Random Walk and Value Determination of a Call Option

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Introduction

In this report we will determine the fair price of a call option on ING shares. This will be done using discrete random walks, on which we will do a Monte Carlo simulation to determine the value of the call option more precisely. This report will conclude with a estimation of how much a buyer of the option would play for it.

Methods

Matlab is used for the programming and creation of figures. The functions `randomWalk.m` and `optionPrice.m` are used for this. The parameters used are presented in Tab. 1.

Table 1: Parameters used in the random walks.

Parameter	**Value**
Expiring date	June 17$^{\text{th}}$ 2016
Start time series	February 19$^{\text{th}}$ 2016
Strike price	€12,-
Starting price	€11,-
Drift (μ)	2% per year
Volatility (σ)	20% per year
Timestep (dt)	1 day

It is assumed that there are no transaction costs or dividend. The time series between February 19$^{\text{th}}$ 2016 and June 17$^{\text{th}}$ 2016 contains 82 stock-working days, where the weekends and the national holidays are already excluded. There are in total 252 days in a year that the American stock market is open, so we used that to downscale volatility and drift from yearly to daily. This resulted in daily parameters $\mu_d = 0.02/252$ and $\sigma_d = 0.2/\sqrt{252}$. This processing from yearly to daily is done within the function, though, so when running the function, one should still use the parameters presented in Tab. 1, while using a running time in days.

Random Walk

For the Random Walk, we used Eq. 2.1 from Willmott (1995)[1]:

$$\frac{dS}{S} = \sigma_d dX + \mu_d dt, \tag{1}$$

where S is the price of the asset, dt is the timestep (taken 1 day) and dX is a random value extracted from a normal distribution with mean 0 and variance dt. This means that although the effect of drift per timestep is dependent on dt, the volatility per timestep is too, by the influence of dt on dX.

[1] Paul Wilmott, Howison, Dewynne, The Mathematics of Financial Derivatives, Camebridge University Press, 15th edition 2009.

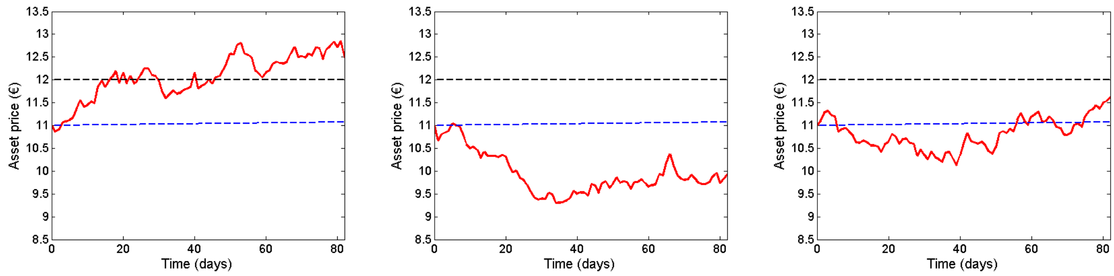
Monte Carlo simulation

If the generated asset price of a random walk would be smaller than €12,- (i.e. the exercise price), the value of the option would be zero, as the owner of the option would not exercise. But if the generated asset price would be larger than €12,- (say €12.30), the value of the option would be that asset price minus €12,- (e.g. €0.30), being a number larger than zero. So to calculate the value of the option, we need a large number of random walks. This leads to the usage of a Monte Carlo simulation. We did 10,000 random walks, and after each random walk, the option value was noted, being either €0,-, or a value greater than zero. The mean of all these values is an approximation of the value of the call option.

Results and Discussion

Random Walk

The results of three individual random walks are plotted in Fig. 1. The run time of each random walk was between 0.05-0.1 s each. In Fig. 1a the random walk ended at €12,4949, which means that in this run buying the call option would lead to a profit (i.e. it is greater than €12,-). In Fig. 1b the stock price is lower than the exercise price, which means that buying the stocks via the call option would be irrational, meaning a option value of €0,-. In Fig. 1c there is a rise of the stock since February and the volatility even drove the stock price higher than solely the drift would (blue line), but the price is still lower than the exercise price.



(a) €12.4949 **(b)** €9.9260 **(c)** €11.6235

Figure 1: Random walks with different resulting asset prices (red), the asset price evolution solely by drift (blue) and a horizontal line along the exercise price of €12,- (black).

Monte Carlo simulation

Results were outputted as a histogram (Fig. 2). The mean value of an option, i.e. the expected profit, remains at around €0,1932, as can be seen from the red line. As explained previously, negative option values will be set to zero as the owner of the call option would not exercise in those cases. This obviously explains the high frequency of €0,- in Fig. 2. For visualization purposes, the y-axis is cut at a frequency of 500. Furthermore, the runtime of the program was 29.55 s.

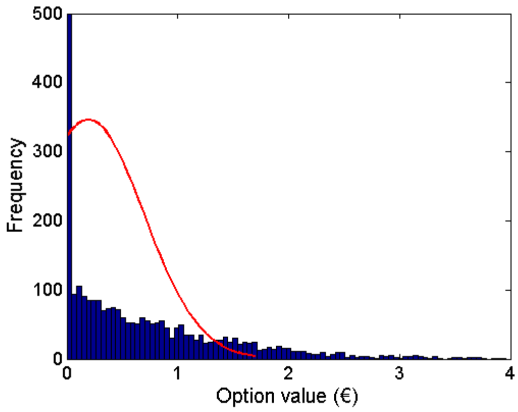


Figure 2: Distribution of option value frequencies (N=10.000). The red line shows a fitted normal distribution.

Conclusion

The call option value calculated by a 10,000-runs Monte Carlo simulation is €0,1932. One could call this amount the value of having such an option; the profit one can gain on average by exercising. However, of course the option is not free. The buyer of the option would likely pay a little less than this amount for the option to gain some profit. Say the buyer wants to have a profit of 1% of his total investment (exercise price + option price), then he would be prepared to pay an option price P of:

$$1\% = \frac{\text{Profit}}{\text{Investment}} \tag{2}$$

$$1\% = \frac{12.1932 - 12.00 - P}{12.00 + P} \tag{3}$$

$$P \approx €0.072 \tag{4}$$