*University of British Columbia, Vancouver*
GEOS 300 - Microscale Weather and Climate
Knox, January 16, 2023

# Answers to Study Questions - Lecture 5

1. (a) The highest yearly total $K_{Ex}$ (extraterrestrial irradiance) is found at the Equator ($\theta = 0°$) with $13.2\,\mathrm{GJ\,m^{-2}\,year^{-1}}$. This means that there is an energy gradient from the Equator to the Poles (lowest input with $5.5\,\mathrm{GJ\,m^{-2}\,year^{-1}}$ at the Poles $\theta = 90°$) that creates a global circulation exchanging energy from low to high latitudes (ocean currents, general atmospheric circulation).
   (b) The highest daily total $K_{Ex}$ is found at the poles with 47 $\mathrm{MJ\,m^{-2}}$ $\mathrm{day^{-1}}$ during the summer solstice. This is because the solar irradiance reaches the poles during the full 24h cycle.
   (c) For Vancouver, the highest instantaneous $K_{Ex} \approx 1190\,\mathrm{W\,m^{-2}}$ is found on the day of the summer solstice (Jun 22) at 12 LAT. The lowest value ($0\,\mathrm{W\,m^{-2}}$) during any night.

2. Using the energy conservation equation (reading package, equation 2.12) we know that radiation of wavelength $\lambda$ incident upon a substance must either be *transmitted* through it, be *reflected* from its surface, or be *absorbed*:

   $$\Psi_\lambda + \alpha_\lambda + \zeta_\lambda = 1 \tag{1}$$

   If the body is opaque, then $\Psi_\lambda = 0$ and:

   $$\begin{aligned} \alpha_\lambda + \zeta_\lambda &= 1 \\ \alpha_\lambda &= 1 - \zeta_\lambda \end{aligned} \tag{2}$$

   so the reflectivity is $\alpha_\lambda = 1 - 0.75 = \underline{0.25}$.

3. We use Equation (1) and solve for absorptivity:

   $$\zeta_{\mathrm{PAR}} = 1 - (\Psi_{\mathrm{PAR}} + \alpha_{\mathrm{PAR}}) \tag{3}$$

   This results in $\zeta_{\mathrm{PAR}} = 1 - (0.08 + 0.11) = 0.81$. Hence, the fraction of the incident radiation of $800\,\mu\mathrm{mol\,s^{-1}\,m^{-2}}$ that is absorbed is 81%:

   $$0.81 \times 800\,\mu\mathrm{mol\,s^{-1}\,m^{-2}} = \underline{648\,\mu\mathrm{mol\,s^{-1}\,m^{-2}}}$$

4. We use the bulk transfer formula from Lecture 5 and rearrange for the transmissivity $\Psi_a$:

$$K_{\downarrow} = K_{Ex}\Psi_a^m \tag{4}$$

$$\Psi_a = \left(\frac{K_{\downarrow}}{K_{Ex}}\right)^{\frac{1}{m}} \tag{5}$$

$K_{Ex}$ has been calculated for the same time and location in the Study Question Set 4 (Question 5) and is $K_{Ex} = 562\,\mathrm{W\,m^{-2}}$ (for the calculation see Answers of Study Question Set 4). We further need the optical air mass number $m$ which is:

$$m = \frac{1}{\cos Z} = \frac{1}{\sin\beta} \tag{6}$$

we use $\beta = 22.40°$ and $\sin\beta = 0.381$ from Study Question Set 4, Question 4 and get $m = 2.62$. We insert this in Equation (5):

$$\Psi_a = \left(\frac{298\,\mathrm{W\,m^{-2}}}{534\,\mathrm{W\,m^{-2}}}\right)^{\frac{1}{2.62}} = \underline{0.80} \tag{7}$$

5. We use $\Psi_a = 0.80$ from Question 4 and solve the bulk transfer formula:

$$K_{\downarrow} = K_{Ex}\Psi_a^m \tag{8}$$

$K_{Ex}$ at 10:00 is calculated according to the procedure in Study Question Set 4 and is $474\,\mathrm{W\,m^{-2}}$, and the optical air mass number is $m = 2.96$ ($Z = 70.24°$), hence:

$$K_{\downarrow} = 474\,\mathrm{W\,m^{-2}} \times 0.80^{2.96} = \underline{245\,\mathrm{W\,m^{-2}}} \tag{9}$$