University of British Columbia, Vancouver
GEOG 300 - Microscale Weather and Climate
Knox

Answers to Study Questions - Lecture 21

1. The air temperature gradient $\Delta\overline{T}/\Delta z$ is

$$\frac{\Delta\overline{T}}{\Delta z} = \frac{\overline{T}_2 - \overline{T}_1}{z_2 - z_1} = \frac{10 - 15}{11 - 1} = -\frac{5}{10} = -0.5\,\mathrm{K\,m^{-1}}$$

$\Delta\overline{\theta}/\Delta z$ is the gradient of potential temperature, and potential temperature is defined as (see prerequisite courses GEOG 200 or ATSC 201, and T.R. Oke, 'Boundary Layer Climates' p. 58):

$$\overline{\theta} = \overline{T} + \Gamma_d\,\Delta z_n$$

Γ_d is the dry adiabatic lapse rate which is approximately $1\,\mathrm{K}/100\,\mathrm{m} = 0.01\mathrm{K\,m^{-1}}$ and Δz_n is the height between the measurement level and the normalized (100 kPa) level. Because we are looking at vertical differences (i.e. gradients) only, we can 'bring down' the higher level to the lower one, i.e. Δz_n cancels out and we do not need to care where the 100 kPa level is, i.e.

$$\begin{aligned}
\Delta\overline{\theta} = \overline{\theta_2} - \overline{\theta_1} &= \Big(\overline{T}_2 + \Gamma_d\,(\Delta z + \Delta z_n)\Big) - \Big(\overline{T}_1 + \Gamma_d\,\Delta z_n\Big) \\
&= \Big(\overline{T}_2 + \Gamma_d\,\Delta z\Big) - \overline{T}_1 \\
&= \Big(10.0^\circ\mathrm{C} + 0.01\mathrm{K\,m^{-1}} \cdot 10\,\mathrm{m}\Big) - 15.0^\circ\mathrm{C} \\
&= 10.1 - 15.0 = -4.9\,\mathrm{K}
\end{aligned}$$

and

$$\frac{\Delta\overline{\theta}}{\Delta z} = -\frac{-4.9}{10} = -0.49\,\mathrm{K\,m^{-1}}$$

A negative (potential) temperature gradient means a decreasing (potential) temperature with height, hence unstable conditions, a situation typically found during day.

2. Same approach, $\Delta\overline{T}/\Delta z$ is

$$\frac{\Delta\overline{T}}{\Delta z} = \frac{\overline{T}_2 - \overline{T}_1}{z_2 - z_1} = \frac{11.0 - 10.0}{6 - 1} = \frac{1}{5} = +0.2\,\mathrm{K\,m^{-1}}$$

Increasing temperature with height means inversion (stable stratification). Now, for portantial temperature:

$$\begin{aligned}\Delta\overline{\theta} &= \left(\overline{T}_2 + \Gamma_d\,\Delta z\right) - \overline{T}_1 \\ &= \left(11.0°\mathrm{C} + 0.01\mathrm{K\,m^{-1}} \cdot 5\,\mathrm{m}\right) - 10.0°\mathrm{C} \\ &= 11.05 - 10.0 = +1.05\,\mathrm{K}\end{aligned}$$

and

$$\frac{\Delta\overline{\theta}}{\Delta z} = \frac{1.05}{5} = 0.21\,\mathrm{K\,m^{-1}}$$

We conclude, the difference between $\Delta\overline{T}/\Delta z$ and $\Delta\overline{\theta}/\Delta z$ is small in both cases. The positive (potential) temperature gradient means a increasing (potential) temperature with height, hence stable conditions, a situation typically found during night.

3. The K-Theory says:

$$Q_H = -C_a\,K_H\,\frac{\Delta\overline{\theta}}{\Delta z}$$

Look up ρ_a and c_p on p. 392 and p. 398 of Oke, T. R. 'Boundary Layer Climates' to calculate C_a. At 10°C ρ_a is 1.230 kg m^{-3} (the closest temperature value in the table), hence C_a is

$$C_a = \rho_a\,c_p = 1.230\,\mathrm{kg\,m^{-3}} \cdot 1010\,\mathrm{J\,kg^{-1}\,K^{-1}} = 1242.3, \mathrm{J\,m^{-3}\,K^{-1}}$$

For the gradient in question 1:

$$Q_H = -1242.3, \mathrm{J\,m^{-3}\,K^{-1}} \cdot 0.2\,\mathrm{m^2\,s^{-1}} \cdot (-0.5\,\mathrm{K\,m^{-1}}) = +124\,\mathrm{W\,m^{-2}}$$

A positive sensible heat flux density Q_H transports energy away from the surface.

For the gradient in question 2:

$$Q_H = -1242.3, \mathrm{J\,m^{-3}\,K^{-1}} \cdot 0.2\,\mathrm{m^2\,s^{-1}} \cdot 0.21\,\mathrm{K\,m^{-1}} = -52\,\mathrm{W\,m^{-2}}$$

A negative sensible heat flux density Q_E transports energy towards the surface.

4. The gradient of carbon dioxide concentration $\Delta\overline{\rho_c}/\Delta z$ is

$$\frac{\Delta\overline{\rho_c}}{\Delta z} = \frac{\overline{\rho_{c2}} - \overline{\rho_{c1}}}{z_2 - z_1} = \frac{15-14}{11-1} = \frac{1}{10} = +0.1\,\mathrm{mmol\,m^{-3}\,m^{-1}}$$

   This means carbon dioxide concentration increases with height. There might be plants assimilating carbon dioxide by photosynthesis at the ground. Inserting into the K-Theory, and setting $K_H = K_C$ (K_H from question 3, Reynolds analogy):

$$F_c = K_C\,\frac{\Delta\overline{\rho_c}}{\Delta z} = -0.2\,\mathrm{m^2\,s^{-1}} \cdot 0.1\,\mathrm{mmol\,m^{-3}\,m^{-1}} = -0.02\ \mathrm{mmol\,m^{-2}\,s^{-1}}$$

   Because 1 mol $CO_2 \equiv 44$ g CO_2 (see any periodic table of elements), we can convert this to a mass flux density by:

$$-0.002\,\mathrm{mmol\,m^{-2}\,s^{-1}} \cdot 44\,\mathrm{g\,mol^{-1}} = -88\,\mu\mathrm{g\,m^{-2}\,s^{-1}}$$

   The negative mass flux density means there is a net transport towards the surface.

5. The gradient of absolute humidity $\Delta\overline{\rho_v}/\Delta z$ is

$$\frac{\Delta\overline{\rho_v}}{\Delta z} = \frac{\overline{\rho_{v2}} - \overline{\rho_{v1}}}{z_2 - z_1} = \frac{4-5}{11-1} = -\frac{1}{10} = -0.1\,\mathrm{g\,m^{-3}\,m^{-1}}$$

   The K-Theory says

$$E = -\,K_E\,\frac{\Delta\overline{\rho_v}}{\Delta z} = -0.2\,\mathrm{m^2\,s^{-1}} \cdot (-0.1\,\mathrm{K\,m^{-1}}) = 0.02\,\mathrm{g\,m^{-2}\,s^{-1}}$$

6. The mass flux density of water vapour E and the latent heat flux density Q_E are related by the latent heat of vaporization L_v (p. 392 in Oke, T. R. 'Boundary Layer Climates') by:

$$Q_E = L_v\,E$$

   Inserting values form question 4 (make sure units agree):

$$Q_E = 2.476 \cdot 10^6\,\mathrm{J\,kg^{-1}} \cdot 2 \cdot 10^{-5}\,\mathrm{kg\,m^{-2}\,s^{-1}} = 49.4\,\mathrm{W\,m^{-2}}$$

   A positive latent heat flux density Q_E transports energy away from the surface.