Definition: A Skolem sequence of order n is a permutation of the sequence $S = x_1, y_1, x_2, y_2, ..., x_n, y_n$ of $2n$ integers such that

1. $x_i = y_i = i$
2. $y_i = x_i + i$.

Problem Statement: We want to find for which $n \in \mathbb{N}$ there exists a Skolem sequence.

Proof: Let S be a Skolem sequence, consider the sum of its $2n$ integers:

$$\sum S = \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i = 1 + 2 + ... + 2n$$

Since $y_i = x_i + i$. we can rewrite the left side:

$$\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i = \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} (x_i + i) = 2\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} i$$

So now our equation can be written as:

$$2\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} i = 1 + 2 + ... + 2n$$

We can use the fact that the sum of numbers 1, 2, .. n = $\frac{n(n+1)}{2}$.

$$2\sum_{i=1}^{n} x_i + \frac{n(n+1)}{2} = \frac{2n(2n+1)}{2}$$

We can rearrange terms to isolate the sum of the x_is.

$$\sum_{i=1}^{n} x_i = \frac{2n(3n+1) - n(n+1)}{4}$$

Multiplying out the right side of the equation we get:

$$\sum_{i=1}^{n} x_i = \frac{4n^2 + 2n - n^2 - n}{4} = \frac{3n^2 + n}{4}$$

The $\sum_{i=1}^{n} x_i$ will be an integer, therefore so must $\frac{3n^2+n}{4}$. Now, with out new equation, lets consider $1, 2, ...n \bmod 4$.

If $n \bmod 4 = 0$, then n is a multiple of 4 and so is $3n^2 + n$ since we can factor out a 4 from each term.

If $n \bmod 4 = 1$, then $3n^2 + n = 3(1+4i)^2 + 1 + 4i = 12i^2 + 28i + 4$ is also a multiple of 4.

If $n \bmod 4 = 2$, then $3n^2 + n = 3(2+4i)^2 + 2 + 4i = 12i^2 + 52i + 14$ is not divisible by 4.

If $n \bmod 4 = 3$, then $3n^2 + n = 3(3+4i)^2 + 3 + 4i = 12i^2 + 76i + 30$ is not divisible by 4.

So we can conclude that Skolem sequences only exist for $n \bmod 4 = 0, 1$.