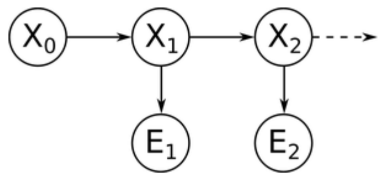
CS486: Artificial Intelligence

Homework 7 (15 pts)
Hidden Markov Models
Due 30 October @ 1630

Consider the HMM shown below.



The prior probability $P(X_0)$, dynamics model $P(X_{t+1}|X_t)$, and sensor model $P(E_t|X_t)$ are as follows:

X_0	$P(X_0)$
0	0.75
1	0.25

X_{t+1}	X_t	$P(X_{t+1}\|X_t)$
0	0	0.65
1	0	0.35
0	1	0.0
1	1	1.0

E_t	X_t	$P(E_t\|X_t)$
a	0	0.35
b	0	0.5
c	0	0.15
a	1	0.85
b	1	0.1
c	1	0.05

We perform a first dynamics update, and fill in the resulting belief distribution $B'(X_1)$.

X_1	$B'(X_1)$
0	0.4875
1	0.5125

We incorporate the evidence $E_1 = a$.We fill in the evidence-weighted distribution $P(E_1 = a|X_1)B'(X_1)$, and the (normalized) belief distribution $B(X_1)$.

X_1	$P(E_1 = a\|X_1)B'(X_1)$
0	0.170625
1	0.435625

X_1	$B(X_1)$
0	0.281443298969
1	0.718556701031

You get to perform the second dynamics update. Fill in the resulting belief distribution $B'(X_2)$.

X_2	$B'(X_2)$
0	0.18293814433
1	0.81706185567

Now incorporate the evidence $E_2 = c$. Fill in the evidence-weighted distribution $P(E_2 = c|X_2)B'(X_2)$, and the (normalized) belief distribution $B(X_2)$.

X_2	$P(E_2 = c\|X_2)B'(X_2)$
0	0.0274407216495
1	0.0408530927835

X_2	$B(X_2)$
0	0.401803909729
1	0.598196090271

$$
\begin{aligned}
B'(x_2 = 0) &= P(X_2|X_1)B(X_1) \\
&= P(x_2 = 0|x_1 = 0)B(x_1 = 0) + P(x_2 = 0|x_1 = 1)B(x_1 = 1) \\
&= 0.65 \cdot 0.281443298969 \\
&= 0.18293814433 \\
B'(x_2 = 1) &= 1 - B'(x_2 = 0) \\
&= 0.81706185567
\end{aligned}
$$

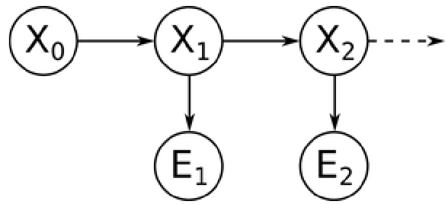
$$
\begin{aligned}
P(E_2 = c|x_2 = 0)B'(x_2 = 0) &= 0.15 \cdot 0.18293814433 \\
&= 0.97255927835
\end{aligned}
$$

$$
\begin{aligned}
P(E_2 = c|x_2 = 1)B'(x_2 = 1) &= 0.05 \cdot 0.81706185567 \\
&= 0.0408530927835
\end{aligned}
$$

$$
\begin{aligned}
B(x_2 = 0) &= \frac{0.0274407216495}{0.0274407216495 + 0.0408530927835} \\
&= 0.401803909729
\end{aligned}
$$

$$
\begin{aligned}
B(x_2 = 1) &= \frac{0.0408530927835}{0.0274407216495 + 0.0408530927835} \\
&= 0.598196090271
\end{aligned}
$$

For the following HMM:



The prior probability $P(X_0)$, dynamics model $P(X_{t+1}|X_t)$, and sensor model $P(E_t|X_t)$ are as follows:

X_0	$P(X_0)$
0	0.0
1	1.0

X_{t+1}	X_t	$P(X_{t+1}\|X_t)$
0	0	0.7
1	0	0.3
0	1	0.35
1	1	0.65

E_t	X_t	$P(E_t\|X_t)$
a	0	0.4
b	0	0.25
c	0	0.35
a	1	0.2
b	1	0.55
c	1	0.25

Assume the sensor is broken and we get no more evidence readings. We are forced to rely on dynamics updates, only, going forward.

What is the stationary distribution for X_∞?

X_∞	$\tilde{B}(X_\infty)$
0	0.53846
1	0.46153

$$P(x_\infty = 0) = P(x_\infty = 0|x_\infty = 0)P(x_\infty = 0) + P(x_\infty = 0|x_\infty = 1)\big(1 - P(x_\infty = 0)\big)$$

$$X = P(x_\infty = 0|x_\infty = 0)X + P(x_\infty = 0|x_\infty = 1)(1 - X)$$

$$X = 0.7X + 0.35(1 - X)$$

$$X = 0.7X + 0.35 - 0.35X$$

$$X - 0.7X + 0.35X = 0.35$$

$$0.65X = 0.35$$

$$X = 0.53846$$

$$\tilde{B}(x_\infty = 0) = 0.53846$$

$$\tilde{B}(x_\infty = 1) = 1 - 0.53846 = 0.46153$$