Project Work on SMT Modeling

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1 SMT Model

1.1 No Rotation Model

1.1.1 Decision variables

The decision variables of the model are:

- X_i for $i \in 0, \ldots, N-1$: the x-coordinate of circuit i
- Y_i for $i \in 0, \ldots, N-1$: the y-coordinate of circuit i
- H: the height of the plate

All decision variables are integers. The problem is modeled in the integer theory using the Z3 SMT solver.

1.1.2 Objective function

Since we want to minimize the total area used and the width W is fixed by the instance, the objective is to minimize the height H of the plate. The lower bound of H is defined by the constraint

$$H \geq \max(heights)$$

which constrains H to be greater than or equal to the maximum rectangle height. The upper bound is defined by the constraint

$$H \leq \sum_{i=0}^{n} height_i$$

which constrains H to be lower than or equal to the sum of all rectangle heights.

1.1.3 Constraints

The constraints encode:

Domain constraints on Coordinates

$$0 \leq X_i \leq W - \min(widths), \quad 0 \leq Y_i \leq H - \min(heights)$$

Boundary Constraints

$$X_i + \text{width}_i \leq W, \quad Y_i + \text{height}_i \leq H$$

Non-overlapping Constraints for each pair of rectangles i, j where $i < j$, at least one of the following must hold:

$$X_i + \text{width}_i \leq X_j, \quad X_j + \text{width}_j \leq X_i, \quad Y_i + \text{height}_i \leq Y_j, \quad Y_j + \text{height}_j \leq Y_i$$

1.2 Rotation-Allowed Model

1.2.1 Decision Variables

In addition to the original X_i, Y_i, H variables, this model introduces:

- rot_i: a binary variable indicating if rectangle i is rotated
- $w\text{real}_i$: the actual width of rectangle i after applying any rotation
- $h\text{real}_i$: the actual height of rectangle i after applying any rotation

1.2.2 Constraints

Rotation Constraints

- Rectangles that are wider than the plate width cannot rotate:

$$(w_i > W) \implies (r_i = \text{False})$$

- w_{real} and h_{real} are set based on rot using if-then assignments:

$$w_{ri}, h_{ri} = \begin{cases} h_i, w_i & \text{if } r_i = \text{True} \\ w_i, h_i & \text{if } r_i = \text{False} \end{cases}$$

Non-overlapping Constraints Non-overlapping constraints are modified to use the actual width and height values after rotation.

Boundary constraints The boundary constraints now ensure the actual bounding box fits within the container dimensions.

1.2.3 Symmetry Breaking constraints:

Squares cannot rotate if their width equals height:

$$(w_i = h_i) \implies (r_i = \text{False})$$

1.2.4 Objective

The objective remains the same, minimize height H.

This extended formulation allows exploiting additional packing opportunities through rotation.

1.3 Validation

The models are implemented and solved using the Z3 SMT solver. No additional solvers or modeling languages are used. Computation time is reported to evaluate solver performance on both models. The bar plot shows increasing difficulty for both models, particularly for the rotation-allowed model, which exceeds the 5 minute time limit on instances after the 9th. This surely indicates a poor constraints optimization and room for improvement and suggests to explore approaches to better reduce the size of the search space.

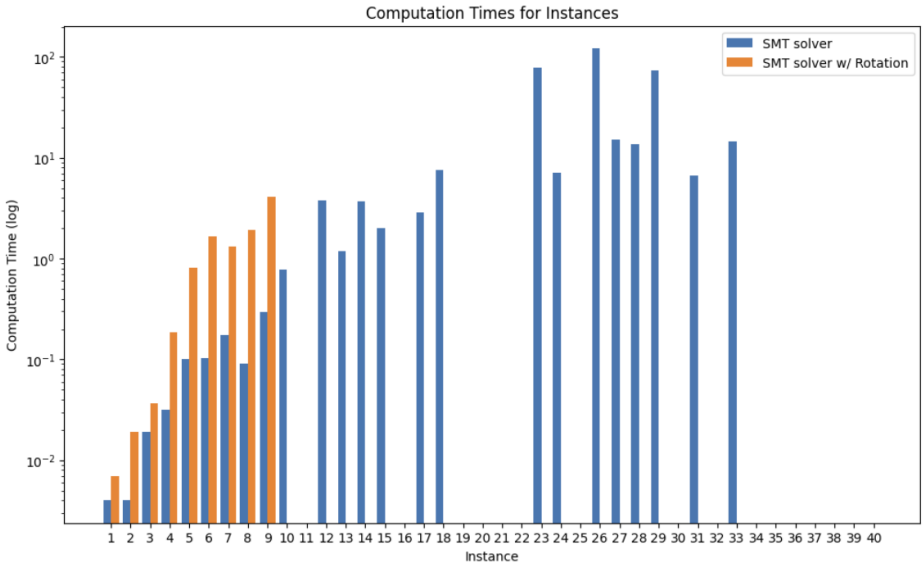

Figure 1: Bar plot showing computation time