relation between Sam and Sue, and that it is possibly a relation of recognition.

3.3. <u>Active Adjectives</u>

In an article entitled <u>Stative Adjectives and Verbs in English</u> (Lakoff, 1966), Lakoff cites evidence that the feature of stativity cross classifies adjectives and verbs. Just as there are stative and non-stative (or active) verbs, so, Lakoff concludes, there are stative and non stative adjectives. This cross classification Lakoff adduces as evidence that adjectives and verbs should be analyzed as belonging to a single syntactic category.

The non-stativity of an adjective is revealed, Lakoff argues, by essentially three syntactic tests. Like active verbs, active adjectives occur in imperatives:

(1) Close the door!

(2) Be polite!

They also permit the progressive:

(3) John is closing the door

(4) John is being polite

Finally, they also appear to allow do-something replacement:

(5) What John did to please me was close the door

(6) What John did to please me was be polite

These phenomena are crucial for the theory that I have maintained. I have claimed that active verbs may be recognized as a distinct class, semantically, by virtue the fact that they contain an implicit assertion of the relation "Do". In particular, I have claimed that from any sentence of the form i+active verb it is possible to conclude that i does something. But this is merely a semantic statement of what is involved in the syntactic rule of do-something replacement. If it is true that some adjectives also undergo this rule, it follows that the distinction I have drawn between adjectives and verbs does not always hold, and that some adjectives also incorporate an assertion of the realtion "Do". The same conclusion would seem to follow from the occurrence of some adjectives in imperatives since intuitively it makes sense to give an order to someone only if that person is expected to do something. Finally, a progressive verb form makes sense only if we have to do with an activity which can be conceived of as stretched out over a span of time.

Clearly, then, if our theory is to be maintained, the phenomena in (1)-(6) must be considered very carefully. Let us begin with a closer examination of do-something replacement.

In the first place, it must be pointed out that the do-something replacement test and the progressive test are at odds with each other, at least when the former applies in contexts like that of (5) and (6). Thus, locatives such as in church and at home must be classified as stative according to the progressive test, in view of the absence of sentences like the following:

(7) *John is being in church every day for a month

(8) *My wife is being at home every day when I return

However, these same locatives would have to be classified as active in view of the acceptability of

(9) What John did to please his pastor was be in church
every day for a month

(10) What my wife does to please me is be at home every day when I return

Otherwise-stative verbs also occur in a context like that of (5) and (6). Thus

(11) *John is believing in God

but (12) What John did to please God was believe in him

Finally, otherwise-stative adjectives also seem to behave in this way:

(13) *John was being rich and famous by the time he was 21

(14) What John did to please his mother was be rich and famous by the time he was 21.

The context of (5) and (6) is a very special one. Do-something replacement, or something very much like it, also applies in a number of other contexts, as illustrated by the following pairs:

(15) Jesse robs banks, but Frank would never rob banks
(16) Jesse robs banks, but Frank would never do that

(17) Westmoreland kills people for a living. Can you imagine killing people for a living?

(18) Westmoreland kills people for a living. Can you imagine doing that for a living?

(19) Max killed a pig, but why he killed a pig I'll never know

(20) Max killed a pig, but why he did it I'll never know

(21) Tom greets everyone in a supercilious manner, but Bill greets everyone graciously

(22) Tom greets everyone in a supercilious manner, but Bill does it graciously

If Lakoff's conclusions were correct, then one would predict that non-stative adjectives would function just like the non-stative verbs in (15)-(22). This prediction is not fulfilled, however:

(23) Jesse is polite, but Frank would never be polite
(24) *Jesse is polite, but Frank would never do that

(25) Westmoreland is professionally merciless. Can you imagine being professionally merciless?

(26) *Westmoreland is professionally merciless. Can you imagine doing that professionally?

(27) Max was polite with that pushy salesman, but why he was polite with him I'll never know

(28) *Max was polite to that pushy salesman, but why he did

it I'll never know

(29) Tom is openly hostile, while Bill is inwardly hostile
(30) *Tom is openly hostile, while Bill does it inwardly

Note that the adjectives which appeared to undergo do-something replacement in (14) also do not behave in this way:

(31) John was rich and famous at 21, but Bill was not rich and famous at 21

(32) *John was rich and famous at 21, but Bill did not do that

The above are clear counter-examples to Lakoff's claim that active adjectives behave like active verbs with respect to do-something replacement. The only example which Lakoff provides in which this is true is an example in which stative adjectives also behave in the manner of active verbs. We are properly suspicious, therefore, of the efficacy of the do-something replacement test in detecting non-stative adjectives. Still to be explained, of course, is the fact that any adjective at all can occur in frames like (5) and (6), while they cannot appear in frames like those of (23)-(32). Below, we will see that this can be traced to the fact that the logical structure of frames like (5) and (6) is entirely different from that of frames like (23)-(32). We turn our attention now to an examination of the progressive and imperative tests.

Unfortunately for Lakoff's position, these tests are also at odds with each other. Witness the following:

(33) Joe is bleeding from the mouth
(34) *Bleed from the mouth

(35) Sturdley is sweating profusely
(36) *Sweat profusely

(37) Max's eyes are tearing
(38) *Tear

(39) George is seething with anger
(40) *Seethe with anger

If we decide that the progressive test is to be our standard, then bleed, sweat, tear, and seethe must be taken as active verbs. On the other hand, if the imperative test is our standard, they must be regarded as stative.

There is a legitimate sense in which all of the verbs in

(33)-(40) express activities. All of them are biological activities which one suffers rather than activities which one performs. In this respect they are like the activities into which inanimate objects enter. Inanimate objects are presumptively involuntary in their behavior. Thus, what the moon *does* when it revolves around the Earth, or what snow *does* when it melts, is not a performance, but merely a happenstance.

Orders are sensible only if the act which is ordered is an act which can be done voluntarily. One cannot order a person to bleed or to sweat for the same reason that one cannot order snow to melt. Addressing the order "sweat profusely" to a person is as infelicitous as addressing the order "revolve around the Earth" to the moon, since in neither case could there be any true compliance.

The non-stativity of a predicate is therefore not a sufficient condition for its appearance in imperatives. Nor, as the following examples will show, is it a necessary condition.

As we have already noted, the progressive form of the verb *be* does not ordinarily occur before locatives. Thus, the rules of English do not allow sentences such as

(44) *Big John is being in Laredo

(42) *The troll is being under the bridge

This fact is compatible with the conclusion reached by Lakoff in *On the Nature of Syntactic Regularity* (Lakoff, 1965. p. IX-15), namely that prepositions like *in* and *under*, at least in uses like the above, belong to the class of stative adjectives. However, this conclusion is itself incompatible with the quite normal imperatives

(43) Don't be in Laredo when the sun comes up

(44) Be under the bridge when I get back

Similarly, there are adjectives which do not take progressives, but occur quite readily in imperatives:

(45) *John is being ready for anything

(46) *John is being able to answer the questions on page 2.

(47) Be ready for anything, men!

(48) Be able to answer the questions on page 2.

Now being in Laredo or under a bridge, or being ready or able, are clearly states and not activities. But it makes sense to order someone to be in these states, since being in them is subject to volition. In other words, it is possible for someone to do something voluntarily to bring these states about. Thus, (43), (44), (47), and (48) can be paraphrased as follows:

(49) Bring it about that you are not in Laredo when the sun comes up

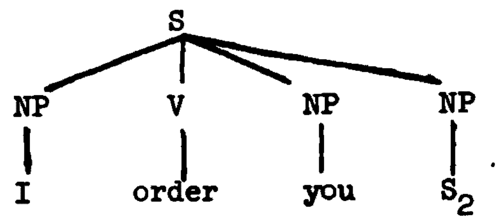
(50) Bring it about that you are under the bridge when I return

(51) Bring it about that you are ready for anything, men

(52) Bring it about that you are able to answer the questions on page 2

In his performative analysis of imperatives, Ross (Ross, 1968) argues convincingly that an imperative sentence, S_2, is derived transformationally from a structure like that of (53):

(53)



Adopting this analysis for our present purposes, and letting "!" represent the performative part of an imperative and "Y" represent the second person pronoun, (49)-(52) will have the schematic representation

(54) $!(\exists u)(YDu\&Ca(u;S_2))$

where "$Ca(u;S_2)$" means that the activity u is causally related to the state of affairs predicated in S_2.

Note that the non-occurrence of imperatives of the type

(55) *Be tall

(56) *Be six years old

parallels exactly the non-occurrence of sentences like

(57) *Bring it about that you are tall

(58) *Bring it about that you are six years old

The deviance of (57) and (58) has nothing to do with the stativity of the predicates they involve, but only with the fact that <u>being tall</u> and <u>being six years old</u> are not states which one can voluntarily acquire. The deviance of (55) and (56) can be explained naturally on the assumption that their semantic representations are essentially the same as those of (57) and (58), i.e., that all of these sentences have semantic representations corresponding generally to (54).

The above analysis of imperatives can be extended to a variety of cases. Consider the following examples:

(59) Know this theorem by Tuesday

(60) Have your own I.C.B.M. first-strike capability

(61) Be a member of the Mickey Mouse Club

Each of these examples can be paraphrased in the manner of (49)-(52). In each case, fulfillment of the order, request, admonition, or whatever, which is expressed requires some sort of activity. It does not follow, however, that knowing a theorem, owning an I.C.B.M. first-strike capability, or being a member of the Mickey Mouse Club are themselves activities.

On this analysis, we are not forced to revise our original estimate of adjectives to account for their occurrence in imperatives of the above type. We can still maintain that they correspond to semantically elementary terms, and that they do not, like active verbs, incorporate the relation "Do". In general, the fact BE+Adj imperatives involve activities is a feature of the logical structure of such imperatives as a whole, and it has nothing to do with the adjectives which they contain.

The above analysis does not apply, however, to all BE+Adj imperatives. It is clear that

(62) Bring it about that you are polite

and (63) Bring it about that you are not coy

are not happy paraphrases of

(64) Be polite

and (65) Don't be coy

An alternative is suggested by the following fact. Virtually every adjective which can occur with the progressive form of <u>be</u> and can be predicated of individuals can also be predicated of the <u>behavior</u> of individuals. The following are just a few such cases:

(66) Pollyanna is being unduly optimistic

(67) Pollyanna's behavior is unduly optimistic

(68) Hoppity Hooper is being manic-depressive again
(69) Hoppity Hooper's behavior is manic depressive

(70) Agnew is being megalo-maniacal again
(71) Agnew's behavior is megalo-maniacal

(72) Nixon is being incoherent again
(73) Nixon's behavior is incoherent

(74) Alley Oop is being troglodytic again
(75) Alley Oop's behavior is troglodytic

This is certainly no accident. The noun behavior denotes a generalized activity. It appears, then, that we have a very natural way of characterizing the set of non-stative adjectives. They are, namely, those which can be predicated of activities as well as of individuals. The class of stative adjectives, on the other hand, is just that class of adjectives which are not predicable of activities. Note that we do not get anything like the following:

(76) *Tom's behavior is fat

(77) *John's behavior is tall

(78) *George's behavior is blue

(79) *Bill's conduct is thin

(80) *Arisotle acts greekly

etc.

Granted that

(81) John is being polite

and (82) Be polite

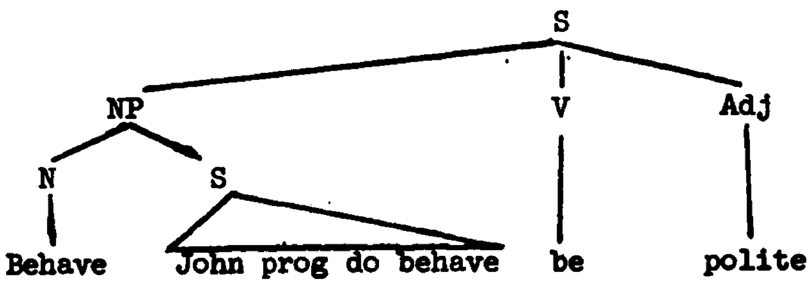
can be paraphrased as

(83) John is behaving politely

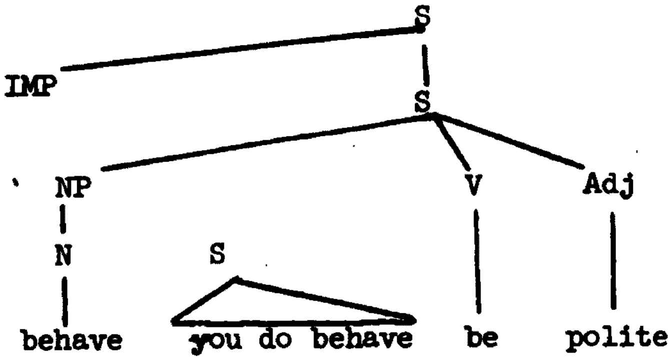
and (84) Behave politely

the possibility arises that (81) and (82) are derived from proximate structures like those of (83) and (84), respectively. Let us represent (83) and (84) by means of the following somewhat simplified tree-structures:

(85)



(86)



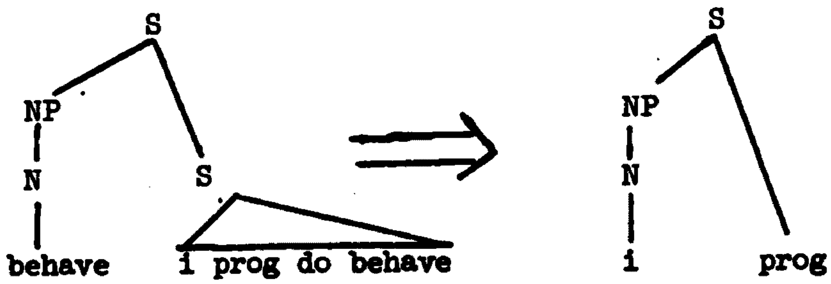
These trees have essentially the same semantic content as the following formulas of *S, in which, however, the feature of tense and aspect have been ignored:

(87) $(\exists u)(Beh(u) \& JDu \& Po(u))$

(88) $!(\exists u)(Beh(u) \& JDu \& Po(u))$

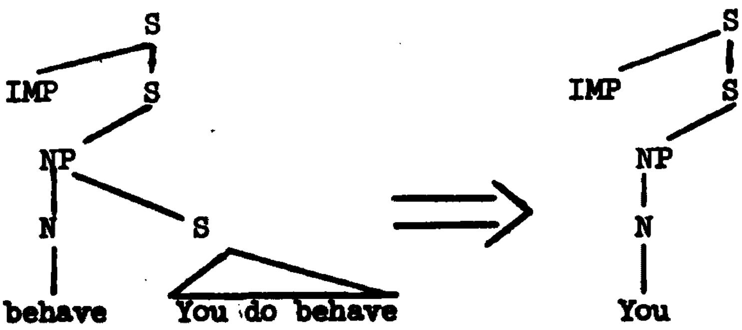
To account now for structures like that of (81) and (82), we can posit a transformation which we will call <u>Behavior Deletion</u>. This will function in the following manner:

(89)



and

(90)



Independent justification for a transformation like the above is provided by sentence pairs like the following:

(91a) Shakespeare will be virtually unreadable in 1994
(91b) The works of Shakespeare will be virtually unreadable in 1994

(92a) Russia did not attend the meeting
(92b) Representatives of Russia did not attend the meeting

(93a) The cabinet of Minister Chomunley was very frugal
(93b) The members of the cabinet of Minister Chomunley were very frugal

(94a) The court of Louis the Bald was very impious
(94b) The people associated with the court of Louis the Bald were very impious

(95a) _Playboy_ gets more ribald every month
(95b) The articles, photographs, cartoons, etc. appearing in _Playboy_ get more ribald every month

In each of the above pairs, the (a) sentence is related to the (b) sentence by what is known in classical rhetoric as metonymy. Viewed as a process, metonymy consists of transferring a predication on one noun to another which is closely associated with the first. Thus, what appears superficially to be predicated of the magazine _Playboy_ in (95a) is in a deeper sense predicated of the material which appears in it.

The relation between the sentence pairs

(96) John is being polite

and (97) John is behaving politely

may also be considered one of metonymy. Thus, the transformation which we have posited to relate sentences like (96) and (97) does not apply merely in isolated cases, but formalizes a fairly productive process of English.

Now let us examine the theoretical consequences of the above analysis. If sentences like (96) are derived from syntactic representations like that which underlies (97), then the occurrence of the progressive in such sentences will be in no way out of the ordinary, since we may expect the progressive to occur with a verb like _behave_. The felicity of imperatives on the order of _be polite_ will be accounted for on the same grounds. None of this will force us to conclude that adjectives like _polite_ predicate activities of their subjects. The fact that we may conclude that

John is doing something from the sentence *John is being polite* will follow from the general analysis of such progressives, and it will have no bearing on the logical structure of adjectives. Similarly, the fact that some activity is required to fulfill the order *be polite* will issue from the analysis of such imperatives, and it will not require us to conclude that the adjective *polite*, like the verb *eat*, contains an implicit reference to some activity. If we must single out some element on which to pin the "activeness" of sentences like *John is being polite* and *be polite*, it will have to be the verb *be* itself. From a logical point of view, it is accidental that the progressive and the imperative *be* are related phonologically to the ordinary copula occurring in sentences like *John is tall*.

The sentence

(98) John is polite

will be represented in *S in the same way that other sentences of the form NP+BE+ADJ have been represented, viz.

(99) $Po(J)$

The rules of *S will allow us to conclude from (99) that politeness is a property of John, i.e.,

(100) $JH(\iota p)(y)(yHp \equiv Po(y))$

It is significant that the imperative *be polite* cannot be interpreted as an order to possess a property, or to acquire one, or that the sentence *John is being polite* cannot be interpreted to mean that John is possessing some property. These facts follow naturally from the analysis given above, according to which *polite* is predicated of some activity in progressives and imperatives, and not of an individual. These facts also lend considerable support, however, to the view that a sentence like (98) has a logical structure entirely different from those of (81) and (82), in spite of their superficial resemblance.

Purely syntactic evidence for the same conclusion comes from the following considerations. In the examples of (15)-(22) and (23)-(30) we observed that adjectives and verbs did not behave in the same fashion with respect to do-something replacement. To the above examples we can add the below:

(101) What did Sam do? He skinned a snake
(102) *What did Sam do? He was petulant

(103) What does Max do? He buys cheaply and sells dear
(104) *What does Max do? He is optimistic

In these examples, however, and in (23)-(30), the adjectives appear only in non-progressive constructions. When they appear in progressives, adjectives frequently behave just like active verbs:

(105) What is Max doing? He is dancing the frug
(106) What is Max doing? He is being careful not to step on the cracks

(107) What are the neighbors doing? They're fighting again
(108) What are the neighbors doing? They're being noisy just to annoy us

(109) What is Mary doing to that cat? She's fondling it
(110) What is Mary doing to that cat? She's being affectionate with it

Be+Adj imperatives manifest a similar behavior, insofar as they can occur as appropriate responses to questions like "What should I do":

(111) What should I do when the tax collector comes? Be polite

(112) What should I do when they start shooting? Be brave

(113) What should I do if the sky falls in? Be optimistic

This difference in the behavior of BE+Adj imperatives and NP+Be+Being +Adj constructions on the one hand and NP+Be+Adj constructions on the other can be explained very naturally, and is in fact predicted, by the analysis which I have proposed.

One final bit of evidence. At first glance, it may appear that the relation between the two sentences

(114) John is polite

and (115) John behaves politely

is exactly parallel to the relation between the sentences

(116) John is being polite

(117) John is behaving politely

and

(118) Be polite

(119) Behave politely

Thus, it would appear that there is a good prima facie case for deriving (114) from a structure like that of (115), in the same way as (116) and (118) are derived from (117) and (119). This parallelism is destroyed, however, by the fact that (114) and (115), unlike the other two pairs, are not true paraphrases. Consider the sentence

(120) John was born polite

This does not mean that John behaved politely during his birth, or that he began to behave politely immediately afterward. If there is any connection at all with behavior it is something like this: John manifested politeness so early in his behavior that one must conclude that he was congenitally predisposed to behave politely. Thus, (120) may be paraphrased by

(121) John was predisposed to behave politely from birth

What this example shows, crucially, is that there is a distinction to be drawn between a personal characteristic which disposes one to behave in a certain fashion and the behavior itself. It appears, therefore, that (122) below is a better paraphrase of (114) than (115) is:

(122) John is predisposed to behave politely

On the other hand, (116) and (118) cannot be paraphrased in this manner:

(123) *John is being predisposed to behave politely

(124) *Be predisposed to behave politely

Thus, it appears that (114) is a proposition of an entirely different sort than (116) and (118). The former attributes a property to an individual, but the latter do not. Conversely, the latter are concerned with activities, while the former is not. In short, (114) has a stative interpretation, while (116) and (118) have active interpretations.

It remains now to explain a fact noted near the beginning of this section, that active adjectives undergo do-something replacement in contexts like the following:

(125) What John does to please his mother is be polite

When sentences of this sort were first considered, it was suggested that the unusual occurrence of do-something replacement could be traced to the fact that they had unusual logical structures. We are now prepared to expand on this suggestion.

It is generally thought that a sentence such as (125) is derived transformationally by a process know as pseudo-clefting from a structure like that of

(126) John is polite (in order) to please his mother

The details of this process are not important here. What is important is that it involves do-something replacement in some cases, and not in others. In particular, sentences with active verbs, like the following:

(127) John eats persimmons

(128) John blows everybody's mind

are converted by pseudo-clefting to

(129) What John does is eat persimmons

(130) What John does is blow everybody's minds

while sentences with adjectives, such as

(131) John is tall

(132) John is polite

are converted to

(133) What John is is tall

(134) What John is is polite

At the same time, (126) is converted to (125), and not to

(135) *What John is to please his mother is polite

Superficially, the only difference between (126) and (132) is the presence of a purpose adverb in the former. But this difference alone would not appear sufficient to account for the fact that pseudo-clefting introduces <u>be</u> in (132) and <u>do</u> in (126).

A very natural explanation of this fact can be provided, however, on the basis of the analysis presented in this section. Above, we noted that (132) could be paraphrased in the following ways:

(136) John has the property of being polite

(137) John is predisposed to behave politely

while the sentence

(138) John is being polite

could not be so paraphrased:

(139) *John is having the property of being polite

(140) *John is being predisposed to behave politely

In this respect, (126) behaves like (138) rather than like (132):

(141) *(In order) to please his mother, John has the property of being polite

(142) *(In order) to please his mother, John is predisposed to behave politely

It seems warranted to conclude, therefore, that (126) must have an underlying structure similar to that of (138). But as we have seen, (138) is to be derived by the transformation of Behavior Deletion from a structure corresponding to that of

(143) John is behaving politely

Accordingly, the immediate source of (126) would be a structure like that of

(144) John behaves politely (in order) to please his mother

If Behavior Deletion applies immediately to (144), then (126) will result. Suppose, however, that pseudo-cleft formation applies first. Since (144) contains an active verb, this will involve do-something replacement, and the result will be

(145) What John does (in order) to please his mother is behave politely

If Behavior Deletion applies at this point, we will derive

(146) What John does (in order) to please his mother is be polite

which is, of course, the correct result. The absence of the sentence

(147) *What John is (in order) to please his mother is polite

can likewise be fully explicated. To produce (147), it would be necessary to apply pseudo-cleft formation to the sentence

(148) John is polite (in order) to please his mother

But if, as we have assumed, (148) must have the underlying structure of (144), and if Behavior Deletion never applies before pseudo-cleft formation, (147) could never be produced.

3.4. Conclusions

On the basis of the analysis presented in this chapter, the following general conclusions emerge:

(1) Sentences with active main verbs share a minimum logical structure of the form

(a) iDu&...u...

while those with stative main verbs have the structure

(b) ihj&...j...

The conditions "...u..." and "...j..." are expressed in terms of primitive descriptive predicates. Thus, both active and stative verbs have semantically complex structures which include a variable and a descriptive predicate.

(2) Adjectives correspond directly to primitive descriptive predicates. Non-stative adjectives correspond to predicates which are predicable of activities, while stative adjectives correspond to predicates which are not.

(3) The grammatical distinction between adjectives and verbs reflects a corresponding distinction in their semantic structure. Since adjectives and verbs do not bear the same semantic relationship to their subjects, there is no justification for introducing them into semantically interpreted structure under a common node.

IV. Generic Noun Phrases

4.0. Introduction

In Chapter I, we alluded to a suggestion of Bach's according to which all English noun phrases are to be viewed as having logical representations of the form "(Qx)(f(x))", where "(Qx)" is either a quantifier or an operator, and "f" is some predicate. As we have seen, a variety of English noun phrases can be successfully analyzed in this manner. However, it is not at all obvious how this analysis can be applied to generic noun phrases. Consider, for example, the sentence

(1) Graduate students are poor

The surface structure of (1) offers no clue as to what quantifier or operator, if any, must be present in its logical representation. It is clear that the reference of the noun phrase graduate students cannot be correctly represented by universal quantification over the entire set of graduate students, i.e., the truth conditions for (1) are not the same as those for

(2) $(x)((x=(\iota y)(Gra(y)))\supset P(x))$

The existence of an occasional rich graduate student is not taken as a disconfirmation of (1), but (2) is false if there is so much as a single rich graduate student. In some sense, the reference of the noun phrase in (1) is restricted to those graduate students whom the assertor wishes to regard as "typical." But how is this restriction to be indicated formally? In his article Nouns and Noun Phrases, Bach speaks of the necessity of positing a "generic quantifier" for the analysis of English, but he offers no concrete proposals (cf. Bach, 1968, p. 106).

The analysis of generic noun phrases is complicated by the fact that there are several different kinds. In Chapter II, we examined one particular kind--namely, those exemplified in the following sentences:

(2) The automobile was invented in 1889

(3) Dinosaurs are extinct

(4) The dandelion is my favorite weed

If we take "invented in 1889" and "my favorite weed" as well as "extinct" as unanalyzed predicates, then according to the analysis offered in Chapter II, (2)-(4) express propositions of the general form

(5) $G(\gamma_f)$

which is in turn an abbreviation of the schema

(6) $G((\imath x)(f(x)\&(y)(f(y)\supset y\leq x)))$

Thus, the generic noun phrases in (2)-(4) correspond to logical structures of the form "(Qx)(...x...)", where "(Qx)" is the definite description operator "$(\imath x)$". This conforms fairly closely with Bach's proposal. Unfortunately, sentences like (1) are very different from sentences like (2)-(4), and the same analysis will not apply to both. The essential distinction can be made apparent in the following way. As a more or less accurate paraphrase of (1) we can have

(7) All typical graduate students are poor

However, the attempt to paraphrase (2)-(4) in the same manner changes their meanings completely, or produces nonsense:

(8) All typical automobiles were invented in 1889

(9) All typical dinosaurs are extinct

(10) All typical dandelions are my favorite weeds

In spite of the fact that (3) has a grammatically plural subject, it is evident that its logical subject is singular. That is, like (2) and (4), it is a predication about a whole genus of things, and it does not concern the members of this genus individually. Thus, from (2)-(4) we cannot conclude any of the following:

(11) My 1968 Buick is a typical automobile, and it was invented in 1889.

(12) Dino is a typical dinosaur, and he is extinct

(13) This is a typical dandelion, and it is my favorite weed

However, from (1) we can safely infer

(14) Henry is a typical graduate student, and he he poor

In the following discussion, we will refer to generic noun phrases of the sort exemplified in (2)-(4) as collective, and to those exemplified by (1) as distributive. As we will see below, there is, in spite of their differences, a very close connection between the two, and that an adequate analysis of the latter presupposes an analysis of the former.

A third type of generic noun phrase appears in sentences like the following:

(15) Triangles have three sides

(16) Human beings are mortal

(17) Atoms are decomposable

Like (1), these sentences have a distributive sense. That is, they predicate something of individual triangles, human beings, and atoms. Unlike (1), however, they do not admit of exceptions. Thus, they cannot be qualified as follows:

(18) All typical triangles have three sides

(19) All typical human beings are mortal

(20) All typical atoms are decomposable

To distinguish the generic noun phrases in (15)-(17) from the previous types, we will call them categorical.

It must be observed that there is an important distinction to be drawn between sentences such as (15)-(17) and ordinary universal affirmative sentences like the following:

(21) All surviving sea serpents live in Loch Ness

(22) Soon, all former U.S. Presidents will be dead

(23) All roads lead to Rome

If the universal quantifier all is prefixed to (15)-(17), they do not change in truth value. Moreover, it is possible to substitute singular generic noun phrases for the plural noun phrases in these sentences without disrupting them. Thus, we can have

(24) All triangles have three sides
(25) Triangles have three sides
(26) The triangle has three sides

(27) All human beings are mortal
(28) Human beings are mortal
(29) The human being is mortal

(30) All atoms are decomposable
(31) Atoms are decomposable
(32) The atom is decomposable

On the other hand, the sentences in (21)-(23) are changed radically if the quantifier all is deleted, and they do not permit substitution of the singular generic noun phrase at all:

(33) All surviving sea serpents live in Loch Ness
(34) Surviving sea serpents live in Loch Ness
(35) The surviving sea serpent lives in Loch Ness

(36) Soon, all former U.S. Presidents will be dead
(37) Soon, former U.S. Presidents will be dead
(38) Soon, the former U.S. President will be dead

(39) All roads lead to Rome
(40) Roads lead to Rome
(41) The road leads to Rome

In standard applications of the predicate calculus to English, the distinction between sentences like (15)-(17) and those like (21)-(23) is ignored, and all of them are translated in terms of the schema

(42) $(x)(f(x) \supset g(x))$

In the system *S, however, it is possible to assign them distinct kinds of representations. In addition to the schema (42), *S also makes available the following:

(43) $(x)((x=(\iota y)(f(y))) \supset g(x))$

Below, we will see that the difference in behavior between (15)-(17) and (21)-(23) can be fully explicated if the former are assigned representations corresponding to (42) and the latter are assigned representations corresponding to (43).

4.1. Distributive Generics

As we have noted, the sentence

(1) Graduate students are poor

may be regarded as true even though there may be an exceptional graduate student who is rich. In this respect, (1) contrasts very sharply with the related sentence

(2) All graduate students are poor

which is clearly intended to be true without exception.

In spite of the obviousness of the above distinction, it has often gone unnoticed, or has been deliberately ignored, by logicians who have been interested in the representation of natural language by means of symbolic logic. Thus, Reichenbach, to name only one example, suggests that the sentence

(3) Oranges taste good

is to be represented in the same way as the sentence

(4) All oranges tast good

that is, by means of

(5) $(x)(O(x) \supset G(x))$ (Cf. Reichenbach, 1965, p. 435)

This is clearly mistaken,however. In ordinary usage, all of the following assertions can be simultaneously true:

(6) Oranges taste good
(7) Some oranges are rotten
(8) Nothing rotten tastes good

If we understood the subject of (6) to be <u>all</u> oranges, as in (5), then (6)-(8) would be contradictory. There is no contradiction in ordinary usage, however, because the reference of <u>oranges</u> in (6) is understood to be restricted to those oranges which are normal or typical, and this apparently excludes rotten ones.

The question now is what alternative to (5) we must choose as a representation of sentences like (6). As we have already observed, (1) can be paraphrased in the following manner:

(9) All typical graduate students are poor

Similarly, (6) can be paraphrased as

(10) All typical oranges taste good

This suggests that we might try incorporating a predicate like typical in all distributive generics like (1) and (6). As a representation of (6), therefore, we might have something like the following:

$$(11)\quad (x)((x=(\iota y)(Gra(y))\&Ty(x))\supset P(x))$$

This analysis turns out to be unsatisfactory, however. Although typical appears to be a monadic predicate in the surface structure of sentences like (9) and (10), it cannot be interpreted semantically unless it is taken to be implicitly binary. That is to say, it is meaningless to say that something is typical unless we are prepared to say of what. If the noun phrase "a typical graduate student" were represented as above, then we would have to suppose that sentences of the form "typical(x)" were freely generated by our base rules. But this would allow all sorts of nonsense like the following:

(12) Gravity is typical
(13) Eternity is typical
(14) The square root of two is typical

These sentences are unintelligible because it is impossible to tell what their subjects are supposed to be typical of. By contrast, a sentence such as

(15) John is a typical boy

is perfectly intelligible. In this case, we have no trouble supplying the missing argument. A typical boy is one who is representative of the type boy. Similarly, a typical graduate student is one who is representative of the type graduate student, and a typical orange is one which is representative of the type orange.

What it takes to be representative of a given type varies, of course, as the type itself varies. Moreover, conceptions of what constitutes a type are likely to depend upon rather private judgments and rather private concerns. Granted, however, that we have an antecedently provided notion of a given type, we can define a representative of that type as anything which conforms closely to it in point of the characteristics it possesses, or is judged to possess.

Let us introduce, therefore, a binary predicate <u>similar</u> <u>to</u>, which will express the relation which exists between something and a given type when that thing possesses the salient properties of the type. For a given predicate "f" the corresponding type is expressed as "γ_f". Thus, an expression of the form

$$(16)\quad Si(x;\gamma_f)$$

will mean that whatever is denoted by x is similar to the type γ_f. Whenever this relation holds, x will have the properties which are criterial for the type. Hence, we will need the meaning postulate

$$(17)\quad (x)(Si(x;\gamma_f)\supset f(x))$$

For reasons which will become apparent below, we will also assume that the predicate "Si" is reflexive, i.e., that everything is, in the intended sense, similar to itself. Thus, in addition to (17), we will also have

$$(18)\quad (x)(Si(x;x))$$

As a representation of (1) we can now write

$$(19)\quad (x)(Si(x;\gamma_{Gra})\supset P(x))$$

It is evident that (19) will not be equivalent to

$$(20)\quad (x)[\ x=(\iota y)(Gra(y))\supset P(x)]$$

that is, to the assertion that <u>all</u> graduate students are poor. The contradictory of (20) is

$$(21)\quad (\exists x)(x=(\iota y)(Gra(y))\&-P(x))$$

Thus, (20) will be false if there is at least one non-poor graduate student. The contradictory of (19), however, is

$$(22)\quad (\exists x)(Si(x;\gamma_{Gra})\&-P(x))$$

Thus, in order to falsify (19), it will be necessary to show not only that there is a non-poor graduate student, but that this same individual is representative of the type.

There is a regular relationship in English between distributive generics such as (1) and collective generics. This relationship is illustrated in the following sentence pairs:

(23) Graduate students are poor
(24) The graduate student is poor

(25) Lions are rapacious
(26) The lion is rapacious

(27) Human beings are mortal
(28) The human being is mortal

It appears that whenever an assertion is true of the representatives of a given type, it is true of the type itself. The con verse, however, does not hold, as the following sentences indicate:

(29) The lion is the king of beasts
(30) *Lions are kings of beasts

(31) Britain and France launched an attack against the dollar
(32) *Britain and France launched an attack against dollars

This situation is reflected precisely in *S. The sentence

(33) The graduate student is poor

may be represented in *S as

(34) $P(\gamma_{Gra})$

But given (18) and (19) above, (34) is deducible. In general, any proposition of the form

(35) $(x)(Si(x;\gamma_f) \supset (\ldots x \ldots))$

will entail

(36) $(\ldots \gamma_f \ldots)$

Proof:

(1)	$(x)(Si(x;\gamma_f) \supset (\ldots x \ldots))$	Hyp
(2)	$Si(\gamma_f;\gamma_f) \supset (\ldots \gamma_f \ldots)$	UI
(3)	$Si(\gamma_f;\gamma_f)$	(18)
(4)	$(\ldots \gamma_f \ldots)$	MP

The converse of this implication will not hold in general. That is, it will not be the case that

(37) $(\ldots \gamma_f \ldots) \supset (x)(Si(x;\gamma_f) \supset (\ldots x \ldots))$

Thus, the relationship between distributive and collective generics noted in examples (23)-(32) can be correctly characterized in *S.

4.2. Categorical Generics

Categorical generics differ from distributive generics by virtue of claiming turth without exception. Thus, the sentence

(1) Triangles have three sides

entails

(2) All triangles have three sides

whereas

(3) Graduate students are poor

does not entail

(4) All graduate students are poor

In the introduction to this chapter it was noted that categorical generics like (1) were to be represented in terms of the schema

$$(5)\quad (x)(f(x) \supset g(x))$$

while universal affirmative sentences like (2) and (4) were to be represented by means of

$$(6)\quad (x)((x=(\iota y)(f(y))) \supset g(x))$$

It is easy to show, however, that (5) entails (6), i.e.,

$$(7)\quad (x)(f(x) \supset g(x)) \supset (x)((x=(\iota y)(f(y))) \supset g(x))$$

Proof:

(1)	$(x)(f(x) \supset g(x))$	Hyp
(2)	$f(x) \supset g(x)$	UI
(3)	$x=(\iota y)(f(y)) \supset f(x)$	T21
(4)	$x=(\iota y)(f(y)) \supset g(x)$	R1, MP
(5)	(7)	UG, DED

Thus, if representations are assigned as indicated, the relation between sentences like (1) and (2) can be accounted for.

It might be expected that the converse of (7) would also be true, i.e.,

$$(8)\ (x)\{(x=(\iota y)(f(y))\supset g(x))\supset(x)(f(x)\supset g(x))\}$$

(8) does not hold, however. To falsify (8), we would have to show that there is an interpretation under which its antecedent is true and its consequent false. Suppose, therefore, that "$x=(\iota y)(f(y))$" means that x is a road, and that "$g(x)$" means that x leads to Rome. Then if it were literally true that all roads lead to Rome, the antecedent of (8) would be true. But what of the consequent, "$(x)(f(x)\supset g(x))$"? This will be false if there is some x for which $f(x)$ holds and $g(x)$ does not. In *S, however, it is possible to define an abstract entity corresponding to the type of all roads, and by definition, $f(x)$ will hold of this entity. Thus, the assertion "$(x)(f(x)\supset g(x))$" will be false if $g(x)$ does not hold of the type of all roads, or in other words, if we can deny that the type of all roads leads to Rome. In *S, assertions which hold of all of the exponents of a given type do not necessarily hold of the type itself. Thus, even though it may be true that all individual roads lead to Rome, it does not follow that the type of all roads does.

The above reflects exactly the situation existing in ordinary English. The antecedent and consequent of (8) correspond to (9) and (10), respectively:

(9) All roads lead to Rome

(10) Roads lead to Rome

It is clear that (10) does not follow from (9). In ordinary English, of course, (9) is not regarded as literally true. In fact, it is usually regarded as a statement about Rome, and not a statement about roads at all. Thus, it is paraphrasable somewhat as follows:

(11) It is in the nature of Rome that all roads should lead to it

On the other hand, (10) seems to be understood the other way around, that is, in the same way as

(12) It is in the nature of roads that they should lead to Rome

Hence, even if (9) were literally true, (10) would not follow, because it is evident that there is nothing in the nature of roads which would dictate that they must lead to a particular place, even if it happens to be true that all of them do.

In the previous section it was noted that every distributive generic entails a corresponding collective generic. The same relation holds between categorical and collective generics. Let us assume, for example, that the following is true:

(13) Flying saucers are fictitious

From (13), we will be able to infer the corresponding collective generic, that is,

(14) The flying saucer is fictitious

Generally, we can show

(15) $(x)(f(x) \supset g(x)) \supset g(\gamma_f)$

Proof:

(1)	$(x)(f(x) \supset g(x))$	Hyp
(2)	$f(\gamma_f) \supset g(\gamma_f)$	UI
(3)	$f(\gamma_f)$	T16
(4)	$g(\gamma_f)$	MP
(5)	$(x)(f(x) \supset g(x)) \supset g(\gamma_f)$	DED

In the introduction to this chapter, we adduced the following triad of simultaneously true assertions:

(16) Triangles have three sides

(17) All triangles have three sides

(18) The triangle has three sides

This triad was contrasted with another of which only the first sentence could conceivably be true:

(19) All roads lead to Rome

(20) Roads lead to Rome

(21) The road leads to Rome

On the analysis of generic noun phrases presented in this chapter, we have seen that (17) and (18) follow from (16), but that (20) and (21) do not follow from (19). In this respect, then, it is clear that our analysis predicts the correct results.

V. Mass Nouns and Count Nouns

The obvious difference between mass nouns and count nouns is that the latter can be either singular or plural, while the former are always singular. But this difference depends on a further and much more significant one. Ultimately, the distinction is semantic rather than syntactic, and it extends to adjectives and verbs as well as to nouns. To adopt Quine's terminology, some English words are "cumulative" and others are "reference dividing." (Quine, 1960; p. 91). The mass noun water is cumulative in the sense that given any two bits of reality which are water, the sum of them is also water. On the other hand, a noun like dog is reference dividing. A composite entity consisting of two or more dogs does not merit the appelation dog, since only the individual components of such a composite possess all of the requisites of doghood. The composite possesses some of them, to be sure; for example, it likes bones, it barks, and it may even bite some exceptionally unfortunate mailman. But for one thing, it lacks a dog-like shape, and this lack is enough to disqualify it of the title dog. The appropriateness of the term "reference dividing" should be clear. To ascribe dog-qualities to a given region of space is to imply that that region is discrete, that it is countably distinct from anything else which has these qualities.

There are a great many adjectives, which, like mass nouns, are cumulative. The sum of two things which are red, blue, heavy, or hairy is again red, blue, heavy or hairy. These adjectives frequently have very closely related mass nouns. Thus, we can say that a certain painting contains a lot of red, that a certain girl wears a lot of blue, and, of a hairy-headed man, that he has a lot of hair. As examples of reference dividing adjectives, we can adduce spherical, triangular, and square. Not surprisingly, the nouns which are cognate with these are count nouns--viz., sphere, triangle, and square.

In the case of verbs, the cumulative-reference dividing distinction is more or less covert. It may be detected, however, by various syntactic tests. The quantifier "a little" occurs with mass nouns, but not with count nouns. Thus, we can have

(1) A little beer never hurt anyone

but not (2) *A little cigarette never hurt anyone

This same quantifier also divides verbs into two classes:

(3) Spiro talked a little and then sat down
(4) *Spiro existed a little and then croaked

(5) Sandy Koufax pitched a little
(6) *Sandy Koufax departed a little

(7) Nixon's hopes shrunk a little
(8) *Nixon's bubble burst a little

There are many other quantifiers which display the same co-occurrence possibilities. Consider the following examples:

(9) Fred drank too much beer
(10) *Fred smoked too much cigarette
(11) Spiro talks too much
(12) *Spiro exists too much

(13) You're in a heap of trouble, boy
(14) *You're in a heap of fix, boy
(15) Sandy Koufax pitched a heap last night
(16) *Sandy Koufax departed a heap last night

(17) Sturdley drank an appreciable amount of gin
(18) *Sturdley smoked an appreciable amount of cigarette
(19) Nixon's hopes shrunk an appreciable amount
(20) *Nixon's bubble burst an appreciable amount

Granted that there are quantizable and unquantizable nouns, then the fact that there are also quantizable and unquantizable verbs follows naturally from the analysis of verbs presented in Chapter III. According to that analysis, all verbs have potential noun phrases in their logical histories. The difference between the verb pitch and the verb depart can therefore be explicated by noting that the former incorporates a mass noun, while the latter incorporates a count noun. These incorporated nouns can frequently appear on the surface; to pitch is to do some pitching, and to depart is to make a departure. Thus, (15) and (16) above can be paraphrased as follows:

(21) Sandy Koufax did a heap of pitching last night

(22) *Sandy Koufax did a heap of departure last night

The ill-formedness of (16) can be explained on the basis of the ill-formedness of (22). Similarly, the ill-formedness of (12) and

(20) can be explained on the basis of the ill-formedness of

(23) *Spiro has too much existence

and (24) *Nixon's bubble did an appreciable amount of bursting

The problem of distinguishing cumulative verbs from reference dividing verbs is therefore reduced to the problem of finding the same distinction between certain nouns. A minimal act of pitching consists of delivering a single pitch. But the sum of any two of these minimal acts of pitching constitutes a further act of pitching. Thus, we can have a series like the following:

(25) In the ninth, Koufax pitched to one batter, and got him to fly out on one pitch

(26) Koufax pitched in the ninth and struck out the only batter he faced on three pitches

(27) Koufax pitched for the Dodgers last night, and recorded his eighth complete game in nine starts

(28) Last season, Koufax pitched for the Dodgers and won 26 games

(29) Koufax pitched for the Dodgers for 12 years

Acts of departing are not cumulative in this way. That is to say, two or more departures cannot be added together to constitute another departure. Hence, while the verb <u>pitch</u> is cumulative, the verb <u>depart</u> is reference dividing.

Since the cumulative-reference dividing distinction applies to all three of the categories noun, adjective, and verb, one would hope to be able to characterize that distinction in a general way, that is, in terms of some element which is common to these three categories. We have just seen that cumulative and reference dividing verbs can be distinguished once we have drawn the same distinction for nouns. But the semantic properties of nouns depend in turn upon those of adjectives, that is, upon the semantic properties of the primitive predicates which they incorporate. Our problem reduces, therefore, to one of providing a formal means of distinguishing cumulative and reference dividing predicates.

Cumulative predicates can be defined very simply in the system *S. In Chapter II, we introduced the notation "xUy" to stand for the sum of two referents. To say that a given predicate is cumulative is to say that if it is predicable of a given x and y, then it is predicable of xUy--i.e., if F is cumulative, then

(30) $(x)(y)((F(x)\&F(y))\supset F(xUy))$

If *S were supplemented with an appropriate metalanguage, then we could provide a general definition of cumulative predicates on the order of the following:

(31) f is cumulative ≡ df. $(x)(y)((f(x)\&f(y))\supset f(xUy))$

A highly interesting result follows immediately from (31). In an empty universe of discourse, and in a universe of discourse containing only one entity, the assertion

(32) $(f(x)\&f(y))\supset f(xUy)$

is automatically true. In the empty universe,

(33) $-(\exists x)(f(x))$

is true for all values of x and for any predicate, f. Hence, f(x) and f(y) are false for all values of x and y, and (32) is true. In a universe containing a single entity, say x, we will have

(34) $(y)(y=x)$

Thus, any predicate which holds of x will also hold of y. Furthermore, the sum of x and any y will be identical to x itself--i.e.,

(35) $xUy=x$

Hence, if any predicate holds of x, it will also hold of y, and of the sum xUy, and (32) will again be true. It follows that every predicate is cumulative in a universe of discourse which is empty or contains only one entity. The moral is that cumulative predicates -i.e., mass type predicates--do not logically require a pluralistic universe. Only if we are to have non-cumulative predicates--i.e., predicates for which (32) does not hold--must we have a universe of discourse which contains at least two entities.

It is evident that (32) will not hold for any predicate which is reference dividing. Thus, the existence of a plurality of designata is a necessary condition for a predicate's being reference dividing, i.e.,

(36) f is reference dividing $\supset -(x)(y)((f(x)\&f(y))\supset f(xUy))$

This does not amount to a definition, however. To have a proper definition, we must have both a necessary and a sufficient condition. To provide the latter, we will have to review the theory of indefinite descriptions developed in Chapter II, and effect some minor revisions.

Indefinite descriptions are expressions of the form "(1y)(f(y))", "(2y)(f(y))", "(3y)(f(y))", etc. These are intended to translate English noun phrases such as "a dog", "two dogs", "three dogs", etc. It is clear that mass mouns, in their normal meanings, cannot occur in such constructions. Due allowance must be given, of course, to sentences like the following:

(37) There are three beers that Bill will drink--Schlitz, Blatz, and Ballantine

In this sentence, the noun <u>beer</u> does not refer to the stuff itself, but rather to <u>kinds of beer</u>; it is the implicit count noun <u>kind</u> (or <u>type</u>, <u>sort</u>, <u>brand</u>, etc.) to which the quantifier <u>three</u> is attached in the underlying structure. A different problem is presented by sentences such as

(38) Joe had three beers and then went home and kicked his cat

Here, the reference of the noun <u>beer</u> is to bottles or glasses of beers. In this usage, <u>beer</u> is a legitimate count noun, and it must be carefully differentiated from the ordinary mass noun.

With objections of the above sort out of the way, we can say that a sufficient condition for a noun's being called a count noun is its ability to replace "such-and-such" in expressions of the form "x is a such-and-such." Since the latter are to be translated by means of "x=(1y)(f(y))", it should follow that a sufficient condition for being reference dividing is the ability of a predicate to occur in indefinite descriptions.

In Chapter II, expressions of the form "x=(1y)(f(y))" were defined contextually as follows:

Df.11 $x=(1y)(f(y))$ for
$(\exists y)\{f(y)\&(\exists w)(f(w)\&-w=y)\&(z)((f(z)\&z*y)\supset y\leq z)\&x=y\}$

A sentence such as

(39) Rover is a hound

is assigned the representation

(40) $R=(1y)(H(y))$

Expanding (40) by means of definition (11), we have

(41) $(\exists y)\{H(y)\&(\exists x)(H(x)\&-x=y)\&(z)((H(z)\&z*y)\supset y\leq z)\&y=R\}$

Various logical maneuvers will allow us to isolate the following three statements from (41):

(a) $H(R)$

(b) $(\exists x)(H(x)\&-x=R)$

(c) $(z)((H(z)\&z*R)\supset R\leq z)$

Df.11 was sufficient for our purposes in earlier cases. However, at this point we will need to provide a somewhat stronger definition. In particular, to exclude the possibility of construing hound as a cumulative predicate, it will be necessary to revise conditions (b) and (c). It will be convenient to examine condition (c) first.

Condition (c) states that Rover is included in every houndlike thing from which he is not countably distinct. Now, by definition, a referent is not countably distinct from anything which it properly includes. Hence, by (c), there can be nothing properly included in Rover which is houndlike. In other words, Rover constitutes a minimal unit of houndhood. But this does not exclude the possibility that Rover may be combined with some other minimal unit of houndhood, say Bowser, to form an additional, though larger unit. To avoid this possibility, we must define Rover in such a way that he constitutes a maximal as well as a minimal unit. This can be accomplished by revising (c) in the following way:

(c') $(z)((H(z)\&z*R\&-z=\gamma_H)\supset R=z)$

Since identity can be defined in terms of mutual inclusion, (c') is equivalent to the following two assertions:

(d) $(z)((H(z)\&z*R\&-z=\gamma_H)\supset R\leq z)$

(e) $(z)((H(z)\&z*R\&-z=\gamma_H)\supset z\leq R)$

According to (e), everything houndlike thing from which Rover is not countably distinct, except the type of all hounds, is included in Rover. Thus, he is a maximal unit of houndhood. In effect, (d) is the same as (c). When we have excepted the generic hound, (d) states that Rover is included in every houndlike thing from which he is not countably distinct. But given that Rover himself is houndlike, that is, given condition (a), to follows that Rover is included in the generic hound in any case. Thus, given (a) and (c'), (c) follows. The validity of previous proofs involving Df.11 will not be affected, therefore, if it is revised to lead to (c') instead of to (c).

If (a) and (b) are simultaneously true, it follows that there are at least two houndlike things in the world, or rather, that the existence of at least two houndlike things is possible. In Chapter II, we saw that this much, at least, was required to capture the sense of English "x is a such-and-such" constructions. In the light of the cumulative-reference dividing distinction, however, this requirement will have to be strengthened somewhat. Condition (b) will be satisfied if there is any houndlike entity which is not identical to Rover. But the type of all hounds is such an entity. Hence, the assertion that Rover is a hound will be true even if Rover is the only individual hound which exists over and above the type. By definition, Rover is included in the type--i.e.,

(42) $R \leq \gamma_H$

But in view of (42), we will also have

(43) $R \cup \gamma_H = \gamma_H$

Since the proposition

(44) $H(\gamma_H)$

is a tautology, it will follow that

(45) $(x)(y)((H(x)\&H(y))\supset H(x \cup y))$

will be true, in other words, that <u>hound</u> will be a cumulative predicate. To avoid this result, it will suffice to impose the same condition on the entity predicated by (b) as we imposed on Rover himself, that is, to revise (b) as follows:

(b') $(\exists x)(H(x)\&-x=R\&(z)((H(z)\&z*x\&-z=\gamma_H)\supset x=z))$

It is evident that (b') entails (b). Hence, if Df.11 is modified to lead to (b') instead of to (b), none of the proofs involving Df.11 which have been previously presented will be materially affected. Accordingly, Df.11 will be amended as follows:

Df.11' $x=(\iota y)(f(y))$ for

$(\exists y)\{f(y)\&(\exists w)(f(w)\&-w=y\&(z)((f(z)\&z*w\&-z=\gamma_f)\supset z=w))$

$\&(z)((f(z)\&z*y\&-z=\gamma_f)\supset z=y)\&x=y\}$

Df.11 resulted in the following conclusion:

(46) $(\exists x)(x=(\iota y)(f(y)))\supset(\underset{2}{\exists} x)(f(x))$

The revised definition will result in the somewhat stronger conclusion:

T26 $(\exists x)(x=(\iota y)(f(y)))\supset(\underset{2}{\exists} x)(x=(\iota y)(f(y)))$

Proof:

(1)	$(\exists x)(x=(\iota y)(f(y)))$	Hyp
(2)	$a=(\iota y)(f(y))$	EI
(3)	$f(a)\&f(b)\&-a=b\&(z)((f(z)\&z*a\&-z=\chi_f)\supset z=a)$ $\&(z)((f(z)\&z*b\&-z=\chi_f)\supset z=b)$	Df.11', EI, SUBS
(4)	$(\exists y)(y=b)$	T11, UI
(5)	$c=b$	EI
(6)	$(\exists x)(f(x)\&-x=b\&(z)((f(z)\&z*x\&-z=\chi_f)\supset z=x)$ $\&f(b)\&(z)((f(z)\&z*b\&-z=\chi_f)\supset z=b)\&c=b\&-a=b)$	EG, R1, MP
(7)	$b=(\iota y)(f(y))\&-a=b$	EG, Df.11'
(8)	$a=(\iota y)(f(y))\&b=(\iota y)(f(y))\&-a=b$	R1, MP
(9)	$(\underset{2}{\exists} x)(x=(\iota y)(f(y))$	EG, Df.19
(10)	T26	DED

A few observations on T26 are perhaps in order. First, it should be noted that T26 does not claim that the existence of one entity of a given description implies the existence of two entities of that description. This assertion would be obviously false. What T26 does claim is that if it is legitimate to assert that x is a such-and-such, then it is legitimate to speak of two such-and-suches, or in other words, that the existence of an indefinite singular nominal implies the existence of a corresponding plural noun.

Suppose that the f in T26 is predicable of something if and only if it has the properties of a Model-T Ford. At one time, there was only one of these, and the existence of that single example did not, and does not, entail the existence of any others. That is, it is logically possible that Henry Ford might have quit the whole business after assembling his first Model-T, instead of founding the Ford Motor Company and going on to build many more.

Now suppose that a friend had visited Henry Ford shortly after he had assembled the first Model-T and had asked, "What is that funny contraption parked in your garage?" To this question, Henry Ford might well have answered, "It's a Model-T Ford." The point of T26 is that this would be a legitimate answer only if Henry Ford considered the existence of further Model-T's to be possible. This situation is to be contrasted with the following: a person who believes that God is the only ruler of the universe cannot, consistent with his own belief, answer an inquiry into the nature of God by saying, "He is a ruler of the universe."

A noun has a plural form if and only if it can occur as the head noun in a noun phrase which has the logical form

(47) $(\iota y)(f(y))$

The fact that mass nouns do not have plurals can be accounted for, then, if we can demonstrate that cumulative predicates are logically excluded from structures like (47)--i.e., if

(48) $(\exists x)(x=(\iota y)(f(y)))\supset\sim(x)(y)((f(x)\&f(y))\supset f(xUy))$

Before (48) can be demonstrated, however, it will be necessary to discuss one further complication.

The sum of two referents a and b is defined as follows:

(49) $aUb=(\imath x)(y)(y{*}x\equiv(y{*}a \vee y{*}b))$

Similarly, the sum of any finite number, n, of referents, is given by

(50) $((a_1Ua_2)\ldots Ua_n)=(\imath x)(y)(y{*}x\equiv(y{*}a_1 \vee y{*}a_2\ldots \vee y{*}a_n))$

Now suppose that we wish to define the sum of <u>all</u> referents for which a given condition holds. Such a sum cannot be defined in the manner of (50), because there may be infinitely many such referents, and we may not know how to specify them in advance. This difficulty can be overcome, however, in the following manner. If the variable x ranges over an infinite set of entities, $a_1, a_2, a_3\ldots,$

(51) $(\exists x)(f(x))$

may be considered equivalent to the infinite disjunction

(52) $f(a_1) \vee f(a_2) \vee f(a_3) \vee \ldots$

Similarly, the assertion

(53) $(\exists x)(f(x)\&g(x))$

may be regarded as equivalent to

(54) $(f(a_1)\&g(a_1)) \vee (f(a_2)\&g(a_2)) \vee (f(a_3)\&g(a_3)) \vee \ldots$

It is evident, then, that the sum of all of the referents for which a given predicate holds can be written as

(55) $(\imath x)(y)(y{*}x \equiv (\exists w)(f(w)\&y{*}w))$

It will now be demonstrated that for a given predicate the above expression denotes the same thing as the generic of that predicate, i.e., that

T27 $\gamma_f = (\imath x)(y)(y{*}x \equiv (\exists w)(f(w)\&y{*}w))$

Proof:

(1)	$y{*}\gamma_f$	Hyp
(2)	$f(\gamma_f)$	T16
(3)	$f(\gamma_f)\&y{*}\gamma_f$	R1, MP
(4)	$(\exists w)(f(w)\&y{*}w)$	EG
(5)	$y{*}\gamma_f \supset (\exists w)(f(w)\&y{*}w)$	DED
(6)	$(\exists w)(f(w)\&y{*}w)$	Hyp
(7)	$f(a)\&y{*}a$	EI
(8)	$a \leq \gamma_f$	R1, MP, A5, EI, UI
(9)	$y{*}a \supset y{*}\gamma_f$	Df.5, UI
(10)	$y{*}\gamma_f$	R1, MP
(11)	$(\exists w)(f(w)\&y{*}w) \supset y{*}\gamma_f$	DED
(12)	$y{*}\gamma_f \equiv (\exists w)(f(w)\&y{*}w)$	R1, MP
(13)	$(y)(y{*}z \equiv (\exists w)(f(w)\&y{*}w))$	Hyp
(14)	$y{*}z \equiv y{*}\gamma_f$	R1, MP
(15)	$z = \gamma_f$	Df.7, UI
(16)	$(y)(y{*}z \equiv (\exists w)(f(w)\&y{*}w)) \supset z = \gamma_f$	DED
(17)	$(z)\{(y)(y{*}z \equiv (\exists w)(f(w)\&y{*}w)) \supset z = \gamma_f\}$	UG
(18)	(12)&(17)	R1, MP
(19)	T27	EG, Df.19

If the sum of any two things which are water is again water, it should follow that the sum of all things which are water is water. This is exactly what T27 predicts. Suppose that there is a primitive predicate, W, which holds of something if and only if it has the property of being water. Then the sum of all the things which have this property will be denoted by the generic γ_W. But the predicate to which a given generic corresponds is always predicable of the generic itself, by T16. Hence, by T27, the appropriate result follows. This result has a direct application in the translation of sentences like the following:

(56) Uranium is rare

It is clear that the subject of (56) is not particular pieces of uranium, since it would be highly infelicitous to say any of the following things:

(57) All lumps of uranium are rare

(58) All typical lumps of uranium are rare

(59) Most lumps of uranium are rare

In Chapter IV, we saw that the failure of paraphrases of this sort was a sure indication of a collective generic. Rarity is a notion which applies only to a collectivity, and not to the component terms of that collectivity. The appropriate translation of (56) into *S is therefore the following:

(60) $R(\gamma_{Ur})$

where R is the predicate rare and γ_{Ur} denotes the entity which consists of everything which is uranium-like. Thus, as we anticipated in Chapter II, mass nouns in such uses turn out to be a special case of generic noun phrases.

Now suppose that we are dealing with a universe of discourse in which there are only two individual dogs, say Rover and Bowser. In this universe of discourse, as well as in any others, there will be a type of all dogs, and this entity will exist over and above Rover and Bowser themselves. The question then arises as to whether the type of all dogs could be considered to be the same thing as the sum of Rover and Bowser. As matters stand, there would appear to be no way of answering this question formally in *S. Informally, however, it is clear that no native speaker of

English would ever subscribe to this proposition. The concept of <u>the canine familiaris</u> is an abstraction which comprehends the potential existence of indefinitely many dogs. But the sum of Rover and Bowser is forever limited to Rover and Bowser themselves. Moreover, there are a number of things which can be predicated of the latter but not of the former. If Rover and Bowser were indeed the only dogs in the world, they would no doubt be considered to be quite remarkable. It would not be surprising, therefore, if we should find that they were both on exhibit in the same zoo, say the Lucknow zoo. If this were the case, there would be a certain justice in saying that the entity which consisted of Rover and Bowser together was located in the Lucknow zoo. It would not seem to follow, however, that

(61) The canine familiaris is located in the Lucknow zoo

The question of the relationship of a type to its representatives is reminiscent of a number of ancient philosophical disputes, in particular, of the debate between Plato and Aristotle concerning Ideas and their manifestations. Aristotle held that an Idea was inseparable, ontologically, from its particular physical instances, whereas Plato held that Ideas have an independent existence. In this respect, the authority of ordinary language seems to be on the side of Plato. Philosophers who have objected to the ontological presumptions of Platonism have frequently objected to those of natural language as well. However, since our task here is to formalize certain aspects of natural language, and not to formulate an ontology which will be satisfying to some particular philosophical school, these objections will be ignored, and we will adopt the frankly Platonic position that there is something which adheres in a given type or genus which does not adhere in any of its members. This proposition, which will be assumed as an additional axiom of *S, can be expressed in the following way:

A6 $(\exists x)\{x * \gamma_f \& (z)(z=(1y)(f(y)) \supset \sim x*z)\}$

By virtue of A6, it can now be shown that the generic dog is not the same thing as the sum of Rover and Bowser in our above example. More generally, it can be shown that

T28 $(a=(1y)(f(y)) \& b=(1y)(f(y))) \supset \sim(a \cup b=\gamma_f)$

Proof:

(1)	$a=(\iota y)(f(y))\&b=(\iota y)(f(y))$	Hyp
(2)	$c*\gamma_f\&(z)(z=(\iota y)(f(y))\supset -c*z)$	A6, EI
(3)	$a=(\iota y)(f(y))\supset -c*a;\ b=(\iota y)(f(y))\supset -c*b$	R1, MP, UI
(4)	$-c*a;\ -c*b$	MP
(5)	$aUb=(\iota x)(y)(y*x\equiv(y*a \vee y*b))$	Df.13
(6)	$(y)(y*aUb\equiv(y*a \vee y*b))$	Df.9, EI, SUBS
(7)	$c*aUb\equiv(c*a \vee c*b)$	UI
(8)	$-(c*a \vee c*b)$	R1, MP
(9)	$-c*aUb$	R1, MP
(10)	$-(c*\gamma_f\equiv c*aUb)$	R1, MP
(11)	$(\exists x)-(x*\gamma_f\equiv x*aUb)$	EG
(12)	$-(x)(x*\gamma_f\equiv x*aUb)$	Df.4
(13)	$-(aUb=\gamma_f)$	Df.7
(14)	T28	DED

We can now proceed directly to a proof of (48), that is, of the theorem:

T29 $(\exists x)(x=(\iota y)(f(y)))\supset -(x)(y)((f(x)\&f(y))\supset f(xUy))$

Proof:

(1)	$(\exists x)(x=(\iota y)(f(y)))$	Hyp
(2)	$(\exists_2 x)(x=(\iota y)(f(y)))$	T26, MP
(3)	$a=(\iota y)(f(y))\&b=(\iota y)(f(y))\&-a=b$	Df.19, EI
(4)	$-(aUb=\gamma_f)$	T28, R1, MP
(5)	$(z)((f(z)\&z*a\&-z=\gamma_f)\supset z=a)$	Df.11', EI, SUBS, R1, MP
(6)	$(f(aUb)\&aUb*a\&-(aUb=\gamma_f))\supset aUb=a$	UI
(7)	$(y)(y*aUb\equiv(y*a \vee y*b))$	Df.9, EI, SUBS
(8)	$a*aUb\equiv a*a \vee a*b;\ b*aUb\equiv b*a \vee b*b$	UI
(9)	$a*a;\ b*b$	T9, UI
(10)	$a*a \vee a*b;\ b*a \vee b*b$	R1, MP
(11)	$a*aUb;\ b*aUb$	MP
(12)	$-b*a$	T17, MP
(13)	$aUb=a\equiv(y)(y*aUb\equiv y*a)$	Df.7
(14)	$aUb=a$	Hyp
(15)	$(y)(y*aUb\equiv y*a)$	MP

(16)	$b*aUb\equiv b*a$	UI
(17)	$b*a\&-(b*a)$	MP, R1, MP
(18)	$-aUb=a$	DED, RED
(19)	$-(f(aUb)\&aUb*a\&-(aUb=\gamma_f))$	R1, MP, MP
(20)	$aUb*a\&-(aUb=\gamma_f)$	R1, MP
(21)	$-f(aUb)$	R1, MP
(22)	$f(a)\&f(b)$	T21, MP
(23)	$-(f(a)\&f(b)\supset f(aUb))$	R1, MP
(24)	$-(x)(y)((f(x)\&f(y))\supset f(xUy))$	EG, Df.4
(25)	T29	DED

T29 justifies us in introducing the following definition of reference dividing predicates:

(62) f is reference dividing$\equiv$df.$(\exists x)(x=(1y)(f(y))$

Let us examine the significance of this definition. Given that a predicate is interpreted as cumulative according to definition (32), it follows from (62) and T29 that it cannot also be interpreted as reference dividing. But this result has an important application to the semantics of English sentences. Suppose that we wish to assign semantic representations to the following:

(63) Rover is a dog

(64) Schlitz is a beer

Knowing that <u>dog</u> is a count noun, there is no difficulty in interpreting (63) as an assertion of the form

(65) $R=(1y)(D(y))$

To assert that Rover is a dog is to assert that he is one countably distinct unit of dogness among others, and this is exactly what (65) says. On the other hand, (64) does not mean that Schlitz is a unit of beer. Hence, we would not want to assign it the representation

(66) $S=(1y)(B(y))$

Now, granted that <u>beer</u> is to be taken as a mass noun, it follows from (32), (62), and T29 that (64) <u>cannot</u> be represented by means of (66). In other words, the fact that (63) and (64) must have different kinds of interpretations is a logical consequence of our analysis. The correct interpretation of (64), of course, is that Schlitz is a <u>kind</u> of beer. This proposition can be represented in *S as follows:

(67) $S=(\iota y)(K(y;\gamma_B))$

Where γ_B is the generic beer and K is the predicate kind.

The central thesis of this work has been that distinct grammatical structures have distinct semantic correlates. Mass nouns and count nouns are alike in that both correspond to semantic structures which incorporate predicates and variables. Informally, nouns are complexes which perform the functions of reference and description at the same time. In this respect, they are distinct from adjectives, which fulfill the function of description, but not that of reference, and from verbs, which have the additional function of asserting relations. But the distinction between mass nouns and count nouns is no less logically motivated. In this chapter, we have seen that this distinction is not just an arbitrary grammatical phenomenon, but a genuine case of logical complementarity.

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