terminology, Kitchenade would like to minimize the number of errors it would make in predicting which consumers would buy the new food mixer and which would not. To assist in identifying potential purchasers, Kitchenade devised rating scales on three characteristics—durability, performance, and style—to be used by consumers in evaluating the new product. Rather than relying on each scale as a separate measure, Kitchenade hopes that a weighted combination of all three characteristics would better predict purchase likelihood of consumers.

The primary objective of discriminant analysis is to develop a weighted combination of the three scales for predicting the likelihood that a consumer will purchase the product. In addition to determining whether consumers who are likely to purchase the new product can be distinguished from those who are not, Kitchenade would also like to know which characteristics of its new product are useful in differentiating likely purchasers from non-purchasers. That is, evaluations on which of the three characteristics of the new product best separate purchasers from non-purchasers?

For example, if the response "would purchase" is always associated with a high durability rating and the response "would not purchase" is always associated with a low durability rating, Kitchenade could conclude that the characteristic of durability distinguishes purchasers from non-purchasers. In contrast, if Kitchenade found that about as many persons with a high rating on style said they would purchase the food mixer as those who said they would not, then style is a characteristic that discriminates poorly between purchasers and non-purchasers.

Identifying Discriminating Variables To identify variables that may be useful in discriminating between groups (i.e., purchasers versus non-purchasers), emphasis is placed on group differences rather than measures of correlation used in multiple regression.

Table 7.1 lists the ratings of the new mixer on these three characteristics (at a specified price) by a panel of 10 potential purchasers. In rating the food mixer, each panel member is implicitly comparing it with products already on the market. After the product was evaluated, the evaluators were asked to state their buying intentions ("would purchase" or "would not purchase"). Five stated that they would purchase the new mixer and five said they would not.

Table 7.1 Kitchenade Survey Results for the Evaluation of a New Consumer Product

	*Evaluation of New Product**		
Groups Based on Purchase Intention	X_1 **Durability**	X_2 **Performance**	X_3 **Style**
Group 1: Would purchase			
Subject 1	8	9	6
Subject 2	6	7	5
Subject 3	10	6	3
Subject 4	9	4	4
Subject 5	4	8	2
Group mean	7.4	6.8	4.0
Group 2: Would not purchase			
Subject 6	5	4	7
Subject 7	3	7	2
Subject 8	4	5	5
Subject 9	2	4	3
Subject 10	2	2	2
Group mean	3.2	4.4	3.8
Difference between group means	4.2	2.4	0.2

*Evaluations are made on a 10-point scale (1 = very poor to 10 = excellent).

Examining Table 7.1 identifies several potential discriminating variables. First, a substantial difference separates the mean ratings of X_1 (durability) for the "would purchase" and "would not purchase" groups (7.4 versus 3.2). As such, durability appears to discriminate well between the two groups and is likely to be an important characteristic to potential purchasers. In contrast, the characteristic of style (X_3) has a much smaller difference of 0.2 between mean ratings $(4.0 - 3.8 = 0.2)$ for the "would purchase" and "would not purchase" groups. Therefore, we would expect this characteristic to be less discriminating in terms of a purchase decision. However, before we can make such statements conclusively, we must examine the distribution of scores for each group. Large standard deviations within one or both groups might make the difference between means nonsignificant and inconsequential in discriminating between the groups.

Because we have only 10 respondents in two groups and three independent variables, we can also look at the data graphically to determine what discriminant analysis is trying to accomplish. Figure 7.2 shows the 10 respondents on each of the three variables. The "would purchase" group is represented by circles and the "would not purchase" group by squares. Respondent identification numbers are inside the shapes:

- X_1 (Durability) had a substantial difference in mean scores, enabling us to almost perfectly discriminate between the groups using only this variable. If we established the value of 5.5 as our cutoff point to discriminate between the two groups, then we would misclassify only respondent 5, one of the "would purchase" group members. This variable illustrates the discriminatory power in having a large difference in the means for the two groups and a lack of overlap between the distributions of the two groups.
- X_2 (Performance) provides a less clear-cut distinction between the two groups. However, this variable does provide high discrimination for respondent 5, who was misclassified if we used only X_1. In addition, the respondents who would be misclassified using X_2 are well separated on X_1. Thus, X_1 and X_2 might be used quite effectively in combination to predict group membership.
- X_3 (Style) shows little differentiation between the groups. Thus, by forming a variate of only X_1 and X_2, and omitting X_3, a discriminant function may be formed that maximizes the separation of the groups on the discriminant score.

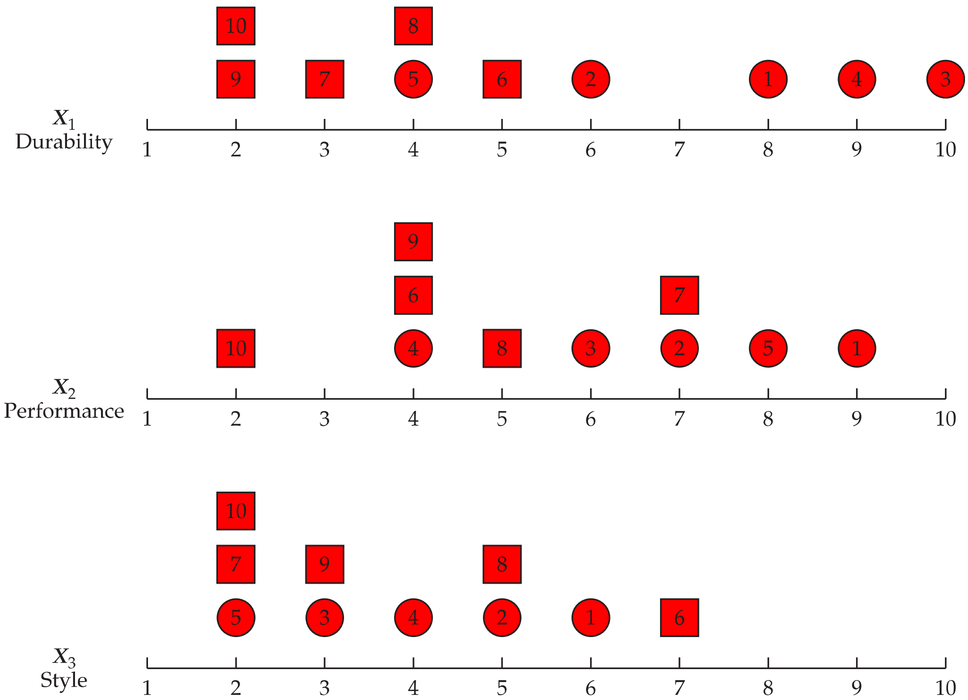

Figure 7.2 Graphical Representation of 10 Potential Purchasers on Three Possible Discriminating Variables

Calculating a Discriminant Function With the three potential discriminating variables identified, attention shifts toward investigation of the possibility of using the discriminating variables in combination to improve upon the discriminating power of any individual variable. To this end, a variate can be formed with two or more discriminating variables to act together in discriminating between the groups.

Table 7.2 contains the results for three different formulations of a discriminant function, each representing different combinations of the three independent variables.

- The first discriminant function contains just X_1, equating the value of X_1 to the discriminant Z score (also implying a weight of 1.0 for X_1 and weights of zero for all other variables). As shown earlier, use of only X_1, the best discriminator, results in the misclassification of subject 5 as shown in Table 7.2, where four out of five subjects in group 1 (all but subject 5) and five of five subjects in group 2 are correctly classified (i.e., lie on the diagonal of the classification matrix). The percentage correctly classified is thus 90 percent (9 out of 10 subjects).
- Because X_2 provides discrimination for subject 5, we can form a second discriminant function by equally combining X_1 and X_2 (i.e., implying weights of 1.0 for X_1 and X_2 and a weight of 0.0 for X_3) to utilize each variable's unique discriminatory powers. Using a cutting score of 11 with this new discriminant function (see Table 7.2) achieves a perfect classification of the two groups (100% correctly classified). Thus, X_1 and X_2 in combination are able to make better predictions of group membership than either variable separately.
- The third discriminant function in Table 7.2 represents the actual estimated discriminant function ($Z = -4.53 + .476X_1 + .359X_2$). Based on a cutting score of 0, this third function also achieves a 100 percent correct classification rate with the maximum separation possible between groups.

Table 7.2 Creating Discriminant Functions to Predict Purchasers Versus Non-purchasers

	Calculated Discriminant Z Scores		
Group	**Function 1: $Z = X_1$**	**Function 2: $Z = X_1 + X_2$**	**Function 3: $Z = -4.53 + .476X_1 + .359X_2$**
Group 1: Would purchase			
Subject 1	8	17	2.51
Subject 2	6	13	.84
Subject 3	10	16	2.38
Subject 4	9	13	1.19
Subject 5	4	12	.25
Group 2: Would not purchase			
Subject 6	5	9	−.71
Subject 7	3	10	−.59
Subject 8	4	9	−.83
Subject 9	2	6	−2.14
Subject 10	2	4	−2.86
Cutting score	5.5	11	0.0

Classification Accuracy:

	Function 1: Predicted Group		**Function 2: Predicted Group**		**Function 3: Predicted Group**	
Actual Group	1	2	1	2	1	2
1: Would purchase	4	1	5	0	5	0
2: Would not purchase	0	5	0	5	0	5

As seen in this simple example, discriminant analysis identifies the variables with the greatest differences between the groups and derives a discriminant coefficient that weights each variable to reflect these differences. The result is a discriminant function that best discriminates between the groups based on a combination of the several independent variables.

A Geometric Representation of the Two-Group Discriminant Function A graphical illustration of another two-group analysis will help to further explain the nature of discriminant analysis [6]. Figure 7.3 demonstrates what happens when a two-group discriminant function is computed. Assume we have two groups, A and B, and two measurements, V_1 and V_2, on each member of the two groups. We can plot in a scatter diagram of the association of variable V_1 with variable V_2 for each member of the two groups. In Figure 7.3 the small dots represent the variable measurements for the members of group B and the large dots those for group A. The ellipses drawn around the large and small dots would enclose some prespecified proportion of the points (discriminant score distributions), usually 95 percent or more in each group. If we draw a straight line through the two points at which the ellipses intersect and then project the line to a new Z axis, we can say that the overlap between the univariate distributions A′ and B′ (represented by the shaded area) is smaller than would be obtained by any other line drawn through the ellipses formed by the discriminant score scatterplots [6].

The important thing to note about Figure 7.3 is that the Z axis expresses the two-variable profiles of groups A and B as single numbers (discriminant scores). By finding a linear combination of the original variables V_1 and V_2,

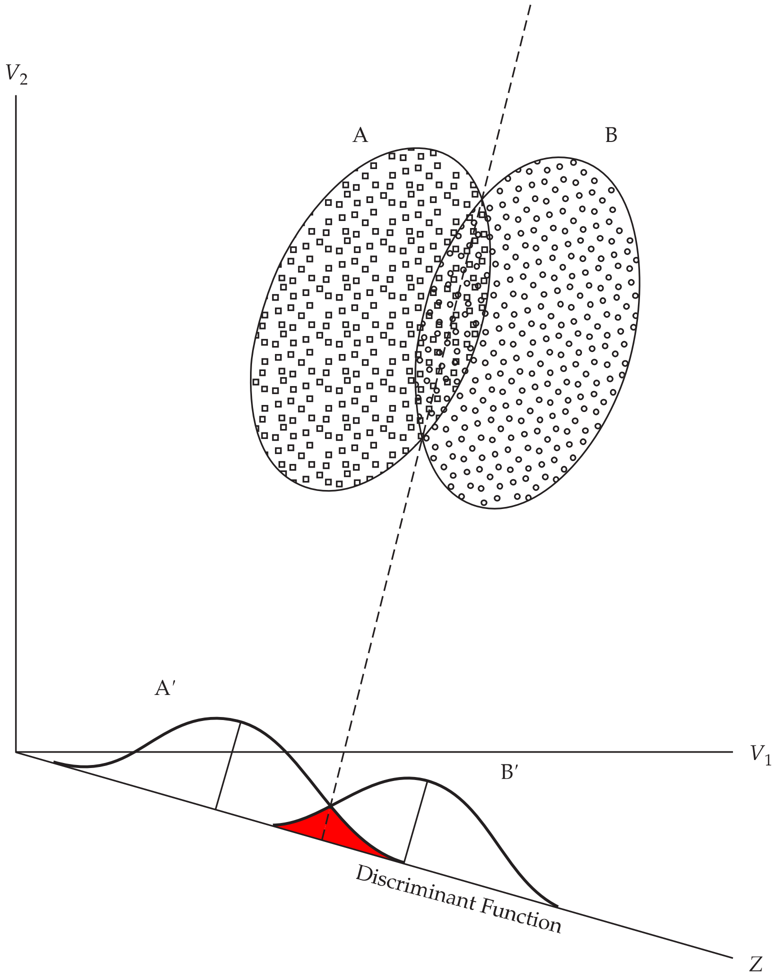

Figure 7.3
Graphical Illustration of Two-Group Discriminant

we can project the results as a discriminant function. For example, if the large and small dots are projected onto the new Z axis as discriminant Z scores, the result condenses the information about group differences (shown in the V_1V_2 plot) into a set of points (Z scores) on a single axis, shown by distributions A′ and B′.

To summarize, for a given discriminant analysis problem, a linear combination of the independent variables is derived, resulting in a series of discriminant scores for each object in each group. The discriminant scores are computed according to the statistical rule of maximizing the variance between the groups and minimizing the variance within them. If the variance between the groups is large relative to the variance within the groups, we say that the discriminant function separates the groups well.

A THREE-GROUP EXAMPLE OF DISCRIMINANT ANALYSIS: SWITCHING INTENTIONS

The two-group example just examined demonstrates the rationale and benefit of combining independent variables into a variate for purposes of discriminating between groups. Discriminant analysis also has another means of discrimination—the estimation and use of multiple variates—in instances of three or more groups. These discriminant functions now become dimensions of discrimination, each dimension separate and distinct from the other. Thus, in addition to improving the explanation of group membership, these additional discriminant functions add insight into the various combinations of independent variables that discriminate between groups.

As an illustration of a three-group application of discriminant analysis, we examine research conducted by HBAT concerning the possibility of a competitor's customers switching suppliers. A small-scale pretest involved interviews of 15 customers of a major competitor. In the course of the interviews, the customers were asked their probability of switching suppliers on a three-category scale. The three possible responses were "definitely switch," "undecided," and "definitely not switch." Customers were assigned to groups 1, 2, or 3, respectively, according to their responses. The customers also rated the competitor on two characteristics: price competitiveness (X_1) and service level (X_2). The research issue is now to determine whether the customers' ratings of the competitor can predict their probability of switching suppliers. Because the dependent variable of switching suppliers was measured as a categorical (nonmetric) variable and the ratings of price and service are metric, discriminant analysis is appropriate.

Identifying Discriminating Variables With three categories of the dependent variable, discriminant analysis can estimate two discriminant functions, each representing a different dimension of discrimination.

Table 7.3 contains the survey results for the 15 customers, five in each category of the dependent variable. As we did in the two-group example, we can look at the mean scores for each group to see whether one of the variables discriminates well among all the groups. For X_1, price competitiveness, we see a rather large mean difference between group 1 and group 2 or 3 (2.0 versus 4.6 or 3.8). X_1 may discriminate well between group 1 and group 2 or 3, but is much less effective in discriminating between groups 2 and 3. For X_2, service level, we see that the difference between groups 1 and 2 is very small (2.0 versus 2.2), whereas a large difference exists between group 3 and group 1 or 2 (6.2 versus 2.0 or 2.2). Thus, X_1 distinguishes group 1 from groups 2 and 3, and X_2 distinguishes group 3 from groups 1 and 2. As a result, we see that X_1 and X_2 provide different *dimensions* of discrimination between the groups.

Calculating Two Discriminant Functions With the potential discriminating variables identified, the next step is to combine them into discriminant functions that will utilize their combined discriminating power for distinguishing between groups.

To illustrate these dimensions graphically, Figure 7.4 portrays the three groups on each of the independent variables separately. Viewing the group members on any one variable, we can see that no variable discriminates well among all the groups. However, if we construct two simple discriminant functions, using just simple weights of 0.0 or 1.0, the results become much clearer. Discriminant function 1 gives X_1 a weight of 1.0, and X_2 a weight of 0.0.