### Opinion dynamics in correlated time-varying social networks

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### ABSTRACT

The study of opinion formation is an important aspect of social group dynamics. In this paper, we have studied the dynamics of popular naming game as an opinion formation model on time-varying social networks. This agent-based model captures the essential features of the agreement dynamics by means of a memory-based negotiation process. We investigate the outcomes of the dynamics on time-varying data of the networks that vary within very short intervals of time (20 seconds). In this study, we attempt to investigate the effect of the order in the temporal structure of the network on the negotiation dynamics. We observe that, quite strikingly, the traversal of opinions in the system is heavily dependent on time order of the network snapshots. A deeper analysis indicates the presence of recurrent community structures across different time points and disturbing the time order actually destroys these recurrence pattern. thereby, significantly affecting the opinion traversal process. For the purpose of completeness of the work, we identify the dependence of the temporal correlation on the temporal resolution of the network. In general, we observe that the correlation diminishes as the resolution is lowered.

### INTRODUCTION

Dynamicity is an intrinsic quality of social networks. For example, in a social gathering, e.g., a conference or an wedding reception, different attendees enter and leave the system at different points in time. In addition one observes variable levels of interaction among the attendees. Such a system can be best modeled using a time-varying social networks where nodes are the attendees and edges denote some means of connectivity among them (e.g., interaction, friendship etc.). Another important dynamics that is inherently coupled with such time-varying social networks is the spread of cultural and linguistic conventions that takes place on top of these networks.

Linguistic opinions can spread all over the network, can get trapped within densely-knit groups or may be completely washed out from the system. This dynamics and consequently the resultant outcome is heavily dependent on the temporal behavior of the underlying network. A good majority of the existing literature related to this area deals with static networks [3–8, 11, 13, 24]. In a recent paper by Maity et. al [15], they have studied the behavior of the opinion traversal in the temporal networks and showed that the outcome is significantly different from the static networks which we believe opens up a new line of research corresponding to opinion dynamics at a temporal scale. In this paper, we have studied the inherent network properties driving the agents' opinion traversal and the dependence of this process on the correlation among networks at different time points. One way of viewing at time-varying networks is as a series of static graphs accumulated over a fixed time interval; however, the order in which each of the static graphs appear in the series is a very important determinant of the course of the spreading dynamics.

In this paper, we investigate the importance of this time order in which the underlying networks appear in the temporal series during the process of opinion spreading. The spreading model we adopt is the wellknown naming game (NG) [4]. The naming game is a simple multi-agent model of non-equilibrium dynamics leading to the emergence of a shared communication scheme/common opinion in a population of agents. The system evolves through local pairwise interactions among artificial agents. This model was devised to explore the role of self-organization in the evolution of languages [22, 23] and has since then acquired an important role in semiotic dynamics that studies evolution of languages through invention of new words, grammatical constructions and more specifically, through adoption of new meaning for different words. It finds widespread applications in various fields ranging from artificial sensor networks as a leader election model [2] to the social media as an opinion formation model. It has also been studied in multi-party communication system [17]. Apart from mean-field case, the model has been studied on regular lattices [3, 13]; small world networks [5, 7, 11, 12]; random geometric graphs [10, 12, 13]; dynamic and adaptive graph [19] and empirical [14] complex netThe minimal naming game consists of a population of N agents observing a single object in the environment (may be a discussion on a particular topic) and opining for that by means of communication with one another through pairwise interactions, in order to reach a global agreement. The agents have an internal inventory, in which they can store an unlimited number of different words or opinions. At the beginning, all the individuals have empty inventories. At each time step, the dynamics consists of a pairwise interaction between randomly chosen individuals. The chosen individuals can take part in the interaction as a "speaker" or as a "listener." The speaker voices to the listener a possible opinion for the object under consideration; if the speaker does not have one, i.e., his inventory is empty, he invents an opinion. In case where he already has many opinions stored in his inventory, he chooses one of them randomly. The listener's activity is deterministic: if she possesses the opinion pronounced by the speaker, the interaction is a "success", and in this case both speaker and listener retain that opinion as the right one, removing all other competing opinions/words from their inventories: otherwise, the new opinion is included in the inventory of the listener, without any cancellation of opinions in which case the interaction is termed as a "failure" (see fig 1). The agents are usually embedded in a social network topology and can therefore interact with the neighboring agents to which they are directly linked by an edge (indicating a channel of communication). It is assumed that there can be potentially huge number of opinions for a particular topic so that the probability that two players will ever invent the same opinion at two different times is practically negligible and that the environment consists of a single topic of discussion.

In [15], the authors studied the impact of time-varying properties of the social network of the agents on the naming game dynamics. In this paper, we concentrate on how the opinion traversal process goes on in a temporal network and how the time correlation existing among the consecutive network snapshots determines the game dynamics. In particular, the consensus formation is much slower in case where there the time order is preserved than in case where it is absent. Next, we investigate the precise reason for the existence of such time correlations and observe that it is mostly driven by the presence of a set of highly recurrent communities that feature across the consecutive snapshots of the network. The opinions get trapped in these community, thereby, slowing down the process of opinion traversal and consensus formation. Remarkably, if the structures of these commu-

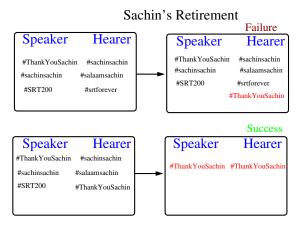


Figure 1: (Color online) Agent's interaction rules in basic NG. Suppose people are tweeting on "Sachin Tendulkar's retirement" using various hashtags (opinions). (Top) The speaker chosen at random, uses "#ThankYouSachin" (also chosen randomly from his inventory (memory) of hashtags). Now, the listener (again chosen at random) does not have this hashtag in her inventory, and therefore she adds the hashtag "#ThankYouSachin" in her inventory and the interaction is a failure. (Bottom) The speaker opines for "#ThankYouSachin" and in this case the hashtag is known by the listener. Therefore, they delete all other opinions except "#ThankYouSachin". The interaction this time is a success.

nities are disturbed, the game dynamics is affected in a similar fashion as in case where the time correlation across the consecutive network snapshots is disturbed. Since the temporal network data that we consider has only a limited number of snapshots, the spreading process does not reach a concensus. In order to investigate if a consensus could be indeed reached, we extrapolate a series of new snapshots using three different techniques. In all three cases, we find that a clear consensus emerges; strikingly the technique that preserves the time-correlation of the original sequence in the extrapolated sequence is found to allow for slowest consensus formation due to the presence of recurrent community structure. Finally, we attempt to establish a relationship between the time correlation and another important component in any temporal series: the time resolution. In particular, we attempt to show that at lower resolutions (i.e., beyond a critical limit) the time correlation is no longer observed and the gaming dynamics is no longer affected by the existence of temporal correlations at this resolution.

The rest of the paper is organized as follows. In Section II, we describe the datasets on which we investigate the naming game dynamics in a time-varying

social scenario. Section III provides the elaborate model description. In Section IV, we identify optimal number of games to be played on each network snapshot. In Section V, we study the effect of time order on the game dynamics and in section VI, we relate this effect with the presence of recurring communities. Section VII is dedicated for the analysis of the impact of extending network structures on achieving consensus. In section VIII, we study the relationship between network correlation on resolution in the context of the spreading dynamics. Finally, in section IX, we conclude this study by summarizing our key contributions and outlining important future directions.

### II DATASETS

For the purpose of the investigation of the naming game dynamics on time-varying networks, we consider two specific real-world face-to-face contact data and present our results for each of them. Both the datasets are obtained from SocioPatterns Collaboration [1]. The data collection infrastructure uses active RFID devices embedded in conference badges to detect and store face-to-face proximity relations of persons wearing the badges. These devices can detect face-to-face proximity (1-1.5 meter) of individuals wearing the badge with a temporal resolution of 20 seconds (see fig 2). A detailed description of the datasets on which we conduct our experiments are as follows:

SCIENCE GALLERY  $(SG_{SECS})$ : The dataset comprises face-to-face interaction record of visitors of the Science Gallery in Dublin, Ireland during the spring of 2009 at the event of art-science exhibition "INFECTIOUS: STAY AWAY" [9]. This dataset consists of time-varying versions of the networks for each of the 69 days. On each day, the evolution of the face-to-face interactions of the agents is captured by varying snapshots of the interaction network obtained at the intervals of 20 seconds. We selected 3 representative days out of the total 69 for this study.

**HYPERTEXT, 2009** ( $HT_{SECS}$ ): The face-to-face interaction data of the conference attendees of ACM Hypertext 2009 held in the Institute for Scientific Interchange Foundation in Turin, Italy, from June  $29^{th}$  to July  $1^{st}$ , 2009, where the SocioPatterns project deployed the Live Social Semantics application. The dataset contains the dynamical network of face-to-face proximity of 113 conference attendees over about 2.5 days.

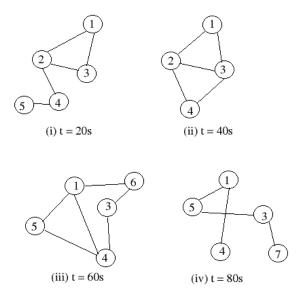


Figure 2: The above figure is a sample temporal network. (i) At t=20s, one can observe that node-5 is connected to node-4 while (ii) at t=40s, node-5 disappears and node-4 makes a new connection with node-3. Similarly, if networks at (iii) t=60s and (iv) t=80s are compared, one can observe that node-6 disappears, node-7 appears for the first time and new connections between node-5 and node-3 are established. These serve as an illustration of how nodes appear or disappear and how new edges are established and old dispensed from the network.

### III THE MODEL DESCRIPTION

The basic naming game model can be summarized as follows. At each time step (t = 1, 2, ...) a negotiation dynamics takes place in a population of artificial agents as follows: two agents are chosen at random, one of them act as a speaker while the other act as a listener. The rules of the interaction are:

- The speaker puts forth an opinion from his inventory of opinions to the listener. If the speaker has multiple opinions in his inventory, he randomly chooses one; if he has none, he invents a brand new opinion <sup>1</sup>.
- If the listener has this opinion present in her inventory, the communication is "successful", and both players delete all other opinions, i.e., shrink their inventory of opinions to this one

<sup>&</sup>lt;sup>1</sup> For implementation purposes this refers to maintaining a word identification number which is incremented by one for every new invention.

winning opinion and hence, arriving at a local agreement.

• If the listener does not have the opinion that's been put forth by the speaker (termed "failure"), it adds the opinion to her own inventory.

Note that in this model any agent is free to interact with any other agent, i.e., the underlying social structure is assumed to be fully connected. For the purpose of our analysis however, we assume that the agents are embedded on realistic social networks (i.e., SG and HT) that are continuously varying over time.

The main quantities of interest which describe the emergent properties of the system are :

- The total number  $N_w(t)$  of words/opinions in the system at the time t (i.e., the sum of the memory sizes of all the agents);
- the number of different words/opinions  $N_d(t)$  in the system at the time t;

In this paper our key observable is  $N_d^{final}$  that is the number of unique words left in the network after the game dynamics has taken place on all snapshots. In the next section, we attempt to estimate the optimal number of games to be played per network snapshot through a detailed analysis of  $N_d^{final}$ .

### IV OPTIMAL NUMBER OF GAMES

In this section, we discuss how the number of games that are played per network snapshot i.e.,  $Games_{snapsho}$  affects the overall dynamics and the final value of  $N_d$  ( $N_d^{final}$ ). Evidently, during the games, the underlying societal structure in the network alters the dynamics. Besides, there is another factor that needs to be considered i.e.,  $Games_{snapshot}$ . The number of games to be played per network snapshot is an indication of how well the corresponding agents interact among themselves.

### 1 EXPERIMENTS

Consider a network snapshot that has been aggregated over 20 seconds. Now, the number of games played corresponding to these 20 seconds intervals determines the effectiveness of agent interactions and strongly drives the advancement of overall game dynamics. In order to make the dynamics independent

of the number of games played per snapshot (no. of interactions in a specific 20 seconds time interval), we attempt to find if there is an optimal value of  $Games_{snapshot}$  beyond which  $N_d^{final}$  remains unaffected by the value of  $Games_{snapshot}$  chosen. We do so to understand the effect of the underlying time-varing strucrure on  $N_d^{final}$  in isolation from the number of games being played per snapshot.

### 2 RESULTS

Fig 3 shows how  $N_d^{final}$  decreases steadily till a value of  $Games_{snapshot}$  and then stabilizes. This indicates that after a certain value of  $Games_{snapshot}$ , the value of  $N_d^{final}$  does not change signicantly. We can summarize that there is a saturation that has been reached and call this value of  $Games_{snapshot}$  as  $Games_{sat}$ . We use value greater than  $Games_{sat}$  to study the effects of network properties on the game dynamics. An important observation is that the value of  $Games_{sat}$  is network dependent and varies across the datasets. For instance, the  $Games_{sat}$  for  $HT_{SECS}$  is around 7000 games per network snapshot and for  $SG_{SECS20}$ , it is 5000 etc.

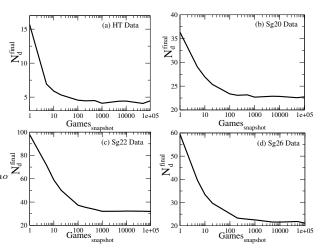


Figure 3: Variation of  $N_d^{final}$  with  $Games_{snapshot}$  (a)  $HT_{SECS}$  (b)  $SG_{SECS20}$  (c)  $SG_{SECS22}$  (d)  $SG_{SECS26}$ .  $Games_{snapshot}$  is represented in log scale on the x-axis.

### V EFFECT OF THE NETWORK TOPOL-OGY ON OPINION DYNAMICS

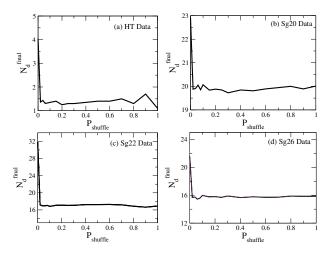
In a temporal network, each timestamp is associated with the respective network snapshot. We have a series of network snapshots which are in order and interestingly, this defines a correlation between consecutive snapshots and strongly influences the course of the game dynamics. In order to validate this hypothesis and show the effect of this temporal order, we devise the following simple experiment.

### 1 EXPERIMENTS

As said earlier, for every timestamp  $T_x$ , there is an associated network snapshot  $N_x$ . Let the total number of timestamps be  $T_N$ . The experiment is described as follows:

- 1. Randomly pick two timestamps from the given temporal network. Let them be  $T_x$  and  $T_y$ . The respective network snapshots associated be  $N_x$  and  $N_y$ .
- 2. With a probability value of  $P_{shuffle}$ , interchange the two network snapshots with each other i.e., in the shuffle, associate the  $T_x$  with  $N_y$  and  $T_y$  with  $N_x$ .
- 3. Do this  $T_N$  number of times on the network.
- 4. Play the naming game  $Games_{sat}$  number of times on each of the shuffled networks and collect the final value of  $N_d$  (i.e.,  $N_d^{final}$ ).

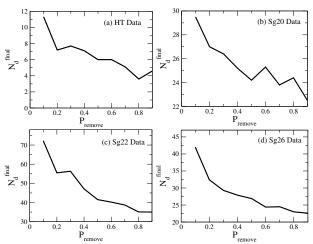
### 2 RESULTS



**Figure 4:** Plot of  $N_d^{final}$  vs Probability of shuffling  $P_{shuffle}$  for (a)  $HT_{SECS}$  ( $Games_{sat}=7000$ ) (b)  $SG_{SECS20}$  ( $Games_{sat}=5000$ ) (c)  $SG_{SECS22}$  ( $Games_{sat}=7000$ ) (d)  $SG_{SECS26}$  ( $Games_{sat}=6000$ )

Therefore as the probability  $P_{shuffle}$  increases from 0.0 (no shuffling) to 1.0 (complete shuffling), the extent to which the temporal order that gets disrupted

also increases. This affects the  $N_d^{final}$  as shown in the fig 4. The figure clearly shows that a minimal disturbance in the temporal order causes a drastic change in the  $N_d^{final}$ . The huge drop in the value of  $N_d^{final}$  indicates that the effect of the underlying network structure on the spreading dynaics changes drastically due to loss of the temporal order information. After this point,  $N_d^{final}$  seems to remain stable since the temporal sequence being disturbed, the network snapshots become equivalent to a series of random graphlets and any sequence would produce roughly the same value of  $N_d^{final}$ . In the next section we investigate the specific change in the network structure that takes place due to the shuffling of the time order thus causing an abrupt fall in the  $N_d^{final}$ .



**Figure 5:** Plot of  $N_d^{final}$  versus proportion of removed edges  $P_{remove}$  with various  $Games_{sat}$  (a)  $HT_{SECS}$  ( $Games_{sat} = 7000$ ) (b)  $SG_{SECS20}$  ( $Games_{sat} = 5000$ ) (c)  $SG_{SECS22}$  ( $Games_{sat} = 7000$ ) (d)  $SG_{SECS26}$  ( $Games_{sat} = 6000$ )

# VI REASONS FOR THE SUDDEN FALL IN $N_D^{FINAL}$

In this section, we attempt to relate the fall in  $N_d^{final}$  observed previously with a crucial structural property of the network. Analysis of the temporal networks reveal the presence of highly recurrent communities i.e., the communities that appear again and again in different network snapshots. The value of  $N_d^{final}$  decreases quite slowly owing to the presence of community structure that causes an entrapment of opinions within the communities. In other words, whenever an agent tries to interact, there is a high probability that it will be with another agent from the same community. A typical example of recurring communities in various network snapshots cor-

responding to different timestamps is shown in 7. The agents that are marked in red form a community structure and they recur in many network snapshots. Now, in the shuffling experiment, this recurrence of the community structure present in the consecutive network snapshots gets destroyed as a result of the shuffling thus causing the  $N_d^{final}$  to drop abruptly. If  $P_{shuffle}$  is more than 0.1, there is no further decrease in  $N_d^{final}$  since the recurrence is completely lost and any arrangement of the snapshots on the timeline is equivalent to a random arrangement. In the following experiment, we identify the highly recurring communities (see fig 6) and check if their removal actually affects the value of  $N_d^{final}$ .

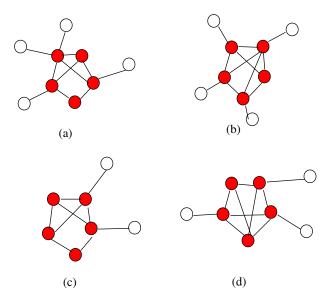
### 1 EXPERIMENTS

The experiment is described below:

- 1. For every network snapshot corresponding to a particular timestamp, apply Newman-Girvan method [20] to find the communities present.
- Update the count of the occurence of each community.
- 3. Sort the communities according to their count in descending order.
- 4. Repeat steps 1, 2 and 3 for all the timestamps and obtain the most recurring communities i.e., communities with largest counts are at the top of the sorted list.
- 5. In all the snapshots that we observe the presence of any one or more of the five most recurrent communities, we randomly remove a fraction of edges from within these communities (=  $P_{remove}$ ). In case there are more than one recurrent community from the list of top five in any snapshot, the community with larger recurrence is chosen for edge removal.
- 6. We then allow the dynamics on these new snapshots with  $Games_{sat}$  number of games played on each snapshot.

### 2 RESULTS

From fig 5, the steady decline of the value of  $N_d^{final}$  can be seen. This experiment provides an explanation for the decline of the  $N_d^{final}$  observed when the snapshots were shuffled. When the snapshots are shuffled in the above experiment, the underlying community



**Figure 6:** (Color online) Recurrence of communities in network snapshots of different timestamps. The red nodes are recurring in different network snapshots.

structure that was present due to the recurring communities is lost. As the community structure is destroyed, the opinions no longer remain trapped within communities for longer times and there is a higher chance for them to disperse faster eventually leading to the decrease of the  $N_d^{final}$ , sometimes reaching consensus

## VII EXTRAPOLATING NETWORKS TO REACH CONSENSUS

The temporal network data that we have at exposure is limited and the dynamics does not reach consensus since the simulations cannot proceed long enough to allow for the consensus to happen. We therefore attempt to extrapolate and append new snapshots to the data through three synthetic approaches outlined in [21]. In that paper, three different approaches to synthetic extension of empirical contact sequences were explained that we use here to synthesize new network snapshots:

1. SRep: Sequence replication. The network snapshots that form the original temporal network are repeated periodically in the same order as they appear, defining a new extended characteristic function such that  $\chi_e^{SRep}(i,j,t) = \chi(i,j,t) \mod T$ . Simply put, an edge (i,j) between agents i and j that appears at time t will be

again available starting at time t' > t after a time t + T.

- 2. SRan: Sequence randomization. The time ordering of the interactions is randomized, by constructing a new characteristic function such that, at each time step t,  $\chi_e^{SRan}(i,j,t) = \chi(i,j,t')$  where t' is a timestamp chosen uniformly at random from the set 1,2,3,...,T. Concisely, we choose a network snapshot out of the temporal sequence of snapshots with equal probability and extend the network till necessary. This method preserves on average all the characteristics of the projected network, but destroys the temporal correlations of successive contacts.
- 3. SStat: Statistically extended sequence. An intermediate level of randomization can be achieved by generating a synthetic contact sequence as follows: We consider the set of all conversations  $c(i, j, \Delta t)$  in the sequence, defined as a series of consecutive contacts of length  $\Delta t$  between the pair of agents i and j. The new sequence is generated, at each time step t, by choosing  $\bar{n}$ conversations ( $\bar{n}$  being the average number of new conversations starting at each time step in the original sequence), randomly selected from the set of conversations, and considering them as starting at time t and ending at time  $t + \Delta t$ , where  $\Delta t$  is the duration of the corresponding conversation. In this method of extension, we avoid choosing conversations between agents iand j which are already engaged in a contact started at a previous time t' < t. This extension preserves all the statistical properties of the empirical contact sequence, with the exception of the distribution of time gaps between consecutive conversations of a single individual.

### 1 EXPERIMENTS

We conducted a series of experiments with the available datasets by synthetically extending and running the naming game on those networks. The synthesis goes on until the population of agents reach consensus i.e.  $N_d(t) = 1$ . The number of games per snapshot is set to  $Games_{sat}$  and the results are shown in fig 7. As one can observe, the network that was synthesized through the SRep technique takes the longest time to reach consensus since this is the only case where time correlation is preserved. To strengthen our observation, we further investigate the pairwise correlation among the edges between every two snapshots to show the effect of preserving the time order.

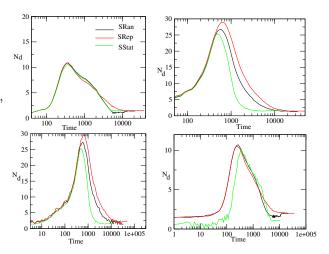


Figure 7: (Color online) Temporal evolution of  $N_d$  for SRan, SRep and SStat networks. (a)  $HT_{SECS}$  ( $Games_{sat} = 7000$ ) (b)  $SG_{SECS20}$  ( $Games_{sat} = 5000$ ) (c)  $SG_{SECS22}$  ( $Games_{sat} = 7000$ ) (d)  $SG_{SECS26}$  ( $Games_{sat} = 6000$ ).

We calculate the edge correlation in a temporal network as follows:

- 1. Consider the network snapshots corresponding to two consecutive timestamps. Identify the set of edges from both the snapshots.
- 2. Calculate the Jaccard coefficient by obtaining the ratio of the intersection of the edges to the union of edges from the snapshot pairs.
- 3. Conduct this for all consecutive timestamps; this produces the temporal variation of the edge correlation in the network.

### 2 RESULTS

The edge correlation varies as shown in fig 10. This temporal variation indicates how the current timestamp's network is different from the next timestamp's and so on. If the edge correlation is lower, one can advocate that the time order of the appearance or the snapshots is not crucial. As we pointed earlier in the paper, if the network is randomized and its time correlation is destroyed, it reaches consensus faster. A similar inference can be made from the edge correlation analysis of SRan, SRep and SStat networks. Also, the reason for SRep network's edge correlation being different from the other two is that the original contact sequence is replicated multiple times, the edge correlation is not really affected and stays way higher than that of SRan and SStat networks. Here

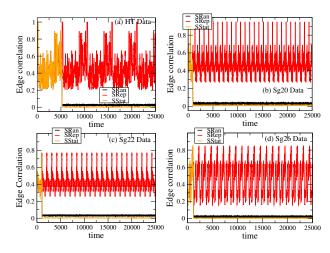


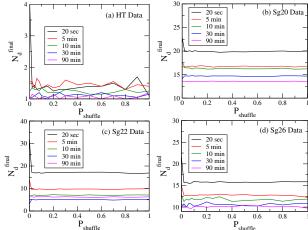
Figure 8: (Color online) Edge correlation vs time for SRan, SRep and SStat networks. (a)  $HT_{SECS}$  (b)  $SG_{SECS20}$  (c)  $SG_{SECS22}$  (d)  $SG_{SECS26}$ . The datapoints are smoothed by taking sliding window moving average with window size 50.

again, we observe that time correlation is a key determinant of the course of  $N_d$ .

## VIII TIME CORRELATION VERSUS TIME RESOLUTION

Discussed above is how the correlation in the temporal networks governs the game dynamics. In temporal networks, the resolution of the timestamps also plays a key role in determining the gaming dynamics. We studied the existence of correlation on the resolution of the temporal networks. We have converted the network from its original form to lower resolutions by aggregating the network snapshots of the adjacent timestamps in a non-overlapping fashion.

The results in the fig 7 show how the correlation behaves as the resolution decreases. The colored lines indicate the various aggregation levels i.e., black indicating that the network snapshots are captured in 20 seconds intervals, red indicating in an interval of 5 minutes or 300 seconds and so on. For lower resolutions, we do not observe any significant drop in the value of  $N_d^{final}$  with increasing  $P_{shuffle}$  as was observed for the higher resolution case in the earlier section. This is an interesting observation that shows that in lower resolution, the correlation is not observed. This indicates that the presence of correlation among the network snapshots is only in the highly resolved networks and as the resolution decreases, i.e., the aggregation level increases, the correlation factor disappears.



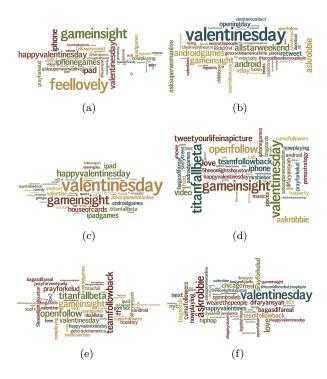
**Figure 9:** (Color online) Effect of aggregation on Correlation. Plot of  $N_d^{final}$  versus  $P_{shuffle}$  at  $Games_{sat}$ . The colored lines indicate different levels of aggregation in the corresponding temporal network(a)  $HT_{SECS}$  ( $Games_{sat} = 7000$ ) (b)  $SG_{SECS20}$  ( $Games_{sat} = 5000$ ) (c)  $SG_{SECS22}$  ( $Games_{sat} = 7000$ ) (d)  $SG_{SECS26}$  ( $Games_{sat} = 6000$ )

### IX DISCUSSIONS

Apart from analyzing the face-to-face interaction data, we have studied the opinion formation phenomenon in Twitter. We have collected 1% random sample of Twitter data stream of  $14^{th}February$ , 2014. We have constructed minutewise hashtag co-occurrence graph of the day. In this setting also, we have found out recurrent hashtags appearing across timescales. Fig 10 shows hashtag cloud of 6 consecutive minutes, where we find quite a few hashtag recurrent across timescales. For example, #gameinsight, #valentinesday, #titanfallbeta, #happyvalentinesday etc. are quite highly frequent as well as recurrent hashtags.

### X CONCLUSIONS AND FUTURE WORK

In this paper, we study the dynamics of naming game as a model of opinion formation on the time-varying social networks. Using the real world datasets i.e.,  $HT_{SECS}$  data and 3 instances of  $SG_{SECS}$  data, we analyzed on how the recurrent communities affect the game dynamics and how to observe their effect more accurately by eliminating the factor of number of games that are played per network snapshot which also has it influence on the value of  $N_d^{final}$ . We also considered the time-evolution of the network in perfect synchronization with the steps of the game (e.g.,  $HT_{SECS}$  and  $SG_{SECS}$  dataset) and found that the



**Figure 10:** Hashtag word cloud for 6 consecutive minutewise hashtag co-occurrence graph on tweet data of  $14^{th}February, 2014$ 

emergent behavior of the most important measure of the opinion traversal (i.e.,  $N_d^{final}$ ) have shown that the dynamics is highly dependent on the underlying societal structure. When the order in how the network snapshots arrive is slightly disturbed, the whole community structure is disrupted which leads to faster dispersal of opinions in the network thereby lowering the  $N_d^{final}$ . We looked at different network synthesis approaches and showed how the absence of time correlation might affect the game dynamics of the network. Also, we concluded that if the resolution of the temporal network is low i.e., more the network snapshots are aggregated, the less visible the correlation becomes i.e., the correlation among the consecutive network snapshots is lost and now shuffling their order does not really have any further effect on the game dynamics.

There are other interesting future directions that can be explored to strengthen the opinion dynamics model presented above. One such direction could be to incorporate the dominance index of the agents into the model. Not all actors in a society are equally dominant; while some of the actors are more opinionated and dominant the others might be more passive. This dominance on naming game dynamics has been studied in [16] and is reported to show faster agreement. Other direction is to incorporate stubbornness in adopting opinions [18] into the model and observe the effect of time correlation on the same. Finally, a thorough analytical estimate of the important dynamical quantities is needed to have a sound understanding of the effect of time correlation on the opinion dynamics.

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