**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

SOL: We have a normal distribution with mean=45 and sd =8.Let X be the amount of time it takes to complete the repair on custumer’s car.To finish in 1 hour we must have X<\_ 50 so to find Pr(X>50).

Pr(X>50)=1-Pr(X<\_50)

Z=(X-)/=(X-45)/8.0

By using normal table to find

Pr(X<\_50)=Pr(Z<\_(50-45)/8.0)=Pr(Z<\_0.625)=73.4%

Probabilty that the service manager cannot meet his demand will be=100-73.4=26.6% or 0.2676.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees

SOL:We have a normal distribution with mean =38 and sd = 6.Let X be the number of employees.

a)Probability of employees greter than age of 44 =Pr(X>44)

Pr (X>44)=1-Pr(X≤44)

Z=(X-)/=(X-38)/6

By using normal table

Pr(X≤44)=Pr(Z≤(44-38)/6)=Pr(Z≤1)=84.134%

Probability that the employee will be greater than the age of 44 = 100-84.135=15.86%

Probability of numbers of employees betwwen 38-44 years of age=Pr(X,44)-0.5=84.1345-0.5=34.1345%

THEREFORE THE STATEMENT IS TRUE.

b)Probability of employees less than age of 30=Pr(X<30).

Z=(X-)/=(30-38)/6)

By using normal table

Pr(X≤30)=Pr(Z≤(30-38)/6)=Pr(Z≤-1.333)=9.12%%

So number of employees with probability 0.912 of them being under age of 30=

0.0912\*400=36.48

THEREFORE THE STATEMENT IS TRUE.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

SOL: As we know that X~N(µ1,σ1^2) and Y~N(µ2,σ2^2) are independent random variable then X+Y~N(µ1+µ2,σ1^2+σ2^2) and X-Y~N(µ1-µ2,σ1^2+σ2^2).

Similarly if Z=aX+bY, where X and Y are as defined above , i.e Z is linear combination of X and Y, Then Z~N(aµ1+bµ2,a^2σ1^2+b^2σ2^2)

Therefore

2X1~N(2µ,4σ^2) and

X1 + X2~N(µ+µ,σ^2+σ^2)~N(2µ,2σ^2)

2X1-(X1+X2)=N(4µ,6σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

SOL:The probability of getting value between a to b should be 0.99.

The probability outside a and b area is 0.01

The probability towards left from a= -0.005

The probability towards right from b = +0.005

We need to cal X of random variable a and b

By finding (Z value) we ca =n calculate the X value

Z=(X-µ)/σ

For probability 0.005 the z value is -2.57

Z\*σ+µ=X

Z(-0.005)\*20+100=-(-2.57)\*20+100=151.4

Z(+0.005)\*20+100=-(-2.57)\*20+100=48.6

So, option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.- Range is Rs (99.00810347848784, 980.9918965215122) in Millions

In [ ]:

Specify the 5th percentile of profit (in Rupees) for the company -5th percentile of profit (in Millions Rupees) is 170.0

In [ ]:

1. Which of the two divisions has a larger probability of making a loss in a given year?

Inference:Probability of dvision 1 making a loss in a given year is more than division 2