

## Answers to Study Questions - Lecture 22

March 13, 2023

1. Read the air density  $\rho_a$  from Oke ‘Boundary Layer Climates’ Table A3.1.

$$\begin{aligned} Q_H &= C_a \overline{w'T'} = \rho_a c_p \overline{w'T'} \\ &= 1.230 \text{ kg m}^{-3} \times 1010 \text{ J kg}^{-1} \text{ K}^{-1} \times -0.031 \text{ m s}^{-1} \text{ K} \\ &= -38.5 \text{ J m}^{-2} \text{ s}^{-1} = \underline{-38.5 \text{ W m}^{-2}} \end{aligned}$$

This is likely a night-time situation, when the surface is cold and that atmosphere is warmer. The profile of potential temperature is likely increasing with height. Excess sensible heat (i.e.  $\rho_a c_p T$ ) is transported from higher levels of the atmosphere down to the layers close to the surface, as indicated by the negative sign of  $Q_H$ .

2. Use the latent heat of vaporization for water,  $L_v$ , from Oke ‘Boundary Layer Climates’ Table A3.1.

$$\begin{aligned} Q_E &= L_v \overline{w'\rho'_v} \\ &= 2.432 \times 10^6 \text{ J kg}^{-1} \times 1.73 \times 10^{-4} \text{ kg m}^{-2} \text{ s}^{-1} \\ &= 420 \text{ J m}^{-2} \text{ s}^{-1} = \underline{420 \text{ W m}^{-2}} \end{aligned}$$

3. The bowen ratio is  $\beta = Q_H/Q_E$ . Analogous to Question 1, calculate  $Q_H$ :

$$\begin{aligned} Q_H &= \rho_a c_p \overline{w'T'} \\ &= 1.188 \text{ kg m}^{-3} \times 1010 \text{ J kg}^{-1} \text{ K}^{-1} \times 0.121 \text{ m s}^{-1} \text{ K} \\ &= 145.1 \text{ W m}^{-2} \end{aligned}$$

Analogous to Question 2, calculate  $Q_E$ :

$$\begin{aligned} Q_E &= L_v \overline{w'\rho'_v} \\ &= 2.453 \times 10^6 \text{ J kg}^{-1} \times 1.21 \times 10^{-4} \text{ kg m}^{-2} \text{ s}^{-1} \\ &= 296.8 \text{ W m}^{-2} \end{aligned}$$

Hence

$$\beta = \frac{Q_H}{Q_E} = \frac{145.1 \text{ W m}^{-2}}{296.8 \text{ W m}^{-2}} = \underline{0.49}$$

4. The relation between specific humidity  $q$  and water vapour density  $\rho_v$  is:  $\rho_v = \rho_a q$ , where  $\rho_a$  is the density of the (moist) air. Because  $\rho_a$  is not changing significantly it can be taken out of the averaging operator:

$$\begin{aligned} Q_E &= L_v \overline{w' \rho'_v} \\ &= L_v \overline{w' (\rho_a q)'} \\ &= \rho_a L_v \overline{w' q'} \end{aligned}$$

rearrange:

$$\begin{aligned} \overline{w' q'} &= \frac{Q_E}{\rho_a L_v} \\ &= \frac{240 \text{ J s}^{-1} \text{ m}^{-2}}{1.188 \text{ kg m}^{-3} \times 2.453 \times 10^6 \text{ J kg}^{-1}} \\ &= 8.24 \times 10^{-5} \text{ m s}^{-1} \text{ kg kg}^{-1} \\ &= \underline{0.0824 \text{ m s}^{-1} \text{ g kg}^{-1}} \end{aligned}$$

5. This covariance is already a mass flux density, because its units are mass per square meter and per second if sorted properly:

$$\begin{aligned} F_{\text{CH}_4} &= \overline{w' \rho'_{\text{CH}_4}} \\ &= 10 \text{ m s}^{-1} \mu\text{g m}^{-3} \\ &= \underline{10 \mu\text{g m}^{-2} \text{ s}^{-1}} \end{aligned}$$