Understanding how learning affects agreement process in social networks

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***Abstract*—In this article, we study how learning affects the dynamics of opinion formation in a population of agents modeled through the so-called naming game. This agent-based model captures the essential features of the agreement dynamics by means of a memory-driven negotiation process. We analyze the impact of learning in such social agreement model through a control parameter α describing the resistance toward learning. We show that there exists a critical α above which the consensus time diverges. In particular, we embed this model on various interaction topologies and real-world face-to-face interaction data and, thereby, point out the important differences in the agreement dynamics that take place in a real time-varying social settings vis-a-vis different static social settings. Remarkably, in all types of topology, we observe that beyond the critical value of α, the number of unique words increase manifold in the system and hence the time to consensus diverges possibly pointing to an universal aspect of language learning. In order to support the simulation results, we further develop a web-based online game - the 'tagging game' - which is a close correlate of the naming game and observe the game dynamics when human subjects are exposed to play the game. Remarkably, similar characteristic properties as that of the modified naming game is observed here even while different people from different geographical location play the game. This shows that synthetic modeling has nowadays reached the maturity to answer certain long-standing questions in cognitive science reasonably well.**

I. Introduction

Learning is an important aspect in society for forming beliefs and opinions. We shape our beliefs and opinions through interactions among friends and foes in our community. Most of us possess a set of beliefs about how others will behave in various social situations, which guide our behavior in social contexts ranging from business meetings to competitive sports. This is essentially an outcome of a learning process [1]–[3]. This aspect of learning plays a vital role in agreement process which involves negotiation and flexibility toward learning reflecting an individual's flexible or irresolute attitude towards adopting one's opinion. In this paper, we shall try to investigate the impact of learning on the agreement dynamics for one such popular model of opinion formation: the naming game (NG) [4]. In particular, we shall try to address the following questions:

- Does flexibility in learning ease the process of agreement/formation of opinion among a population of agents where each agent holds different opinions at the bootstrap? In other words, whether agents' flexible attitude toward learning leads to faster consensus among them?
- Does lack of learning or the irresolute attitude of the agents propel an opinion bias? If so, what is the characteristic of such a bias?
- Is there an impact of the underlying societal structure in shaping up opinion formation process given the component of learning is already present in the system?

The naming game is a simple multi-agent model of non-equilibrium dynamics leading to the emergence of a shared communication scheme/common opinion in a population of agents. The system evolves through local pairwise interactions among artificial agents. This model was devised to explore the role of self-organization in the evolution of languages [5], [6] and has since then acquired an important role in semiotic dynamics that studies evolution of languages through invention of new words, grammatical constructions and more specifically, through adoption of new meaning for different words. NG finds widespread applications in various fields ranging from artificial sensor networks as a leader election model [7] to the social media as an opinion formation model. NG has also been studied in multi-party communication system [8]. Apart from mean-field case, the model has been studied on regular lattices [9], [10]; small world networks [11]–[14]; random geometric graphs [10], [14], [15]; dynamic and adaptive graph [16] and empirical [17] complex networks.

The minimal model of naming game describes a population of N agents trying to agree on assignment of names to a single object (or opining on a particular topic of discussion) in the environment. The agents have at their disposal an internal inventory, in which they can store an unlimited number of different words or names or opinions. At the beginning, all the individuals have empty inventories. At each time step, a pair of individuals are chosen randomly from the population. The chosen individuals can take part in the interaction as a "speaker" or as a "hearer." The speaker voices to the hearer a possible name/opinion for the object under consideration; if the speaker does not have one, he invents one. In case where he already has many names/opinions stored in his inventory, he chooses one of them randomly. The hearer's move is deterministic: if she possesses the opinion pronounced by the speaker, the interaction is a "success", and both speaker and hearer retain that opinion as the right one, removing all

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other competing names/opinions from their inventories (local agreement); otherwise, the new name/opinion is included in the inventory of the hearer (learning) , without any cancellation of opinions in which case the interaction is termed as a "failure".

As learning is an important part of social agreement dynamics [1]–[3], the flexibility/resistance of agents toward learning can influence the agreement process. Hence, in this work we shall attempt to model this learning behavior through a parameter suitably incorporated into the naming game dynamics. We further investigate how such a control parameter can lead to a transition in the dynamical properties of the system on the basis of a critical value of this parameter. The rest of the paper is organized as follows. In section II, we describe in detail the naming game model in presence of the resistance to learning. Section III provides the elaborate impact of learning flexibility for different social structures and points to possible explanations of our findings. In section IV, we indicate how the results obtained from our synthetic model compare with an web-based experiment performed through human subjects. Finally, conclusions are drawn in section V.

II. The model definition

In this section, we attempt to model the irresolute behavior of the agents toward learning in the minimal naming game model. The evolutionary rules of the model are as follows:

- At each time step (t = 1, 2, ..), a speaker i is chosen at random and then the speaker chooses one of his neighbors as the hearer j.
- If the speaker i's inventory is empty, he invents a new opinion. Otherwise, if i already knows one or more opinions for the topic of discussion, then an opinion l, randomly chosen from his inventory, is communicated to the hearer.
- If the hearer j has the opinion in her inventory, the communication is a "success" and both of them keep this opinion and delete other opinions from their inventories.
- If the hearer j does not have the opinion in her inventory, the communication is termed as "failure". There can be two possibilities - a) The hearer is hearing the word/idea/opinion for the first time or b) the hearer already heard of it from some other agent before, however deleted it in course of the game as a result of a subsequent success.

  Case I: The hearer adds this opinion into her inventory in case (a).

  Case II: The hearer with probability α refuses to learn the opinion and with probability $1-\alpha$ learns the opinion.

Thus, the basic idea is that if a hearer had learnt an opinion earlier and had deleted it in a subsequent game due to a success then α models the resistance of the hearer to re-learn the same opinion once again. Note that if $\alpha = 0$, we recover back the original naming game. In the opposite limit, $\alpha = 1$, the opinions that are rejected once are not learnt by the agents ever at all and, thereby, leads to a fragmented state with multiple opinions. Therefore, a non-equilibrium phase transition is expected at some critical value of α (α_c) governing the learning rigidity.

III. Results and discussions

In this section, we will study our model and the role of the parameter α on the dynamics.

A. The Mean-field case

The mean-field case corresponds to a fully connected network in which all agents are in mutual contact. Thus, every individual can, in principle, talk to every other individual. On this topology, we try to investigate the microscopic activity pattern of the game dynamics driven by the parameter α. One of the most important observables of such agreement dynamics is the time to reach a stable uniform state or the consensus (t_{conv}) . The stable state in naming game is always consensus in which every agent has the same opinion in their inventory. However, there can also be possibilities of stable fragmented states where agents are clustered around multiple opinions. Such stable polarized or fragmented states in naming game dynamics has already been observed by Baronchelli et al. [18]. Similar stable fragmented states with coexisting cluster of opinions is also observed in our model as $\alpha \to 1$ (see fig 1(a), (b)). Note that the critical α (α_c) is $\approx$ 0.99. As we increase α beyond α_c, the time to reach consensus (t_{conv}) diverges. Importantly, this divergence of t_{conv} is independent of the population size N. We also examine the time t_x required by the population to reach the fragmented state with x number of opinions in the system. It is evident from fig 1(a) and (b) that for higher x, t_x diverges at some critical $\alpha(x) > \alpha_c$. The consensus time t_{conv} shows a power-law relation with $\alpha_c - \alpha$ for $\alpha < \alpha_c$ (see fig 1(c) and (e)). However, for $\alpha > \alpha_c$, t_{conv} tends to follow an exponential relation with $\alpha - \alpha_c$ (see fig 1(d) and (f)). These two distinct dependency relations of t_{conv} on $|\alpha - \alpha_c|$ signal a transition in the game dynamics at α_c.

The natural question that arises is : What are the microscopic activity patterns that lead to the divergence of the consensus time? To answer this question, one needs to delve deeper into the dynamics. There are two type of opinions/ideas that emerge in the system - (i) the ideas that are accepted by the population without suffering any rejection, i.e., due to refusal to learning by any agent and (ii) the ideas that are rejected at least once by some agents. We propose two metrics to capture each of their behavior - (i) the number of unique words/opinions in the system that have never been rejected by any agent so far ($A_d(t)$) and (ii) the number of unique words/opinions in the system that have been rejected by at least one agent ($R_d(t)$). The dynamics consists of inventions at early stage with new ideas appearing into the system which through pairwise negotiations (failure interactions) are accepted across the population. This corresponds to the rising trend of the $A_d(t)$ curve (see fig 1(g),(h)). Now as invention ceases, the $A_d(t)$ curve shows a fall which corresponds to the fact that the system is having more and more successful games and finally the $A_d(t)$ curve falls and settles to zero. Note that the behavior of the $A_d(t)$ curve remains roughly same irrespective of the

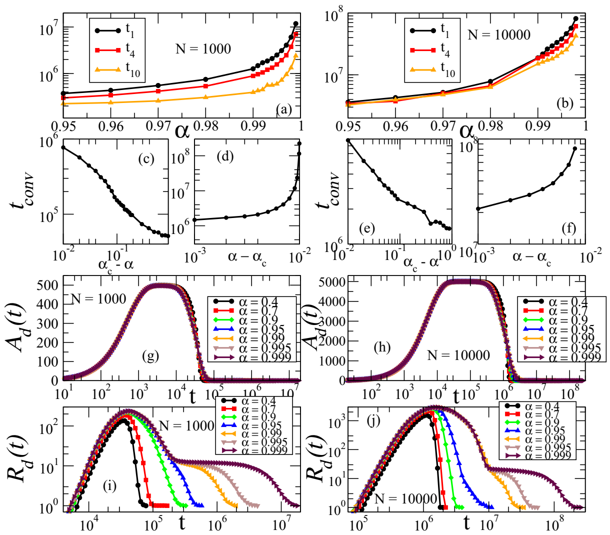

Fig. 1. Time t_x required by a population of size (a) N = 1000 and (b) N = 10000 on a fully-connected graph to reach a (fragmented) stationary state with x different opinions. Convergence time t_{conv} as a function of $\alpha_c - \alpha$ for $\alpha < \alpha_c$ for (c) N = 1000 and (e) N = 10000. Convergence time t_{conv} as a function of $\alpha - \alpha_c$ for $\alpha > \alpha_c$ for (d) N = 1000 and (f) N = 10000. The temporal evolution of $A_d(t)$ for (g) N = 1000 and (h) N = 10000. Time evolution of the number of rejected opinions $R_d(t)$ for (i) N = 1000 and (j) N = 10000. All the data points on the curves are averaged over 100 simulation runs.

value of α one chooses. Now let us look into the evolution of $R_d(t)$. At the early stages there are no rejected words as the agents fail for the first time thus learning a new opinion for the first time. However, once invention stops, the $R_d(t)$ curve (see fig 1(i), (j)) steadily grows which points to the fact that the agents have started to refuse to re-learn the older ideas/opinions once again and agreeing only on the accepted words. In this way the number of rejected ideas/opinions increases in the system and becomes the majority in the system while the accepted words keep decreasing. At some point in time, the number of accepted words become zero ($A_d(t) = 0$). The time required for the system to reach this state ($A_d(t) = 0$) depends on the population size N. As we increase N, the time to reach this state also increases (see fig 2(a)). The dynamics after this state has been reached, becomes a pure monopoly of the rejected words. Hence, the competition for the winner is now within the rejected words themselves. Therefore, an opinion bias already starts forming toward the rejected words/opinions. However, the agents in the system do not accept them easily increasing the number of rejections in the system with the $R_d(t)$ curve getting arrested in a plateau (with very few words in the system) as the majority of the interactions that take place do not change the state of the interacting agents (i.e., games with no outcome). The length of this plateau increases significantly as we increase α slightly beyond α_c thus correctly signaling the divergence of the consensus time which comes after an endless tussle between the words in the plateau and the word that finally becomes the winner (i.e., the word that has been rejected the largest number of times). This phenomenon is an effect of the emergence of opinion bias in the system that force the system align toward the most unlike idea.

We also tried to quantitatively identify the impact of other factors on the consensus time. The time taken by the agents to reach $R_d(t) = 1$ or the consensus time increases with N (see fig 2(b)). We further observe that there exists a positive correlation between the consensus time (t_{conv}) and the total number of rejections (R) that the system incurs for both the regime i.e., within and beyond the critical α (see fig 2(c) and (d) respectively).

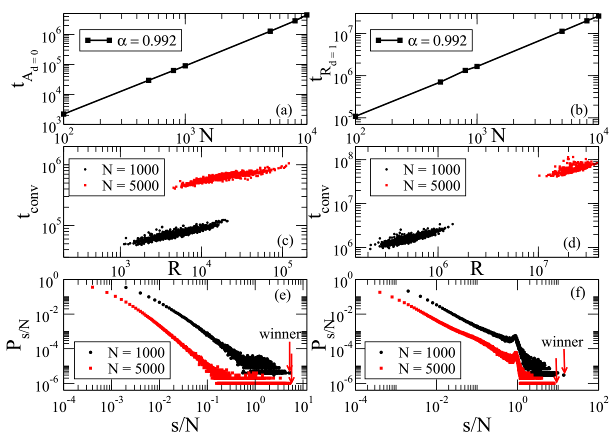

Fig. 2. Time required by (a) $A_d(t)$ curve to settle to zero and (b) $R_d(t)$ curve to settle to 1 for different values of N with $\alpha = 0.992$. (c) and (d) Scatter plot of t_{conv} vs R (total number of rejections) for N = 1000, 5000 with $\alpha < \alpha_c$ ($\alpha = 0.7$) and $\alpha > \alpha_c$ ($\alpha = 0.992$) respectively. (e) and (f) Distribution of the no. of state switchings (s) per agent for N = 1000, 5000 with $\alpha < \alpha_c$ ($\alpha = 0.7$) and $\alpha > \alpha_c$ ($\alpha = 0.992$) respectively. The red arrow indicates the winning word. All the data points on the curves are averaged over 1000 simulation runs.

Furthermore, we analyze the number of switchings (s), i.e., the number of times an initially rejected word switches from rejected to accepted state and back. We plot the distribution of such switchings per agents and observe that for $\alpha < \alpha_c$, a power-law behavior emerges with a heavy tail (fig 2(e)) while for $\alpha > \alpha_c$, there are two distinct regimes of power-law with a kink at $s/N = 1$ indicating that beyond α_c the number of agents experiencing one switching is abruptly higher than expected (fig 2(f)). In other words, this kink signifies a cut-off point at which suddenly a large number of agents in the population start experiencing continuous switchings possibly also reflected in the long plateau for $R_d(t)$ (see figs. 1(i) and (j)). Note that in both cases, the word that withstands the maximum number of switchings in the population emerges as the winner.

B. *Scale-free networks*

Social networks are far from being fully-connected or homogeneous. Most of them show a large skew in the distribution of node-degrees resulting in the so-called scale-free networks. In this section, we shall study the effect of the parameter α on the Barabási & Albert (BA) network [19] which follows a scale-free degree distribution.

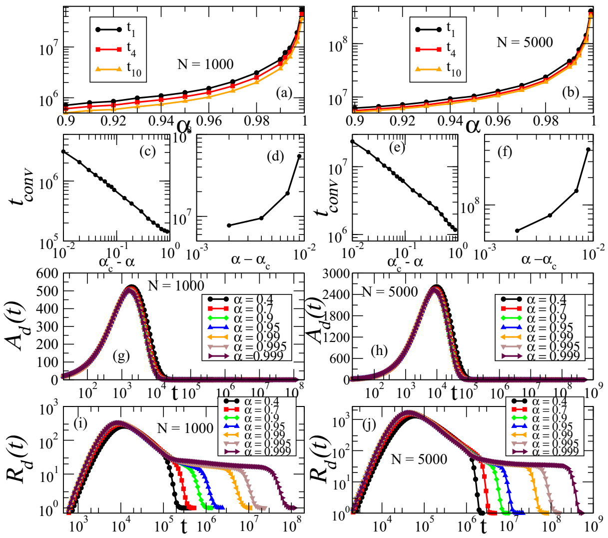

Fig. 3. Time t_x required by a population of size (a) N = 1000 and (b) N = 5000 on a scale-free network with average degree $\langle k \rangle$ = 8 to reach a (fragmented) stationary state with x different opinions. t_{conv} as a function of $\alpha_c - \alpha$ for $\alpha < \alpha_c$ for (c) N = 1000 and (e) N = 5000 and $\langle k \rangle$ = 8. t_{conv} as a function of $\alpha - \alpha_c$ for $\alpha > \alpha_c$ for (d) N = 1000 and (f) N = 5000 and $\langle k \rangle$ = 8. The temporal evolution of $A_d(t)$ for (g) N = 1000 and (h) N = 5000 with $\langle k \rangle$ = 8. Time evolution of $R_d(t)$ for (i) N = 1000 and (j) N = 5000 with $\langle k \rangle$ = 8. All the data points on the curves are averaged over 100 simulation runs for each of 10 network realizations.

The parameter α plays a similar role as in the mean-field case. We find a divergence of the consensus time with critical α (α_c) remaining roughly the same (see fig 3(a) and (b)). The behavior of the consensus time t_{conv} with $|\alpha - \alpha_c|$ also follows a similar characteristic before and after α_c (see fig 3(c), (d), (e), (f)). The $A_d(t)$ curve also behaves in the same way (see fig 3(g) and (h)). The time to reach the $A_d(t) = 0$ also show a positive correlation with population size N (see fig 4(a)). Further, the evolution of the $R_d(t)$ curve shows a similar trend with long-lasting plateau at the late stage of the dynamics for $\alpha > \alpha_c$ with the length of the plateau proportional to the value of α beyond α_c (see fig 3(i) and (j)). Note that for the same value of N as in the mean-field case, here the length of the plateau is much longer (roughly one order of magnitude longer if one compares figs. 1(i) and (j) with figs 3(i) and (j) respectively). This is a straight-forward effect of the social network structure on which the agents are embedded; their neighborhood being restricted, interactions are only possible among limited sets of agents which make it even harder to reach global consensus. In other words the low degree agents remain in a state of disagreement with the other low degree agents for a very long time and these low degree agents being the sheer majority in the population hinder the global consensus. Further, the time for the $R_d(t)$ curve to settle to 1 increases as one increase N (see fig 4(b)). Similarly, t_{conv} also show a positive correlation with the total number of rejections (see fig 4(c) and (d)). We also investigate the state switching behavior of the rejected words across the system which shows similar distribution of number of switchings per agent (see fig 3(e) and (f)) as in the mean-field case. One interesting difference is that at $s/N = 1$, for $\alpha > \alpha_c$, the second regime distribution follows an exponential-like distribution with a heavy tail. Moreover, the kink at $s/N = 1$ appears much earlier here (even while $\alpha < \alpha_c$ (fig 4(e))) which is again the effect of the interactions among only limited sets of agents that we have already indicated to be the reason for a longer size plateau here.

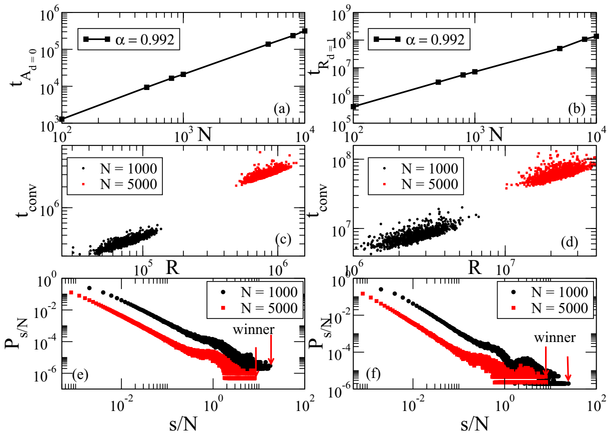

Fig. 4. Time required by (a) $A_d(t)$ curve to settle to zero and (b) $R_d(t)$ curve to settle to 1 for different values of N and $\langle k \rangle$ = 8 with $\alpha = 0.992$. (c) and (d) Scatter plot of t_{conv} vs R for N = 1000, 5000 and $\langle k \rangle$ = 8 with $\alpha < \alpha_c$ ($\alpha = 0.7$) and $\alpha > \alpha_c$ ($\alpha = 0.992$) respectively. (e) and (f) Distribution of no. of state switchings (s) per agent for N = 1000, 5000 and $\langle k \rangle$ = 8 with $\alpha < \alpha_c$ ($\alpha = 0.7$) and $\alpha > \alpha_c$ ($\alpha = 0.992$) respectively. The red arrow indicates the winning word. All the data points on the curves are averaged over 100 simulation runs for each of 10 network realizations.

C. *Time-varying networks*

In all the above discussions, we have considered static graphs where one knows the link structure apriori on which the game is played. However, the real-world social networks show dynamicity. Links appear and disappear over time. For instance, friendship relations keep changing with the due course of time. Thus, the essence of a social network lies in its time-varying nature. With the evolution of time, social conventions, shared cultural and linguistic patterns reshape themselves. Hence, it could be interesting to study the effect of learning in a setup where cultural conventions evolve on time-varying social networks.

For the purpose of the investigation of the naming game dynamics on time-varying networks, we consider two specific real-world face-to-face contact datasets and present our results on each of them. Both the datasets are obtained from the SocioPatterns Collaboration (http://www.sociopatterns.org/datasets/). The first dataset comprises face-to-face interaction data of visitors of the Science Gallery in Dublin, Ireland in Spring of 2009 [20] which consists of time-varying versions of the networks for each of the 69 days. On each day, the evolution of the face-to-face interactions of the agents is captured by varying snapshots of the interaction network obtained after every 20 second time interval. For the purpose of this work, we have shown results on one such representative network (22^{nd} day). This network consists of 240 science gallery visitors. For future invocation of this dataset, we shall refer to it as SG22. The other data is the face-to-face interaction data of the conference attendees of ACM Hypertext 2009 held in the ISI Foundation, Turin, Italy, where the SocioPatterns project deployed the Live Social Semantics application. The dataset contains the dynamical network of face-to-face proximity of 113 conference attendees over about 2.5 days. For future invocation of this dataset, we shall refer to it as HT.

The naming game on time-varying network has already been studied by [21] where they play the game in complete synchronization with the real time, i.e., at each time step $t = 1, 2 \ldots,$ (the elementary unit of time being second) a game is played among those agents that are alive at that particular instant of time in the network.

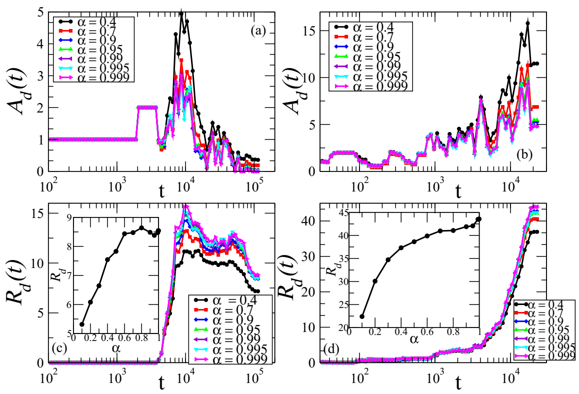

Fig. 5. The temporal evolution of $A_d(t)$ for (a) HT and (b) SG22 dataset. Time evolution of $R_d(t)$ for (c) HT and (d) SG22 dataset . The inset in each case shows the relation of R_d with α where R_d is the number of refused words at the end of the dynamics. All the data points on the curves are averaged over 100 simulation runs.

Unlike the static graphs where it is assumed that a node is alive throughout any process - from the beginning till the end, the time-varying graphs consist of nodes entering, leaving and re-entering into the system at different instances of time which amounts to say that not all nodes are equally responsible for driving a social process. Due to the aforementioned nature of a time-varying network, ideas/opinions keep on entering into the system throughout the dynamics. This is because, even in the late stages of the dynamics, new agents might enter into the system and act as a speaker thus enabling new inventions throughout. From fig 5(a) and (b), it is evident that throughout the process there are non-zero accepted opinions in the system and the refused words show their presence only very late (see fig 5(c) and (d)). Thus, there always exists two competing dominant groups in the system (ones that have been never rejected vis-a-vis ones that have been rejected at least once) and the fact that there is a continuous influx of new ideas and subsequent failures disallows the system to reach a global consensus. However, as we keep increasing α, the number of accepted words decrease (although usually never settling to zero) and the refused words become more and more dominant. Note that since the number of games here is limited by the number of network snapshots, the system might not reach a global consensus within the specified period due to continuous late stage inventions and consequent failures (see [21] for further details). Therefore it is not possible to estimate the exact consensus time (t_{conv}) and hence show the divergence caused due to injection of α into the model. However, in this case also one observes that the agreement becomes increasingly more difficult with α approaching α_c (see insets of fig 5 (c) and (d)) as number of refused words keep increasing manifolds.

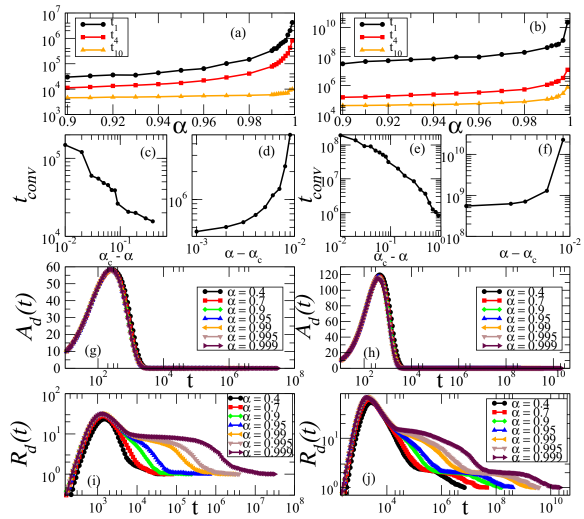

Fig. 6. Time t_x required on (a) HT and (b) SG22 dataset to reach the stationary state with x different opinions. t_{conv} as a function of $\alpha_c - \alpha$ for $\alpha < \alpha_c$ on (c) HT and (e) SG22 dataset. t_{conv} as a function of $\alpha - \alpha_c$ for $\alpha > \alpha_c$ on (d) HT and (f) SG22 dataset. The temporal evolution of $A_d(t)$ for (g) HT and (h) SG dataset and $R_d(t)$ for (i) HT and (j) SG22 dataset respectively. All the data points on the curves are averaged over 100 simulation runs.

For the sake of completeness of the work, we also study the naming game on the composite networks created by accumulating the contact sequences from their time-varying counterparts. The game is played on these static networks until consensus is reached. The basic difference which we had already mentioned is in the behavior of the emergent properties of the system. The key point once again is that the divergence in t_{conv} takes place almost at the same critical value of α as in all other cases (see fig 6(a), (b)). This is well supported by the power-law and exponential behavior of t_{conv} with $|\alpha - \alpha_c|$ for α less than and greater than α_c respectively (see fig 6(c), (d), (e) and (f)). However note that considering t_x (i.e., the time to reach x opinions) for x different from one, the values of α are quite distinct from those observed in the synthetic networks earlier. This is possibly due to the presence of strong community structures in the real networks as has also been reported earlier [17], [21]. Although the presence of community structures accelerates the agreement within communities, it defers the global agreement due to very low propagation of ideas across the communities which are usually interconnected only through weak ties. This factor coupled with the resistance to learning, as we shall see, further delays the dynamics. The $A_d(t)$ curve shows a steady growth reaching a peak and then a steady fall to zero (see fig 6(g) and (h)) whereas the $R_d(t)$ curve starts rising after invention stops until it reaches a peak and finally descends to get arrested into long plateaus (see fig 6(i) and (j)). This behavior of $R_d(t)$ is more prominent and shows existence of multiple plateaus in the late stage of the dynamics (see fig 6(j)) for the real networks which is in contrast to the synthetic static networks discussed earlier. The existence of multiple plateaus in the late stage of the dynamics is possibly a combined effect of the resistance of the agents toward learning coupled with the presence of strong community structures in such networks.

IV. Discussions

Our model has its significance in real-life scenarios apart from opinion formation, in spreading of social conventions where more than one conventions have been adopted by the society. Spreading of social convention/norms has been well studied in literature in terms of models and game-theoretic frameworks [22]–[25]. In fact, spreading of social convention in Online Social Networks has been studied by Kooti et al. in [26], [27]. In [26], Kooti et al. show how from seven retweeting conventions in Twitter, only two (RT and via) become immensely popular. These variations are adopted early by the core members of the Twitter community. These core users are genuinely influential or are the popular ones. However, as the authors also note, this early adoption of all the seven conventions by the core members do not in any way facilitate the spread of only two of the seven conventions over the entire Twitter network. This is because the core members equally learn all the seven conventions without any discrimination. In contrast, the authors observe that the two conventions that become immensely popular are learnt also by an extensive fraction of peripheral Twitter users (less active and less influential) apart from being adopted by the core users. Thus, those variations that cross the boundary of the core users to reach the periphery of the network become the popular ones. However, why a particular convention is adopted over the others remains unexplained in the paper. This study based on real online social data marks the empirical grounding for the model presented here. We conjecture that those conventions that withstand the learning resistance and the maximum number of switchings from the peripheral users finally emerge as the popular conventions. We plan to investigate this hypothesis in further details as a part of our future work.

We have also developed a web-based online tagging game[1] which is a close-correlate of the naming game. We launched this game and invited people to play the game. There were ~ 50 users registered for the game from various geographical locations and we have ~ 2000 game records. The rules of the game are the following: In each game, an user is assigned a role of a "speaker" or of a "listener/hearer". The speaker is shown a random image with already associated tags for the image (if any); now the speaker's task is to the tag the given image with a symbol. The hearer is shown all the images available and the corresponding memory content (already used tags) for the images along with the speaker's tag. From the above information, the hearer has to guess the image tagged by the speaker. If the hearer guesses correctly, the interaction is a success and all the existing tags are deleted from both the speaker's as well as the hearer's memories except the one on which they have agreed upon as is the case for the naming game. However, if the hearer did not have the tag in her memory corresponding to the image but still guesses the image correctly then the hearer is asked whether or not she would add the tag for the particular image under consideration. The appropriate action is taken once response is received. On the other hand, when the hearer's guess is incorrect, she is shown the correct image and asked whether or not she would learn this tag corresponding to the correct image. If yes, the tag is added into her memory else it is discarded (see fig 7).

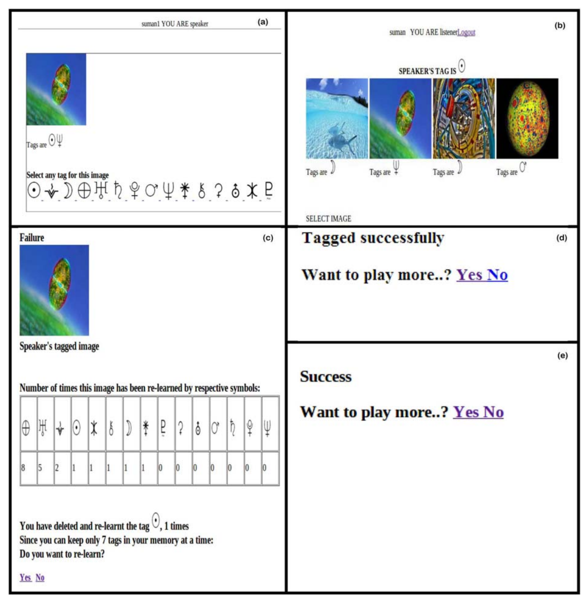

Fig. 7. Snapshots of the tagging game. (a) Speaker's window where speaker need to select a tag for the image. (b) The hearer/listener has to guess the image which the speaker has tagged. (c) **Failure interaction:** The hearer is shown the correct image and asked whether she wants to add the tag used in her memory. (d) The hearer has agreed to learn the tag (e) **Successful interaction:** If the hearer identifies the image correctly then both are shown this same snapshot and all other tags are removed from their memories corresponding to this image keeping only the successful tag.

In fig 8, we show the usage patterns of different symbols used as tags for various images. All the symbols have been used to tag various images; however there are some symbols which has been used much more compared to others. These symbols appeared earlier while arranged in a line (see fig 7(a)) and so probably have been used more. However, all the symbols have been discarded more or less similar number of times except the last few which are low in usage too. We

[1] http://www.abhiask.host22.com/speaker/index.php

have also analyzed the user participation in the games and we found ~ 30 users who have played atleast 20 games and there are some users who have played fewer games or could not continue the game as there were no other players available online. The average rate of refusal to learning turns out to be $\sim 31\%$. Therefore, we observe a rigidity toward learning for real users also playing such a game.

Symbols	No. of times used	No. of times added	No. of times discarded
☉	187	131	27
⚶	178	122	23
☽	185	127	25
⊕	290	192	32
♅	181	106	31
♄	96	74	15
♀	85	65	17
♂	156	108	22
♆	127	93	20
⚵	103	78	19
⚷	62	45	13
⚳	29	25	4
⛢	48	34	10
⚻	43	33	10
♇	34	28	5

Fig. 8. **Symbols and their properties.** The figure shows the symbols and their usage, number of times they have been added in the memories and number of times they have been refused to learn.

In fig 9, we show the distribution of various global quantities across different players. Fig 9(a) shows the distribution of the total number of games played by each player i.e, the fraction of the population that plays at least a given number of games. Similarly in fig 9(b), (c), (d) we show (i) the distribution of the number of symbols used as tags for playing the games, (ii) the number of tags that have been added while playing the game in the role of a hearer, and (iii) the number of tags that have been discarded as a result of refusal to learning respectively. In all the four cases we observe the prevalence of the well-known "80-20 rule" that causes the emergence of power-law structures. In this context, we observe that there are a few players who play a disproportionately large number of games and there are a few symbols that are used/added/discarded a huge number of times as compared to the rest of the symbols.

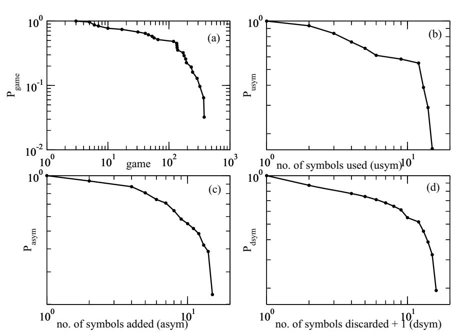

Fig. 9. Cumulative distribution of (a) total number of games played (b) number of symbols used as tags (c) number of symbols added as tag into the memory (d) number of used tags that has been discarded from memory across all the players.

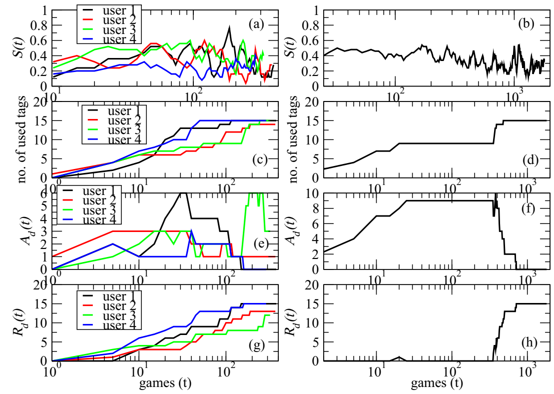

Fig. 10. **Evolution of emergent quantities.** Evolution of (a) success rate ($S(t)$) with games ordered according to increasing timestamp (the data are smoothed by taking window of size 5 for better visualization) (c) number of unique tags with games (e) $A_d(t)$ (g) $R_d(t)$ for the top 4 game players. Evolution of (b) average success rate (the data are smoothed by taking a sliding window of size 10 for better visualization) (d) number of used tags (f) $A_d(t)$ (h) $R_d(t)$ for all the players.

In this section, we shall attempt to provide results for the evolution of various emergent quantities in the tagging game and compare with the findings from the modified naming game presented in the earlier sections. Fig 10(a) shows the evolution of success rate for the top 4 gamers. The behavior is more or less similar with peaks at the end as they are playing more correlated games toward the end. In fig 10(b), we show the behavior of the evolution of average success rates of all the players over time. This typical behavior of average success rate $S(t)$ is also observed in naming game on time-varying networks (see [21] for more details) because the agents can enter and leave at any point in time which is also true for our case. In fig 10(c) and (d), we show the usage patterns of the different symbols used as tags over time for individual players and for all the players respectively. As time proceeds, number of used tags stabilizes to the maximum value. The individual curves also show similar patterns for the players. In fig 10(e) and (f), we observe the behavior of the evolution of number of unique tags that have never been discarded so far i.e., $A_d(t)$ for each of the top 4 players and considering all the players respectively. As also the case in the naming game, we observe here that initially there are lesser number of tags that have been discarded; as more and more games are played the rejections increase hence the count of not-at-all discarded tags falls and stabilizes to 0 (see fig 10(f)). This phenomenon is appropriately supported by the behavior of $R_d(t)$ (see fig 10(h)) which stays at 0 for long times and suddenly players start rejecting tags and, hence, $R_d(t)$ increases and subsequently stabilizes to the maximum value (i.e, the number of symbols present in the system). If we analyze this characteristic behavior for the four individual players, we observe that these players start refusing to learn at much earlier times (see fig 10(g)) than what is reflected from the average behavior of the players (see fig 10(h)). Therefore, it is evident that the real-life games can also reproduce similar characteristic properties of different synthetic models.

V. Conclusions and future work

In conclusion, we have studied the importance of resistance toward learning on the naming game dynamics.

- We have shown that a non-trivial consensus-fragmentation phase transition is observed in terms of a control parameter α representing the extent of resistance toward learning.
- We have elucidated the game dynamics on diverse topological structures from homogeneous fully-connected network to heterogeneous scale-free networks and on real world social networks. There is a strong impact of learning rigidity/stubbornness for all these social networks that generally leads to the divergence of the consensus time. We have measured the critical value of the control parameter describing the divergence of the consensus time and shown that it is independent of the interaction topology. This observation we believe, is very significant since it points to an universal characteristic of language acquisition through learning and has the potential to unfold a new direction of research.
- In addition, we observe that the word/opinion that is able to withstand maximum number of state switchings (from rejected to accepted and vice versa) finally emerges as the winner in the population.
- We also developed tagging game which is a close-correlate of the naming game and compare the emergent quantities of the game with the naming game where we observe a similar behavior of $A_d(t)$ and $R_d(t)$ curve. These interesting similarities with the synthetic model of naming game possibly point to the universality of learning resistance.

There are quite a few other interesting directions that can be explored in future. One such direction could be to incorporate the dominance index of the agents into the model. Not all actors in a society are equally dominant; while some of the actors are more opinionated and dominant the others might be more passive. This dominance on naming game dynamics has been studied by [28] and is reported to show faster agreement. Thus, it will be an interesting study to couple the two apparently opposite factors of dominance and resistance into the system. It will be thus interesting to investigate whether a consensus-fragmentation phase transition phenomena occur in the system under the influence of both of them. Finally, a thorough analytical estimate of the important dynamical quantities and the critical α reported only through empirical evidence here is needed to have a deeper understanding of the emergent behavior of the system.

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