University of British Columbia, Vancouver
GEOB 300 - Microscale Weather and Climate
Knox, January 27, 2020

Answers to Study Questions - Lecture 11

1. Assume a uniform and linear warming rate of the soil:

$$\begin{aligned}\frac{\Delta T}{\Delta t} &= \frac{25.3°\mathrm{C} - 24.8°\mathrm{C}}{1\,\mathrm{h}} \\ &= 0.5\mathrm{K\,h^{-1}} = 1.38 \times 10^{-4}\mathrm{K\,s^{-1}}\end{aligned}$$

   The heat flux at the surface $Q_{G(0)}$ is (Lecture 11, Slide 10):

$$\begin{aligned}Q_{G(0)} &= Q_{G(5\mathrm{cm})} + C\,\frac{\Delta T}{\Delta t}\Delta z \\ &= 25,\mathrm{W\,m^{-2}} + 2\,\mathrm{MJ\,m^{-3}\,K^{-1}} \times 1.38 \times 10^{-4}\mathrm{K\,s^{-1}} \times 0.05\,\mathrm{m} \\ &= 25\,\mathrm{W\,m^{-2}} + 13.8\,\mathrm{W\,m^{-2}} \\ &= \underline{38.8\,\mathrm{W\,m^{-2}}}\end{aligned}$$

2. Now we have a cooling rate (i.e. negative change of temperature over time):

$$\begin{aligned}\frac{\Delta T}{\Delta t} &= \frac{7.0°\mathrm{C} - 7.5°\mathrm{C}}{1\,\mathrm{h}} \\ &= -0.5\mathrm{K\,h^{-1}} = -1.38 \times 10^{-4}\mathrm{K\,s^{-1}}\end{aligned}$$

   Similar to the first example, the heat flux at the surface is:

$$\begin{aligned}Q_{G(0)} &= Q_{G(5\mathrm{cm})} + C\,\frac{\Delta T}{\Delta t}\Delta z \\ &= -12\,\mathrm{W\,m^{-2}} + 2\,\mathrm{MJ\,m^{-3}\,K^{-1}} \times \left(-1.38 \times 10^{-4}\mathrm{K\,s^{-1}}\right) \times 0.05\,\mathrm{m} \\ &= -12\,\mathrm{W\,m^{-2}} + (-13.88\,\mathrm{W\,m^{-2}}) \\ &= \underline{-25.9\,\mathrm{W\,m^{-2}}}\end{aligned}$$

3. Heat sharing refers to how the soil and the atmosphere (or two other materials) share in accepting sensible heat (i.e. Q^* - Q_E) during the

daytime and share in releasing it at night. Heat sharing is determined by the ratio of the thermal admittances of the two materials (Lecture 11, slides 27):

$$\frac{Q_H}{Q_G} = \frac{\mu_a}{\mu_s}$$

where μ_s is the thermal admittance of the soil and μ_a is the thermal admittance of the atmosphere.

4. Rearrange the heat sharing equation from Question 3:

$$Q_H = Q_G \times \frac{\mu_a}{\mu_s}$$

We can calculate μ_s by:

$$\begin{aligned} \mu_s &= \sqrt{k\,C} \\ &= \sqrt{0.27\,\mathrm{W\,m^{-1}\,K^{-1}} \times 2 \times 10^6\,\mathrm{J\,m^{-3}\,K^{-1}}} \\ &= 735\,\mathrm{J\,m^{-2}\,K^{-1}\,s^{-1/2}} \end{aligned}$$

Hence:

$$\begin{aligned} Q_H &= Q_G \times \frac{\mu_a}{\mu_s} \\ &= 38.8\,\mathrm{W\,m^{-2}} \times \frac{5000\,\mathrm{J\,m^{-2}\,K^{-1}\,s^{-1/2}}}{735\,\mathrm{J\,m^{-2}\,K^{-1}\,s^{-1/2}}} \\ &= \underline{264\,\mathrm{W\,m^{-2}}} \end{aligned}$$