University of British Columbia, Vancouver
GEOG 300 - Microscale Weather and Climate
Knox

Answers to Study Questions - Lecture 18

1. (a) $\overline{T}$ is the temporal average of T:

$$\begin{aligned}\overline{T} &= \frac{1}{6}\sum_{t=0}^{5} T(t) \\ &= \frac{1}{6}(12.6 + 11.2 + 11.9 + 13.1 + 12.0 + 11.8) \\ &= \underline{12.1°\mathrm{C}}\end{aligned}$$

(b) T' is the deviation of a single measured value from the temporal average of the time series ($\overline{T}$). For the time step at 40 min we can write:

$$\begin{aligned}T(40\mathrm{min}) &= \overline{T'} + \overline{T}(40\mathrm{min}) \\ T'(40\mathrm{min}) &= T(40\mathrm{min}) - \overline{T} \\ &= 13.1°\mathrm{C} - 12.1°\mathrm{C} \\ &= \underline{1\,\mathrm{K}}\end{aligned}$$

(c) T'^2 is the squared deviation for a a single measured value from the temporal average of the time series. For the time step at 20 min we can write: ($\overline{T}$):

$$\begin{aligned}T'^2(20\mathrm{min}) &= (T(20\mathrm{min}) - \overline{T})^2 \\ &= (11.2°\mathrm{C} - 12.1°\mathrm{C})^2 \\ &= (-0.9°\mathrm{C})^2 \\ &= \underline{0.81\,\mathrm{K}^2}\end{aligned}$$

(d) $\overline{T'}$ is the average deviation in the time series. By definition, this term must be always zero, i.e $\overline{T'} = 0$ (or more general $\overline{a'} = 0$, where a

is any parameter or term). Let's check this:

$$\begin{aligned}
\overline{T'} &= \frac{1}{6}\sum_{t=0}^{5}(T(t)-\overline{T}) \\
&= \frac{1}{6}((12.6-12.1)+(11.2-12.1)+(11.9-12.1) \\
&\quad +(13.1-12.1)+(12.0-12.1)+(11.8-12.1)) \\
&= \frac{1}{6}(0.5-0.9-0.2+1.0-0.1-0.3) \\
&= \underline{0\,\mathrm{K}}
\end{aligned}$$

(e) $\overline{T'^2}$ is the the variance of the time series of T. It is defined as the average of the squared deviations of all time steps. This term is <u>not</u> necessarily zero[1]

$$\begin{aligned}
\overline{T'^2} &= \frac{1}{6}\sum_{t=0}^{5}(T(t)-\overline{T})^2 \\
&= \frac{1}{6}\Big((12.6-12.1)^2+(11.2-12.1)^2+(11.9-12.1)^2 \\
&\quad +(13.1-12.1)^2+(12.0-12.1)^2+(11.8-12.1)^2\Big) \\
&= \frac{1}{6}(0.25+0.81+0.04+1+0.01+0.09) \\
&= \underline{0.37\,\mathrm{K}^2}
\end{aligned}$$

(f) $\overline{T'}^2$ is the temporal average of the deviations squared. Note the fine (but essential) difference of the overbar that does not include the square, and because $\overline{T'}=0$ (see (d)) also its square is zero:

$$\overline{T'}^2 = \overline{T'}\times\overline{T'}$$

[1] Note for those who have taken statistics: You might be used to the *unbiased variance*, where you divide by $N-1$, not N:

$$\overline{T'^2} = \frac{1}{N-1}\sum_{t=0}^{N-1}(T(t)-\overline{T})^2 = \frac{1}{5}\sum_{t=0}^{5}(T(t)-\overline{T})^2$$

However, in atmospheric turbulence studies, we prefer the biased variance (divided by N, as shown in the question above and the lecture slides where we divide by N). The biased variance is a good estimation of the dispersion of a sample of observations, but not necessarily the best measure of the whole population of possible observations.

$$
\begin{aligned}
&= 0\,\mathrm{K} \times 0\,\mathrm{K} \\
&= \underline{0\,\mathrm{K}^2}
\end{aligned}
$$

2. (a) The average of a constant is equal the constant itself

$$\overline{5} = \frac{1}{1}\sum_{i=0}^{0} 5 = 5$$

(b) The average of a constant times a value is equal the constant times the averaged value:

$$\overline{8v} = 8 \times \overline{v}$$

(c) Similarly we can regard temporal averages as constants (they are not changing over time):

$$\overline{\overline{T}\overline{p}} = \overline{T}\overline{p}$$

(d) The average of an averaged value is equal the averaged value

$$\overline{\overline{u}} = \overline{u}$$

(e) Similar to 1(d):

$$\overline{q'} = 0$$

(f) Again, the first term is zero - so is it's product:

$$\overline{T'} \times \overline{w} = 0 \times \overline{w} = 0$$

(g)

$$\overline{3q'} = 3 \times \overline{q'} = 3 \times 0 = 0$$

(h)

$$\overline{w' \times \overline{u}} = \overline{w'} \times \overline{u} = 0 \times \overline{u} = 0$$

(i) Combining the results from (c) and (e):

$$
\begin{aligned}
\overline{\overline{T}p} &= \overline{T} \times \overline{(\overline{p} + p')} \\
&= \overline{T}\overline{p} + \overline{T} \times \underbrace{\overline{p'}}_{=0} \\
&= \overline{T}\overline{p}
\end{aligned}
$$

(j) In a first step apply Reynold's decomposition to both, w and T. One of the resulting terms is a covariance ($\overline{w'T'}$) which is not zero (see lecture):

$$\begin{aligned}
\overline{wT} &= \overline{(\overline{w}+w')\times(\overline{T}+T')} \\
&= \overline{\overline{w}\,\overline{T}} + \underbrace{\overline{\overline{w}T'}}_{=0} + \underbrace{\overline{w'\overline{T}}}_{=0} + \overline{w'T'} \\
&= \overline{w}\,\overline{T} + \underbrace{\overline{w'T'}}_{\text{Covariance}}
\end{aligned}$$

3. Calculate the following parameters if $\overline{u} = 4\,\mathrm{m\,s^{-1}}$, $\overline{v} = 0\,\mathrm{m\,s^{-1}}$, $\overline{w} = 0\,\mathrm{m\,s^{-1}}$, $\sigma_u = 0.4\,\mathrm{m\,s^{-1}}$, $\sigma_v = 0.2\,\mathrm{m\,s^{-1}}$, and $\sigma_w = 0.1\,\mathrm{m\,s^{-1}}$.

(a) I_u is the turbulence intensity of the longitudinal wind component u. I_u has no unit.

$$\begin{aligned}
I_u &= \sigma_u/\overline{u} \\
&= 0.4\,\mathrm{m\ s^{-1}}/4\,\mathrm{m\ s^{-1}} \\
&= \underline{0.1}
\end{aligned}$$

(b) I_w is the turbulence intensity of the vertical wind component w. I_w has no unit.

$$\begin{aligned}
I_w &= \sigma_w/\overline{u} \\
&= 0.1\,\mathrm{m\ s^{-1}}/4\,\mathrm{m\ s^{-1}} \\
&= \underline{0.025}
\end{aligned}$$

(c) $\overline{w'^2}$ is the variance of the vertical wind component w. The variance is the square of the standard deviation σ_w:

$$\begin{aligned}
\overline{w'^2} &= \sigma_w^2 \\
&= (0.1\,\mathrm{m\ s^{-1}})^2 \\
&= \underline{0.01\,\mathrm{m^2\ s^{-2}}}
\end{aligned}$$

(d)

$$\begin{aligned}
\overline{u'^2} + \overline{v'^2} + \overline{w'^2} &= \sigma_u^2 + \sigma_v^2 + \sigma_w^2 \\
&= (0.4\,\mathrm{m\ s^{-1}})^2 + (0.2\,\mathrm{m\ s^{-1}})^2 + (0.1\,\mathrm{m\ s^{-1}})^2 \\
&= 0.16\,\mathrm{m^2\ s^{-2}} + 0.04\,\mathrm{m^2\ s^{-2}} + 0.01\,\mathrm{m^2\ s^{-2}} \\
&= \underline{0.21\,\mathrm{m^2\ s^{-2}}}
\end{aligned}$$

(e) MKE/m is the mean kinetic energy per unit mass (lecture 18, slide 15), i.e.

$$\begin{aligned}
MKE/m &= \frac{1}{2}(\overline{u}^2 + \overline{v}^2 + \overline{w}^2) \\
&= \frac{1}{2}(4^2 + 0^2 + 0^2) \\
&= \frac{1}{2}(16) \\
&= \underline{8\,\mathrm{m}^2\ \mathrm{s}^{-2}}
\end{aligned}$$

(f) $\overline{e}$ is the turbulent kinetic energy per unit mass (lecture 18, slide 15) i.e.

$$\begin{aligned}
\overline{e} &= \frac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}) \\
&= \frac{1}{2}(\sigma_u^2 + \sigma_v^2 + \sigma_w^2) \\
&= \frac{1}{2}\left((0.4\,\mathrm{m\ s}^{-1})^2 + (0.2\,\mathrm{m\ s}^{-1})^2 + (0.1\,\mathrm{m\ s}^{-1})^2\right) \\
&= \frac{1}{2}(0.16\,\mathrm{m}^2\ \mathrm{s}^{-2} + 0.04\,\mathrm{m}^2\ \mathrm{s}^{-2} + 0.01\,\mathrm{m}^2\ \mathrm{s}^{-2}) \\
&= \underline{0.105\,\mathrm{m}^2\ \mathrm{s}^{-2}}
\end{aligned}$$