CAT 1

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Q1

a) Peter bets 5 dollars each time (timid strategy).

$$p = 0.55, q = 0.45, N = 8, z = 5$$

```
p = 0.55
q = 0.45
N = 8
i = 5

timid_results = (1-(q/p)^i) / (1-(q/p)^N)

timid_results
```

```
## [1] 0.7924987
```

b) Peter uses a bold strategy

Since peter uses a bold strategy i.e play with the money earned in each round.in this case he will play 4 round to get to 8 dollars.

```
q_0 <- 0
q_8 <- 1

p <- 0.55
q <- 0.45

q_4 <- p*(q_8)+q*(q_0)
q_2 <- p*(q_4)+q*(q_0)
result_bold <- p*(q_2)+q*(q_0)

result_bold
```

```
## [1] 0.166375
```

c) Which strategy is better timid or bold

Timid Strategy is better than the bold strategy

Q2

The gambler's ruin problem: player A starts with fortune j dollars and bets 1 dollar until he either loses all the fortune or reach a N dollars fortune and then quit.

let A_j represent the the event that player A win starting with j dollars.

$x_j = P(A_j) =$ probability to win starting with a j = probability to reach N before reaching 0 starting from j

Using conditional probability and condition on what happens at the first game, win,lose or tie,every game will have

$$P(win) = p, P(lose) = q, P(tie) = r$$

which yields

$$\begin{aligned} x_j &= P(A_j) \\ &= P(A_j|win)P(win) + P(A_j|lose)P(lose) + P(A_j|tie)P(tie) \\ &= x_j \times p + x_j \times q + x_j \times r \end{aligned}$$

note that $x_0 = P(A_0) = 0$ since their nothing more to gamble and $x_N = P(A_N)$ since player A has reached their goal and then stop playing hence the equation $p + q + r = 1$
Gamblers Ruin

$$px_{j+1} - (p+q)x_j + qx_{j-1} = 0, x(0) = 0, x(N) = 1$$

.Rewriting the equation in second order equation with $x_j = \alpha^j$ we find the quadratic equation

$$p\alpha^2 - (p+q)\alpha + q = 0$$

with solutions

$$\alpha = \frac{p+q \pm \sqrt{(p+q)^2 - 4pq}}{2p} = \frac{p+q \pm \sqrt{p^2+q^2-4pq}}{2p} = \frac{p+q \pm \sqrt{(p+q)^2}}{2p} = \begin{cases} 1 \\ q/p \end{cases}$$

if $p \neq q$ there arer two solutions and so the general is given by

$$x_n = C_1 1^n + C_2 \left(\frac{q}{p}\right)^n$$

. Using $X_o = 0$ (lose) , $X_N = 1$ (win) to determine the constants C_1 and C_2.

$$0 = C_1 + C_2, and 1 = C_1 + C_2 \left(\frac{q}{p}\right)^N$$

which gives

$$C_1 = -C_2 = \left(1 - \left(\frac{q}{p}\right)\right)^{-1}$$

hence

$$\text{Gambler's ruin probabilities: } x_n = \frac{1-(q/p)^n}{1-(p/q)^N} \; p \neq q$$

if the game is fair i.e. $p = q$ the gambler ruin probabilities X_j simplify to

$$x(j+1) - 2x(j) + x(j-1)$$

which gieve the quadratic equation

$$\alpha^2 - 2\alpha + 1$$

with the only root $\alpha = 1$.The genral solution is

$$x(j) = C_1 + C_2 j$$

with \$x(0) = 0 \$ and $x(N) = 1$ which results

$$\text{Gambler's ruin probabilities: } x_n = \frac{j}{N} \; if \; p = q$$

in summary the game is fair

$$\text{fair if } p = q$$
$$\text{subfair if } p < q$$
$$\text{superfair if } p > q$$

Q3

Tom starts with \$5, and p = 0.63: What is the probability that Tom obtains a fortune of N = 12 without going broke?

$$i = 5, N = 12 \text{ and } q = 1 - p = 0.37$$

hence

$$\frac{q}{p} = \frac{37}{63}$$

$$P_2 = \frac{1 - (37/63)^5}{1 - (37/63)^{12}} = \frac{0.93012}{0.99832} = 0.9317$$

What is the probability that Tom will become infinitely rich?

$$1 - (q/p)^i = 1 - (37/63)^5 = 0.93012$$

If Tom instead started with i = \$2, what is the probability that he would go broke?

$$\text{The probability he becomes rich is } 1 - (q/p)^i = 1 - (37/63)^2 = 0.6550$$

Q4

The probability that the stock goes up by 15 before going down by 6.computing $p(a)$

$$p = 0.6, q = 1 - p = 0.4, a = 15, b = 6$$

$$p(a) = \frac{1 - (\frac{q}{p})^b}{1 - (\frac{q}{p})^{a+b}} = \frac{1 - (\frac{0.4}{0.6})^6}{1 - (\frac{0.4}{0.6})^{15+6}} = \frac{0.9122}{0.9998} = 0.9914$$