CAT 2

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Q1

a

$$df = n - l - 1$$

where l is the level in anova test and n is the sample size

$$48 = n - 1 - 1$$

Solve for n

$$n = 50$$

b

$$S = \sqrt{MSE}$$

$$MSE = \frac{SSE}{n-2} = \sqrt{\frac{11354}{48}} = 15.38$$

c

	t-statistic	p-value
$H_0 : \alpha = 0 \, , H_a : \alpha \neq 0$	-2.601	-0.0123
$H_0 : \beta_1 = 0 \, , H_a : \beta_1 \neq 0$	9.464	$1.490 * 10^{-12}$

d

$$MSR = \frac{SSR}{1} = \frac{21186}{1} = 21186$$

$$MSR = \frac{SSE}{n-2} = \frac{11354}{48} = 236.59166$$

$$\text{F-value} = \frac{MSR}{MSE} = \frac{21186}{236.5417} = 89.5656$$

e

$$R^2 = 0.6511$$

Q2

$$\frac{L(\theta_0)}{L(\theta_1)} \leq k$$

$$\frac{\prod_{i=1}^{n} 2 \times x^{2-1}}{\prod_{i=1}^{n} 1 \times x^{1-1}} \leq k$$

$$\frac{1}{2\prod_{i=1}^{n} X_i} \leq k$$

Making x the subject of the formula

$$\frac{1}{2k} \leq \prod_{i=1}^{n} x_i$$

Q3

let $\theta_2 = \sigma^2$ and $\theta_1 = \mu$

$$L(\theta) = L(\mu, \sigma^2) = \left(\frac{1}{\sqrt{2\pi\sigma}}\right)^n exp\left[-\sum_{i=1}^{n}\frac{(X_i - \mu)^2}{2\sigma}\right]$$

The Maximum Likelihood estimator the $L(\theta)$ at point $(\hat{\mu}, \hat{\sigma^2})$ is

$$\hat{\mu} = \overline{X} \text{ and } \hat{\sigma} = \frac{1}{2}\sum_{i=1}^{n}(X_i - \hat{\mu})^2$$

then the $L(\hat{\theta})$ is obtained by replacing μ with $\hat{\mu}$ and σ^2 with $\hat{\sigma}^2$ which gives

$$L(\theta) = \left(\frac{1}{\sqrt{2\pi\hat{\sigma}}}\right)^n exp\left[-\sum_{i=1}^{n}\frac{(X_i - \hat{\mu})^2}{2\hat{\sigma}^2}\right] = \left(\frac{1}{\sqrt{2\pi\hat{\sigma}}}\right)^n e^{-\frac{n}{2}}$$

Therefore the likelihood ratio is calculated as

$$\wedge = \frac{L(\hat{\theta_o})}{L(\hat{\theta})} = \frac{\left(\frac{1}{\sqrt{2\pi\hat{\sigma}}}\right)^n e^{-\frac{n}{2}}}{\left(\frac{1}{\sqrt{2\pi\hat{\sigma}}}\right)^n e^{-\frac{n}{2}}} = \left(\frac{\hat{\sigma^2}}{\sigma_0^2}\right)^{\frac{n}{2}} = \left[\frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{\sum_{i=1}^{n}(X_i - \mu_o)^2}\right]^{\frac{n}{2}}$$

The rejection region is given by

$$\frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{\sum_{i=1}^{n}(X_i - \mu_o)^2} < k^{\frac{2}{n}}$$

Q4

a

$$f(x;\theta) = \theta^x(1-\theta)^{1-x}\; x = 0, 1$$

$$\frac{L(0.5)}{L(\theta)} \leq k$$

$$\frac{\prod_{i=0}^{n} 0.5^x(1-0.5)^{1-x}}{\prod_{i=0}^{n} \theta^x(1-\theta)^{1-x}} \leq k$$

$$\frac{0.5^x \times 0.5^{n-x}}{\theta^{\sum_{i=1}^{n} x}(1-\theta)^{n-\sum_{i=1}^{n}}} \leq k$$

$$\frac{0.5^n}{\theta^{\sum_{i=1}^{n} x}(1-\theta)^{n-\sum_{i=1}^{n}}} \leq k$$

let $Y = \sum_{i=1}^{n}$

$$\frac{0.5^n}{\theta^y(1-\theta)^{5-y}} \leq k$$

$$\frac{0.5^n(1-\theta)^y}{\theta^y(1-\theta)^5} \leq k$$

Move constant to one side

$$\frac{(1-\theta)^y}{\theta^y} \leq 32k(1-\theta)5$$

$$ln\left[\frac{(1-\theta)^y}{\theta^y}\right] \leq ln\left[32k(1-\theta)5\right]$$

$$yln(1-\theta) + yln\theta \leq ln32 + lnk + 5ln(1-\theta)$$

$$y(ln(1-\theta) + ln\theta) \leq ln32 + lnk + 5ln(1-\theta)$$

$$y \leq \frac{ln32 + lnk + 5ln(1-\theta)}{ln(1-\theta) + ln\theta}$$

Q5

H_0 : proposition are equal to the one provided

$$p_1 = \frac{9}{16} = 0.5625 \text{ , } p_2 = \frac{3}{16} = 0.1875 \text{ , } p_3 = \frac{3}{16} = 0.1875 \text{ , } p_4 = \frac{1}{16} = 0.0625$$

H_a : at least on the p_1 is different

Sample size:

$$= 124 + 30 + 43 + 11 = 208$$

$$\chi^2 = \frac{(O-E)^2}{E}$$

Dist.	**O**	$E = np$	$O - E$	$(O-E)^2$	$\frac{(O-E)^2}{E}$
0.5625	124	117	7	49	0.4188
0.1875	30	39	-9	81	2.0769
0.1875	43	39	4	16	0.4103
0.0625	11	13	-2	4	0.3077
Sum	208				3.2137

$df = 4 - 1 = 3$ and at $\alpha = 0.05$ the chi-square value is 7.81

if $\chi^2_{calc} \leq \chi^2_{critical}$ we fail to reject the null hypothesis

$$3.2137 \leq 7.81 \text{ we fail to reject the null hypothesis}$$

Q6

H_o : Choice of major is independent of the hand posture
H_a : Choice of major is not independent of the hand posture

	LH	RH	Totals
RN	89	29	118
LI	5	4	9
LN	5	8	13
Totals	99	41	140

Computing Expectation using the formula $\frac{x_{\text{row total}} \times x_{\text{column total}}}{\text{sum total}}$

	LH	RH
RN	83.44	34.56
LI	6.36	2.64
LN	9.19	3.81

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$\chi^2 = \frac{(89-83.44)^2}{83.44} + \frac{(5-6.36)^2}{6.36} + \frac{(5-9.19)^2}{9.19} + \frac{(29-34.56)^2}{34.56} + \frac{(4-2.64)^2}{2.64} + \frac{(8-3.81)^2}{3.81}$$

$$\chi^2 = 0.37 + 0.29 + 1.91 + 0.89 + 0.7 + 4.6 = 8.76$$

Calculate the degrees of freedom using the formula $df = (r-1)(c-1) = (3-1)(2-1) = 2$

χ^2_α from the table at 0.05 with $df = 2$ is 5.99

$$\chi^2_{calc} > \chi^2_\alpha \text{ reject } \ H_0$$

8.76 > 5.99 hence reject the null hypothesis