University of British Columbia, Vancouver
GEOB 300 - Microscale Weather and Climate
Knox, February 5, 2023

Answers to Study Questions - Lecture 11

1. Assume a uniform and linear warming rate of the soil:

$$\begin{aligned}\frac{\Delta T}{\Delta t} &= \frac{25.3°\text{C} - 24.8°\text{C}}{1\,\text{h}} \\ &= 0.5\text{K}\,\text{h}^{-1} = 1.38 \times 10^{-4}\text{K}\,\text{s}^{-1}\end{aligned}$$

The heat flux at the surface $Q_{G(0)}$ is (Lecture 11, Slide 10):

$$\begin{aligned}Q_{G(0)} &= Q_{G(5\text{cm})} + C\,\frac{\Delta T}{\Delta t}\Delta z \\ &= 25,\text{W}\,\text{m}^{-2} + 2\,\text{MJ}\,\text{m}^{-3}\,\text{K}^{-1} \times 1.38 \times 10^{-4}\text{K}\,\text{s}^{-1} \times 0.05\,\text{m} \\ &= 25\,\text{W}\,\text{m}^{-2} + 13.8\,\text{W}\,\text{m}^{-2} \\ &= \underline{38.8\,\text{W}\,\text{m}^{-2}}\end{aligned}$$

2. Now we have a cooling rate (i.e. negative change of temperature over time):

$$\begin{aligned}\frac{\Delta T}{\Delta t} &= \frac{7.0°\text{C} - 7.5°\text{C}}{1\,\text{h}} \\ &= -0.5\text{K}\,\text{h}^{-1} = -1.38 \times 10^{-4}\text{K}\,\text{s}^{-1}\end{aligned}$$

Similar to the first example, the heat flux at the surface is:

$$\begin{aligned}Q_{G(0)} &= Q_{G(5\text{cm})} + C\,\frac{\Delta T}{\Delta t}\Delta z \\ &= -12\,\text{W}\,\text{m}^{-2} + 2\,\text{MJ}\,\text{m}^{-3}\,\text{K}^{-1} \times \left(-1.38 \times 10^{-4}\text{K}\,\text{s}^{-1}\right) \times 0.05\,\text{m} \\ &= -12\,\text{W}\,\text{m}^{-2} + (-13.88\,\text{W}\,\text{m}^{-2}) \\ &= \underline{-25.9\,\text{W}\,\text{m}^{-2}}\end{aligned}$$

3. Heat sharing refers to how the soil and the atmosphere (or two other materials) share in accepting sensible heat (i.e. Q^* - Q_E) during the

daytime and share in releasing it at night. Heat sharing is determined by the ratio of the thermal admittances of the two materials (Lecture 11, slides 27):

$$\frac{Q_H}{Q_G} = \frac{\mu_a}{\mu_s}$$

where μ_s is the thermal admittance of the soil and μ_a is the thermal admittance of the atmosphere.

4. Rearrange the heat sharing equation from Question 3:

$$Q_H = Q_G \times \frac{\mu_a}{\mu_s}$$

We can calculate μ_s by:

$$\begin{aligned} \mu_s &= \sqrt{k\,C} \\ &= \sqrt{0.27\,\mathrm{W\,m^{-1}\,K^{-1}} \times 2 \times 10^6\,\mathrm{J\,m^{-3}\,K^{-1}}} \\ &= 735\,\mathrm{J\,m^{-2}\,K^{-1}\,s^{-1/2}} \end{aligned}$$

Hence:

$$\begin{aligned} Q_H &= Q_G \times \frac{\mu_a}{\mu_s} \\ &= 38.8\,\mathrm{W\,m^{-2}} \times \frac{5000\,\mathrm{J\,m^{-2}\,K^{-1}\,s^{-1/2}}}{735\,\mathrm{J\,m^{-2}\,K^{-1}\,s^{-1/2}}} \\ &= \underline{264\,\mathrm{W\,m^{-2}}} \end{aligned}$$