Constructing networks from correlation matrices: An application to economical data

Motivation

- Constructing networks by thresholding correlation matrices is widely accepted, e.g., stock networks and functional brain networks.
- Large correlations may be induced by trivial properties, yielding many spurious edges in the generated networks.
- We propose ***Scola***, an alternative to the thresholding which constructs networks using null models for correlation matrices.

Preprint and code

Sadamori Kojaku and Naoki Masuda. *"Constructing networks by filtering correlation matrices: A null model approach"*. Preprint arXiv: 1903.10805.

Python code:
https://github.com/skojaku/scola

scola 0.0.15

```
pip install scola
```

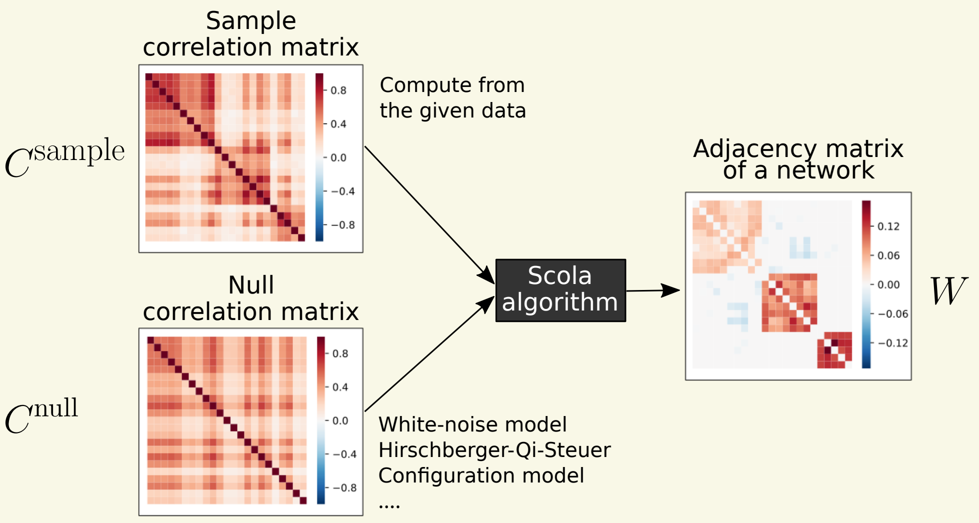
Network construction

Consider N nodes $\mathbf{x} = [x1, x2, \ldots, x_N]$ that follow a multivariate Gaussian distribution, i.e.,

$$\mathbf{x} \sim \mathcal{N}(0, C), \quad C = C^{\text{null}} + W. \tag{1}$$

- C^{null}: Correlation matrix under a null model
- W: Deviation from the null model

Given L observations, $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_L$, our goal is to find W which we regard as the adjacency matrix of a network: we place edges between nodes if the correlations are considerably different from that for the null model.



Find W by maximizing a penalized likelihood

$$\underbrace{\mathcal{L}(W)}_{\text{Likelihood}} - \underbrace{L\sum_{i=1}^{N}\sum_{j=1}^{N}\lambda_{ij}|W_{ij}|}_{\text{L1 penalty}}$$

$$\mathcal{L}(W) \equiv -\frac{L}{2}\ln\det\left(C^{\text{null}}+W\right) - \frac{L}{2}\mathrm{Tr}\left[C^{\text{sample}}\left(C^{\text{null}}+W\right)^{-1}\right] - \frac{NL}{2}\ln(2\pi)$$

L1 penalty encourages W to be sparse, preventing overfitting and yielding a sparse network.

We determine λ values using the adaptive lasso and the extended Bayesian information criterion.

$$(*)\ \lambda_{ij} = \overline{\lambda}|W_{ij}^{\text{MLE}}|^{-\gamma}, \quad \gamma = 2, \quad \overline{\lambda} = \mathrm{argmin}_{\lambda}\ \text{Extended BIC}(\lambda)$$

$$\text{Extended BIC}(\lambda) = -2\mathcal{L}(W(\lambda)) + (M+K)\ln L + 4\beta(M+K)\ln N$$

M: # of edges. $W(\lambda)$: W obtained with $\overline{\lambda} = \lambda$. (*) Adaptive lasso.

References:
[1] N. Masuda, S. Kojaku, and Y. Sano. Configuration model for correlation matrices preserving the node strength. Phys. Rev. E, 98:012312, 2018.
[2] M. Hirschberger, Y. Qi, and R. E. Steuer. Randomly generating portfolio-selection covariance matrices with specified distributional characteristics. Eur. J. Operational Res., 177:1610–1625, 2007.

Results

Construct a network of $N = 988$ products from correlations of $\overline{R}_{cp}^{t}$, *a standardized* (*) country-level yearly export volume

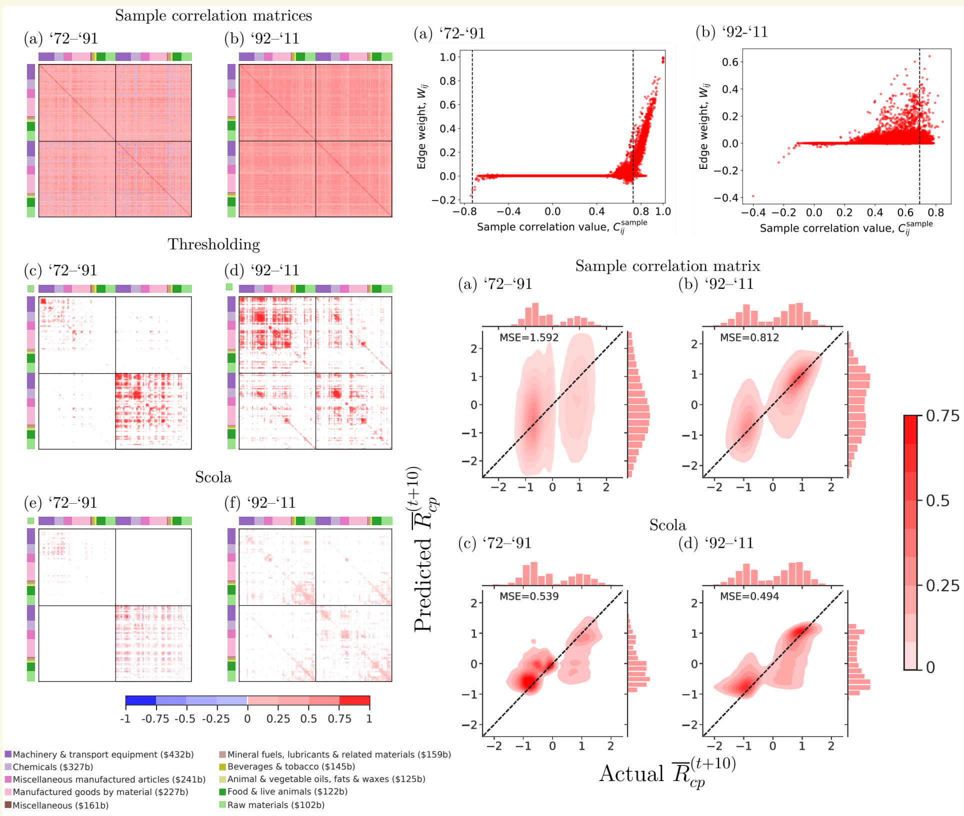
(*)Box-Cox transformation (this transformation makes the distribution closer to a standard Gaussian distribution)

$$C = \begin{bmatrix} C_{0,0} & C_{0,+10} \\ (C_{0,+10})^{\top} & C_{+10,+10} \end{bmatrix}$$

$C_{0,0}$ $C_{+10,+10}$ Correlations between the product within the same year

$C_{0,+10}$ Correlations between the products with a time lag of ten years

C^{null}: Used the configuration model for correlation matrices



Null models for correlation matrices

Extended BIC (smaller is better)

Data	Null model					
	Correlation matrix			Precision matrix		
	C^{WN}	C^{HQS}	C^{con}	C^{WN}	C^{HQS}	C^{con}
Product space						
'72–'91	0.226	0.202	0.187	0.232	0.190	0.208
'92–'11	0.300	0.301	0.226	0.304	0.238	0.256
S&P 500						
'00–'07	0.600	0.562	0.553	0.640	0.557	0.587
'08–'15	0.664	0.558	0.524	0.596	0.539	0.607
Nikkei	0.991	0.874	0.861	1.001	0.837	0.882

C^{WN}: White-noise model (= $\mathbf{I}$) C^{con}: Configuration model [1]
C^{HQS}: Hirschberger-Qi-Steuer model [2]

(*) Sadamori Kojaku and Naoki Masuda
(email) skojaku@rieb.kobe-u.jp (homepage) https://skojaku.github.io/
University of Bristol