Project Work on SMT Modeling

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1 SMT Model

1.1 Decision variables

The decision variables of the model are:

- X_i for $i \in 0, \ldots, N-1$: the x-coordinate of circuit i
- Y_i for $i \in 0, \ldots, N-1$: the y-coordinate of circuit i
- H: the height of the plate

All decision variables are integers. The problem is modeled in the integer theory using the Z3 SMT solver.

1.2 Objective function

Since we want to minimize the total area used and the width W is fixed by the instance, the objective is to minimize the height H of the plate. We first used a more lassive bounds, then we preceded to use a more strict to improve performance.
The old lower bound of H was defined by the constraint

$$H \geq \max(heights) \tag{1}$$

that ensure H to be greater than or equal to the maximum circuits height.
The upper bound was defined by the constraint

$$H \leq \sum_{i=0}^{n} height_i \tag{2}$$

which enforces H to be lower than or equal to the sum of all circuits heights.
The new lower bound of H we used is defined by the constraint

$$H \geq \max\left(\max(heights), \frac{\sum_{i=1}^{n} width_i height_i}{W}\right) \tag{3}$$

which enforces H must be greater than or equal to the maximum of the height of the tallest circuit and the minimum height needed to fit the total area of all circuits.
The new upper bound remained unchanged.

1.3 Constraints

The constraints encode:

Domain constraints on Coordinates

$$0 \leq X_i \leq W - \min(widths), \quad 0 \leq Y_i \leq H - \min(heights)$$

Boundary Constraints

$$X_i + \text{width}_i \leq W, \quad Y_i + \text{height}_i \leq H$$

Non-overlapping Constraints for each pair of circuits i, j where $i < j$, at least one of the following must hold:

$$X_i + \text{width}_i \leq X_j, \quad X_j + \text{width}_j \leq X_i, \quad Y_i + \text{height}_i \leq Y_j, \quad Y_j + \text{height}_j \leq Y_i$$

1.4 Rotation

1.4.1 Decision Variables

In addition to the original X_i, Y_i, H variables, this model introduces:

- rot_i: a binary variable indicating if circuit i is rotated
- $w\text{real}_i$: the actual width of circuit i after applying any rotation
- $h\text{real}_i$: the actual height of circuit i after applying any rotation

1.4.2 Constraints

Rotation Constraints

- Circuits that are wider than the plate width cannot rotate:

$$(w_i > W) \implies (r_i = \text{False})$$

- w_{real} and h_{real} are set based on rot using if-then assignments:

$$w_{real_i}, h_{real_i} = \begin{cases} h_i, w_i & \text{if } rot_i = \text{True} \\ w_i, h_i & \text{if } rot_i = \text{False} \end{cases}$$

Non-overlapping Constraints Non-overlapping constraints are modified to use the actual width and height values after rotation.

Boundary constraints The boundary constraints now ensure the circuits fit within the container dimensions using the right dimensions for rotated ones.

1.4.3 Symmetry Breaking constraints:

Squares cannot rotate if their width equals height:

$$(w_i = h_i) \implies (r_i = \text{False})$$

1.4.4 Objective

The objective remains the same, minimize height H. Also for rotation model we updated the bounds. The older lower bound was

$$H \geq h_{real_i} \tag{4}$$

that constraints H to be greater than or equal to every circuits heights. The old upper was the same of the no-rotation model bound (2).
The lower bound was updated as

$$H \geq \left(\frac{\sum_{i=1}^{n} width_i height_i}{W} \right) \tag{5}$$

where H is greater than or equal to the minimum height needed to fit the total area of all circuits. The upper bound was updated as

$$H \geq \left(\sum_{i=1}^{n} \max(width_i, height_i) \right) \tag{6}$$

where H is lower than or equal to the sum of the maximum between height and width of all circuits, basically the highest stack of circuits possible.

1.5 Validation

Experimental design The models are implemented and solved using the SMT solver (Optimize) provided by the Z3py python library. All the trials were performed on a Samsung GalaxyBook Pro 360 with these specifications: CPU: 11th Gen Intel Core i7-1165G7 @ 2.80GHz, RAM: 16GB, GPU: Intel Xe Graphics. For each run the computation time limit was set as 5 minutes (300000 ms).

Experimental results Computation time was tracked to evaluate solver performance on both models. The two bar plots shown represent the computation with different bounds for H.
In the first one, where we used bounds (1) (2) for no-rotation model and (4) (2) for rotation model, we had poor results; no-rotation model achieved the optimum solution for 24 instances and the rotation-allowed model only for 9. In the second one we used bounds (3) (2) for no-rotation model and (5) (6) for rotation model and the performance were enhanced by a lot; no-rotation model achieved 32 solutions and the rotation-allowed model achieved 25 solutions.
Comparing the two plots we can also notice that the spread between the computation times for each instance of the two models narrowed. Despite the good results, the number of non solved instances suggests that, expecially for the rotation model, there is a wide margin of improving and optimization of the constraints.

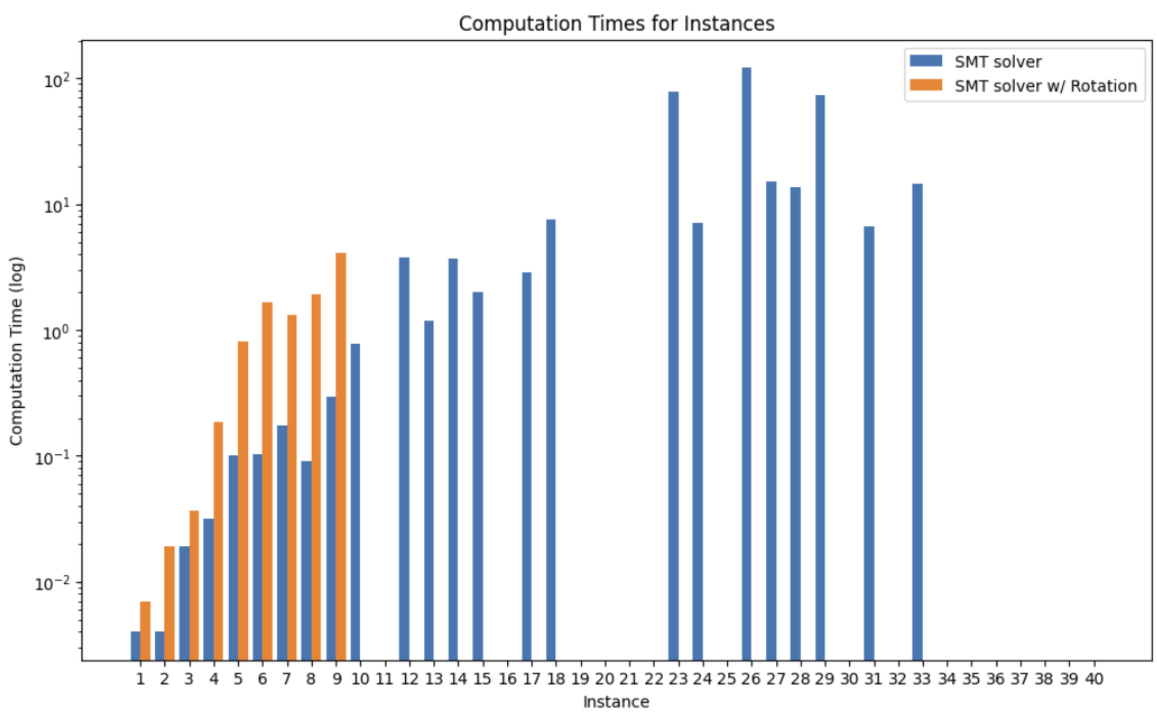

Figure 1: Bar plot showing computation time with old bounds

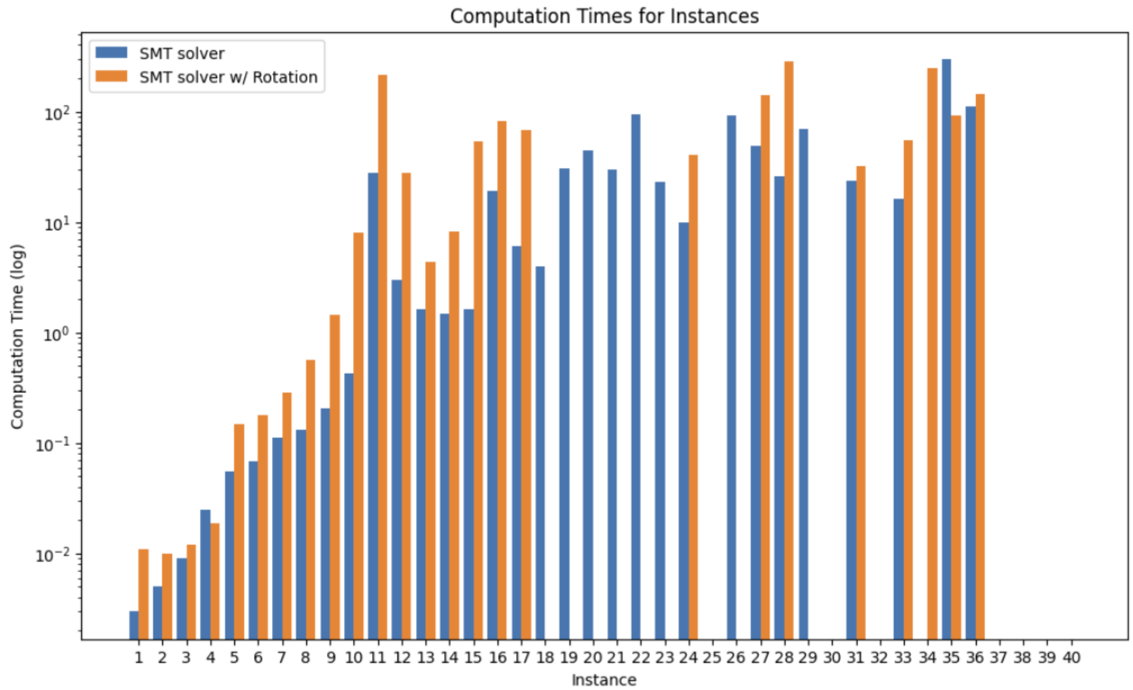

Figure 2: Bar plot showing computation time with updated bounds