CS486: Artificial Intelligence

Homework 8 (15 pts)

Bayes Networks

Due 16 November 2017 @ 1630

Instructions

This is an individual assignment; however, *you may receive assistance and/or collaborate without penalty, so long as you properly document such assistance and/or collaboration in accordance with DAW.*

Answer the questions below and submit a hardcopy with DAW coversheet and acknowledgment statement to your instructor by the due date.

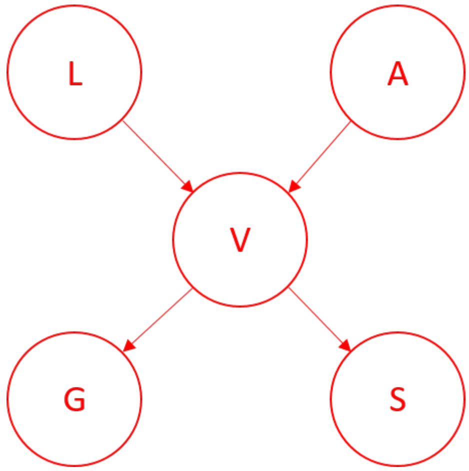
Problem 1: Bayes Network Representation

LTC Sam P. Lingfail thinks cadets are more likely to watch the course videos ($+v$) after he gives a horrible lecture ($-l$) in order to figure out what he was talking about in class. He estimates that the probability of giving a horrible lecture on sampling is 0.95. Unfortunately, the lecture on sampling occurs during Army/Air Force week. The probability of Air Force Week Activities occurring during ESP ($+a$) is 0.9999999, and LTC Lingfail estimates that these activities, if they occur, will have a negative impact on cadets' motivation to watch the videos (V) with the following probabilities based on the interaction of a horrible lecture ($-l$) and Air Force Activities (A):

V	A	$P(V \mid -l, A)$
$+v$	$+a$	0.001
$+v$	$-a$	0.85
$-v$	$+a$	0.009
$-v$	$-a$	0.15

Of course, the probability of good grades ($+g$) if the video is watched ($+v$) is estimated to be 0.99 and a bad grade ($-g$) if the video is not watched ($-v$) is estimated to be 0.6. Finally, if the video is not watched ($-v$) the probability of Scantily-Clad Inline Skating Man$^{\text{TM}}$ making an appearance ($+s$) is only 0.01 because Geoff will be busy, but if he doesn't have to watch the video ($-v$), then the probability of S-CISM appearing goes up to 0.8.

Draw the Bayes network implied by the relationships between random variables ($L, A, V, G,$ and S) described above. Provide *complete* probability tables for each node.



L	$P(L)$
$+l$	0.05
$-l$	0.95

A	$P(A)$
$+a$	0.9999999
$-a$	0.0000001

V	G	$P(G \mid V)$
$+v$	$+g$	0.99
$+v$	$-g$	0.01
$-v$	$+g$	0.4
$-v$	$-g$	0.6

$P(V \mid L, A)$ and $P(S \mid V)$ thrown out due to assignment errors.

Problem 2: D-Separation

For the Bayes net you came up with in Problem 1, answer "yes" or "no" for the following questions:

a. $L \perp\!\!\!\perp A \mid V$? No

b. $G \perp\!\!\!\perp S \mid V$? Yes

c. $A \perp\!\!\!\perp S \mid V$? Yes

d. $A \perp\!\!\!\perp S \mid G$? No

e. $L \perp\!\!\!\perp A \mid G$? No

Problem 3: Variable Elimination

For the Bayes net you developed for Problem 1:

a. What is the size of the full joint probability table for this Bayes net? $2^5 = 32$

b. Given the variable elimination order A, V, G, show the intermediate factors generated using the variable elimination algorithm and state the size of the biggest table generated in the process for the query $P(L \mid +s)$.

Initial factors: $P(L), P(A), P(V \mid A, L), P(G \mid V), P(+s \mid V)$

$$\begin{aligned}
f_1(V, L) &= \sum_a P(a)P(V \mid a, L) \\
f_2(L, G, +s) &= \sum_v P(G \mid v)P(+s \mid v)f_1(v, L) \\
f_3(L, +s) &= \sum_g f_2(L, g, +s) \\
f_3(L, +s) &= P(+s \mid L)
\end{aligned}$$

To find $P(L \mid +s)$ multiply $P(+s \mid L)$ by $P(L)$ to get $P(+s, L)$ and renormalize. Biggest table size: 4.

Problem 4: Sampling

Suppose you are given the following conditional probability tables for alien abduction (A), pregnancy (P), and positive pregnancy test (T):

A	$P(A)$
$+a$	1×10^{-9}
$-a$	0.999999999

A	P	$P(P \mid A)$
$+a$	$+p$	0.4
$+a$	$-p$	0.6
$-a$	$+p$	1×10^{-8}
$-a$	$-p$	0.99999999

P	T	$P(T \mid P)$
$+p$	$+t$	0.9
$+p$	$-t$	0.1
$-p$	$+t$	0.1
$-p$	$-t$	0.9

Answer the following questions (show your work).

a. Based on the CPTs, what is $P(+p, +t)$?

$$\begin{aligned}
P(P) &= P(P \mid A)P(A) \\
P(+p) &= P(+p \mid +a)P(+a) + P(+p \mid -a)P(-a) \\
&= (0.4)(1.0 \times 10^{-9}) + (1.0 \times 10^{-8})(9.99999999 \times 10^{-1}) \\
&\approx (1.04 \times 10^{-8})P(T \mid +p) \\
&\approx 9.36 \times 10^{-9}
\end{aligned}$$

Assume you have generated the following random values in the following order:

0.24113797318029384
0.4784822496122678
0.00000000005
0.8052798971778009
0.9738011727638848
0.9872722255153605
0.0000000001
0.3726587542685793
0.26202639347244017
0.7956605912079744
0.41300512875501694
0.5460497771101764

What probabilities would the following sampling techniques yield for the queries $P(+p, +t)$ and $P(+a \mid +p, +t)$ if you use the random values above to create as many samples as possible for each technique?

For this problem, when converting a random number in the range $[0, 1)$ to an entry in the probability table, follow the technique shown in the example on the Sampling lesson slides where the first row of interest in the probability table gets the appropriate portion of the range starting at 0, then next row of interest gets the next portion of the range, and so on.

b. prior sampling (Hint: You should use all 12 random values above for this answer.)

Sample 1: $-a, -p, +t$ (because $0.2411\ldots > 1 \times 10^{-9} \rightarrow -a$; given $-a$, $0.4784\ldots > 1 \times 10^{-8} \rightarrow -p$; and given $-p$, $5 \times 10^{-11} < 0.1 \rightarrow +t$)
Sample 2: $-a, -p, -t$
Sample 3: $+a, +p, +t$
Sample 4: $-a, -p, -t$
$+p, +t$ appears $1/4$ times: $P(+p, +t) = 0.25$
Given $+p, +t$, $+a$ happens every time: $P(+a \mid +p, +t) = 1$

c. rejection sampling (Hint: You should only use 11 of the random values above for this answer.)

Sample 1: $-a, -p, reject$ ($-a$ and $-p$ are chosen for the same reason as in prior sampling, but we cannot get to $+p, +t$ if we sampled $-p$, so we restart)
Sample 2: $+a, -p, reject$
Sample 3: $-a, -p, reject$
Sample 4: $+a, +p, +t$

Sample 5: $-a, -p, reject$
$+p, +t$ appears 1/5 times: $P(+p, +t) = 0.2$
$P(+a \mid +p, +t) = 1.0$

d. likelihood weighting (Hint: You should use all 12 random values above for this answer.)

In likelihood weighting, you fix $+p$ and $+t$, select unfixed variable A using random values and roulette wheel selection, and then assign a weight to each sample based on the likelihood of the fixed values occurring with the unfixed assignments. Return the sum of the normalized weights for all samples matching the query.
Sample 1: $-a, +p, +t$ ($0.2411 \ldots > 1 \times 10^{-9} \rightarrow -a$; $+p, +t$ are fixed)
Sample 1 weight: $1.0 \times P(+p \mid -a) \times P(+t \mid +p) = 1.0 \times 1 \times 10^{-8} \times 0.9 = 9 \times 10^{-9}$
Sample 2: $-a, +p, +t$, weight: 9×10^{-9}
Sample 3: $+a, +p, +t$, weight: 0.36
Sample 4: $-a, +p, +t$, weight: 9×10^{-9}
Sample 5: $-a, +p, +t$, weight: 9×10^{-9}
Sample 6: $-a, +p, +t$, weight: 9×10^{-9}
Sample 7: $+a, +p, +t$, weight: 0.36
Sample 8: $-a, +p, +t$, weight: 9×10^{-9}
Sample 9: $-a, +p, +t$, weight: 9×10^{-9}
Sample 10: $-a, +p, +t$, weight: 9×10^{-9}
Sample 11: $-a, +p, +t$, weight: 9×10^{-9}
Sample 12: $-a, +p, +t$, weight: 9×10^{-9}
Sum of sample weights: 0.72000009
$P(+p, +t) = 1$
Given $+p, +t$, $+a$ happens 2/12 times with likelihood weight 0.36, so $P(+a \mid +p, +t) = 2 \times 0.36/0.72000009 = 0.999999875$